Computer Algebra
Independent Integration Tests

Summer 2023 edition

4-Trig-functions/4.3-Tangent/102-4.3.1.3-d-sin-^m-a+b-tan-^n

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [91]. This is test number [102].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (91)	0.00 (0)
Mathematica	98.90 (90)	1.10 (1)
Maple	91.21 (83)	8.79 (8)
Fricas	91.21 (83)	8.79 (8)
Mupad	91.21 (83)	8.79 (8)
Giac	90.11 (82)	9.89 (9)
Maxima	86.81 (79)	13.19 (12)
Sympy	8.79 (8)	91.21 (83)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

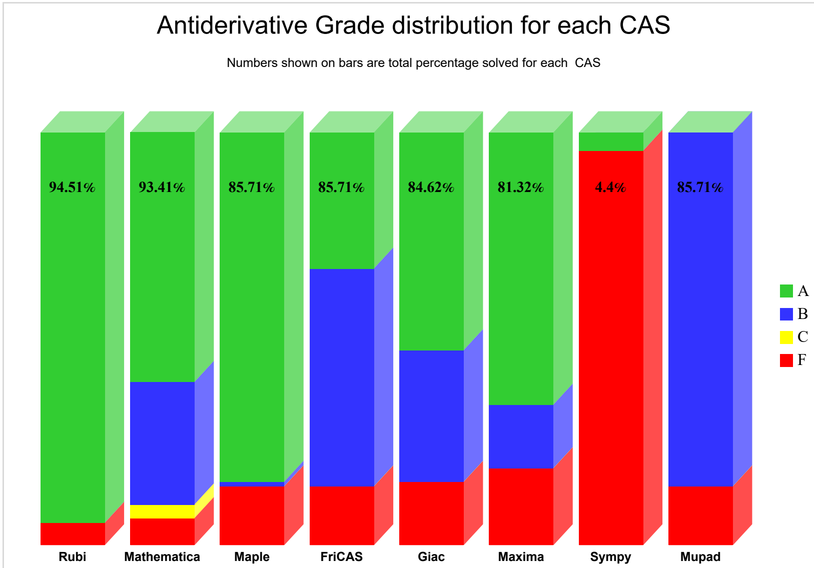
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

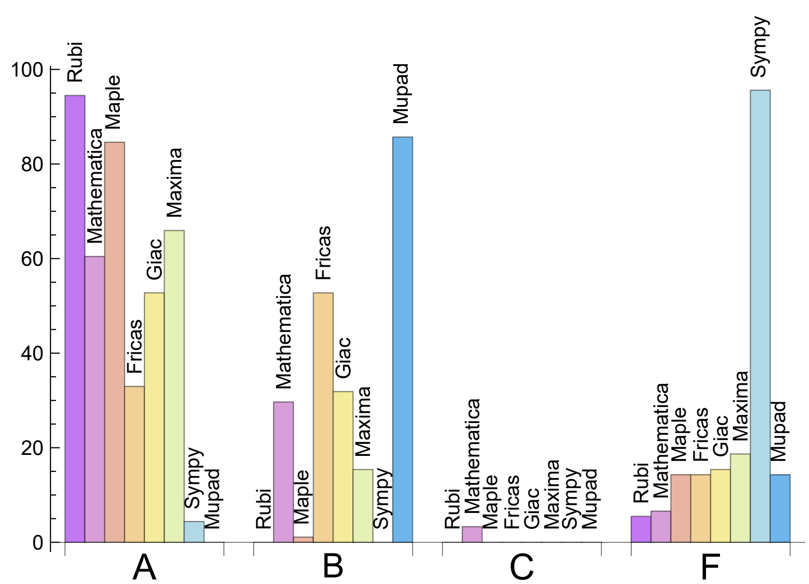
System	% A grade	% B grade	% C grade	% F grade
Rubi	94.505	0.000	0.000	5.495
Maple	84.615	1.099	0.000	14.286
Maxima	65.934	15.385	0.000	18.681
Mathematica	60.440	29.670	3.297	6.593
Giac	52.747	31.868	0.000	15.385
Fricas	32.967	52.747	0.000	14.286
Sympy	4.396	0.000	0.000	95.604
Mupad	0.000	85.714	0.000	14.286

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	1	100.00	0.00	0.00
Fricas	8	100.00	0.00	0.00
Maple	8	100.00	0.00	0.00
Mupad	8	0.00	100.00	0.00
Giac	9	88.89	11.11	0.00
Maxima	12	66.67	0.00	33.33
Sympy	83	84.34	9.64	6.02

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.29
Rubi	0.43
Maxima	0.45
Giac	2.90
Mathematica	3.51
Mupad	5.25
Maple	8.49
Sympy	12.27

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	26.75	0.92	20.00	0.95
Maple	131.22	1.05	106.00	1.00
Rubi	137.51	1.00	109.00	1.00
Maxima	186.62	1.31	120.00	1.10
Mupad	235.73	1.69	146.00	1.25
Mathematica	281.68	1.96	162.00	1.64
Fricas	286.87	2.09	176.00	1.97
Giac	2165.06	15.33	191.00	1.63

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

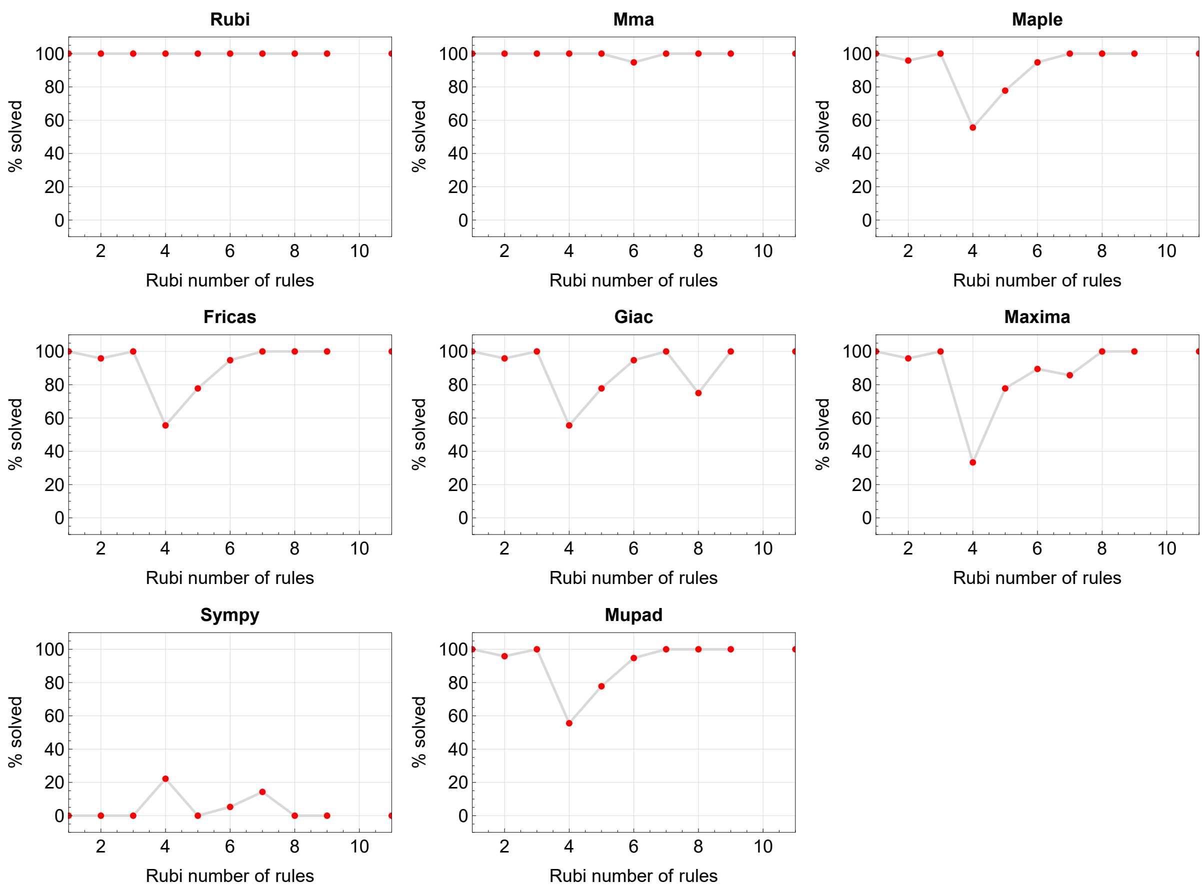


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

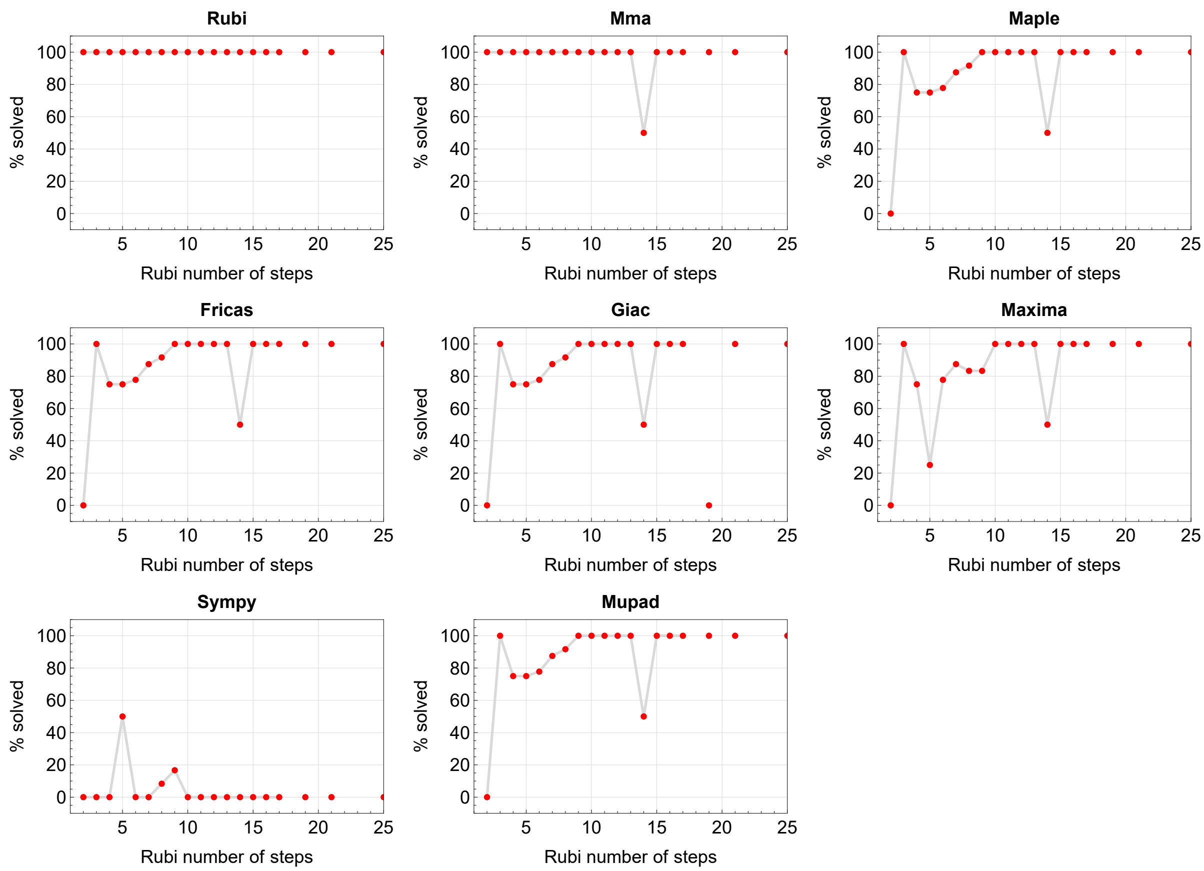


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

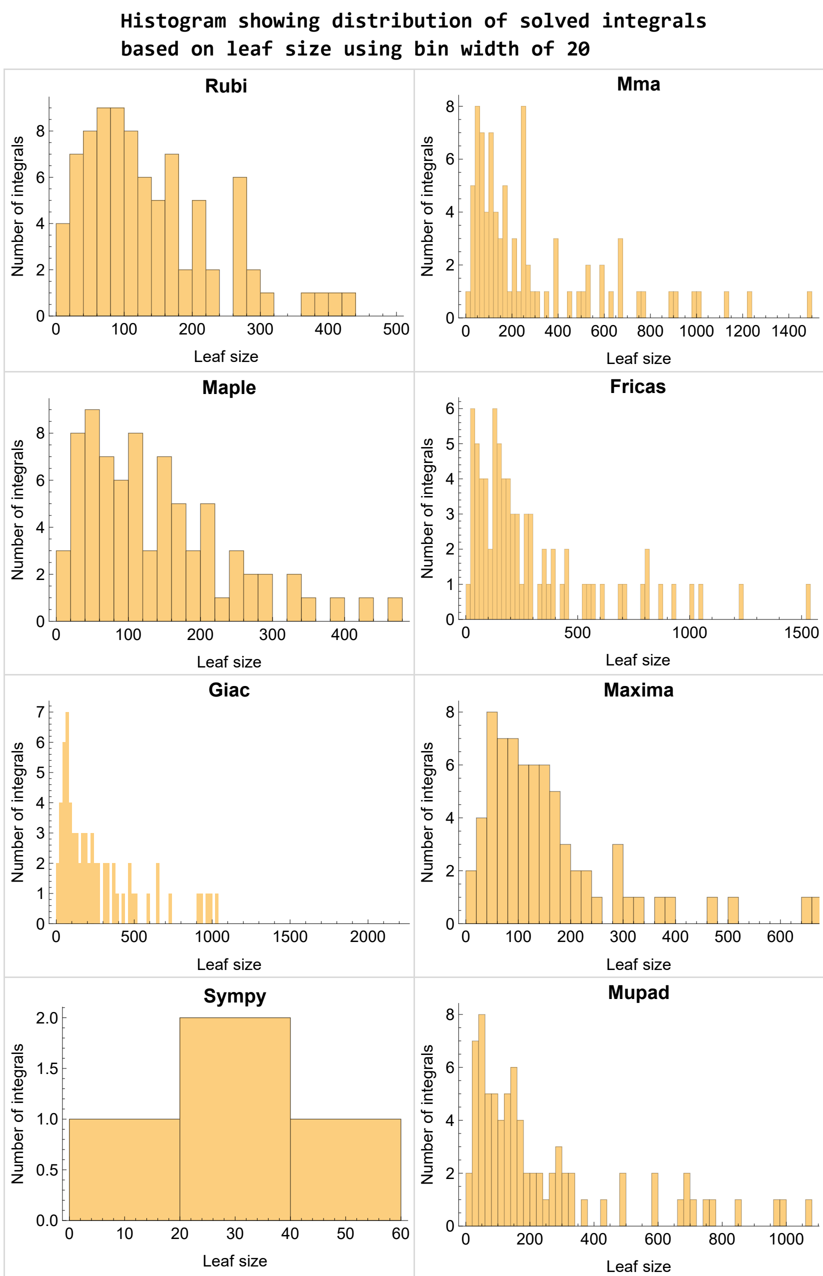


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

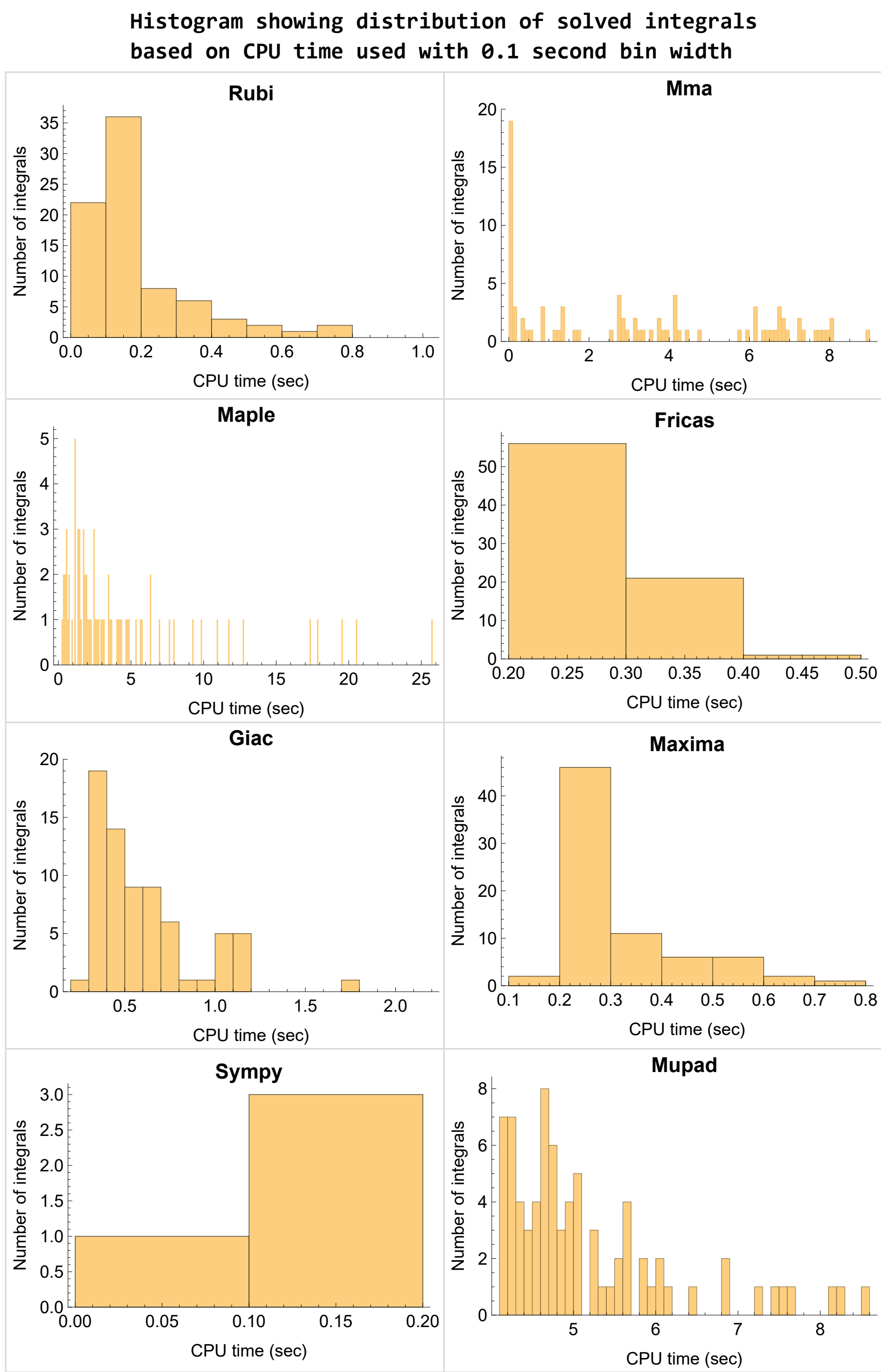


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

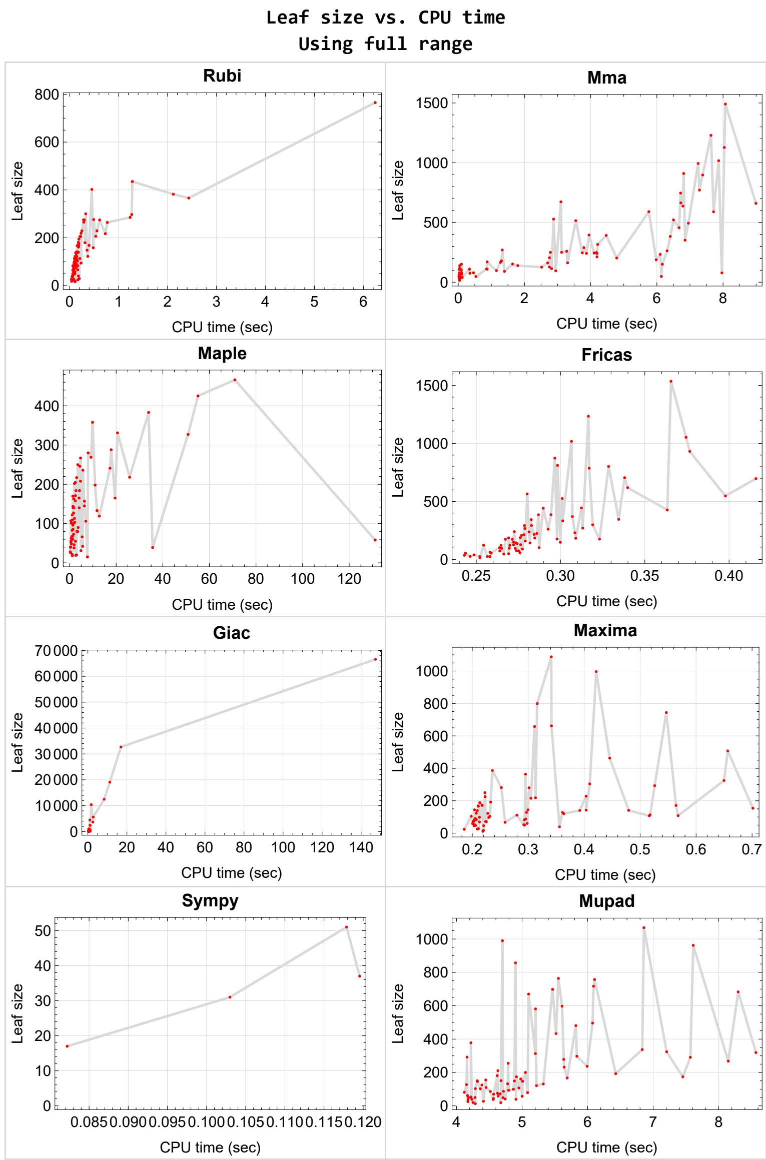


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{83, 88, 89, 90, 91\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {48}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
  x, aa = expr.operator(), expr.operands()
  if x is None:
     return 1
  else:
     return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

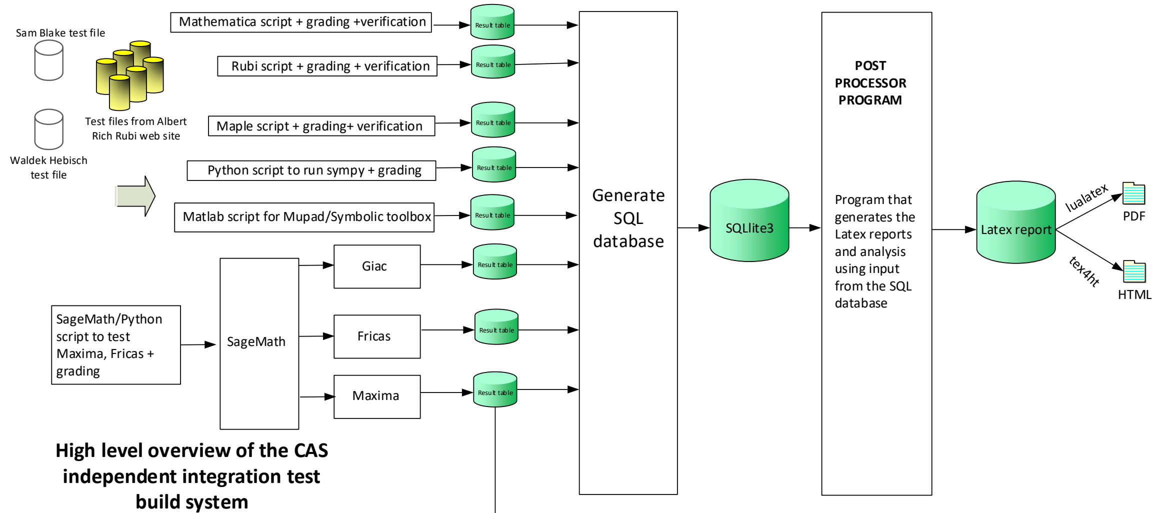
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 8, 10, 11, 12, 13, 14, 15, 16, 17, 19, 21, 23, 25, 29, 31, 33, 36, 38, 42, 45, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 67, 68, 69, 72, 73, 74, 79, 80, 81, 85, 86, 87 }

B grade { 7, 9, 22, 24, 26, 27, 28, 30, 32, 34, 35, 37, 39, 40, 41, 43, 44, 46, 48, 61, 66, 70, 71, 75, 76, 77, 84 }

C grade { 18, 20, 78 }

F normal fail { 82 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78 }

B grade { 7 }

C grade { }

F normal fail { 79, 80, 81, 82, 84, 85, 86, 87 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 6, 11, 12, 13, 14, 15, 22, 23, 24, 25, 32, 33, 34, 35, 41, 42, 43, 44, 45, 50, 51, 52, 53, 54, 57, 59 }

B grade { 5, 7, 8, 9, 10, 16, 17, 18, 19, 20, 21, 26, 27, 28, 29, 30, 31, 36, 37, 38, 39, 40, 46, 47, 48, 49, 55, 56, 58, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78 }

C grade { }

F normal fail { 79, 80, 81, 82, 84, 85, 86, 87 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 6, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 54, 55, 56, 57, 58, 59, 60, 64, 65, 66, 70, 71, 72, 75, 76, 77 }

B grade { 5, 7, 9, 51, 53, 61, 62, 63, 67, 68, 69, 73, 74, 78 }

C grade { }

F normal fail { 79, 80, 81, 82, 84, 85, 86, 87 }

F(-1) timedout fail { }

F(-2) exception fail { 1, 2, 3, 4 }

Giac

A grade { 1, 3, 5, 6, 8, 10, 16, 17, 19, 20, 21, 26, 27, 28, 29, 30, 31, 35, 36, 38, 39, 40, 44, 45, 46, 47, 48, 49, 50, 51, 53, 55, 56, 57, 58, 59, 60, 63, 64, 65, 66, 70, 71, 72, 75, 76, 77, 78 }

B grade { 2, 4, 7, 9, 11, 12, 13, 14, 15, 18, 22, 23, 24, 25, 32, 33, 34, 37, 42, 43, 52, 54, 61, 62, 67, 68, 69, 73, 74 }

C grade { }

F normal fail { 79, 80, 81, 82, 84, 85, 86, 87 }

F(-1) timedout fail { 41 }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78 }

C grade { }

F normal fail { }

F(-1) timedout fail { 79, 80, 81, 82, 84, 85, 86, 87 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4 }

B grade { }

C grade { }

F normal fail { 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 53, 54, 55, 56, 57, 58, 59, 60, 64, 65, 66, 70, 71, 72, 75, 76, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87 }

F(-1) timedout fail { 49, 50, 51, 52, 61, 62, 63, 88 }

F(-2) exception fail { 67, 68, 69, 73, 74 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	67	39	0	39	51	53	49
N.S.	1	1.00	0.86	0.50	0.00	0.50	0.65	0.68	0.63
time (sec)	N/A	0.108	0.105	35.648	0.000	0.249	0.118	0.326	4.707

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	51	31	0	26	37	71	59
N.S.	1	1.00	1.76	1.07	0.00	0.90	1.28	2.45	2.03
time (sec)	N/A	0.198	0.050	4.868	0.000	0.246	0.120	0.328	4.634

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	39	19	0	25	31	41	35
N.S.	1	1.00	0.78	0.38	0.00	0.50	0.62	0.82	0.70
time (sec)	N/A	0.068	0.110	2.675	0.000	0.252	0.103	0.296	4.175

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	33	18	0	14	17	33	39
N.S.	1	1.00	1.74	0.95	0.00	0.74	0.89	1.74	2.05
time (sec)	N/A	0.111	0.030	1.169	0.000	0.252	0.082	0.311	4.906

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	31	21	28	25	0	20	20
N.S.	1	1.00	1.94	1.31	1.75	1.56	0.00	1.25	1.25
time (sec)	N/A	0.111	0.035	1.186	0.211	0.258	0.000	0.307	4.250

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	15	20	17	25	0	18	19
N.S.	1	1.00	0.83	1.11	0.94	1.39	0.00	1.00	1.06
time (sec)	N/A	0.042	0.039	2.969	0.220	0.257	0.000	0.325	4.675

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	75	42	59	73	0	46	41
N.S.	1	1.00	3.12	1.75	2.46	3.04	0.00	1.92	1.71
time (sec)	N/A	0.171	0.042	5.619	0.200	0.264	0.000	0.329	4.742

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	29	15	12	36	0	12	13
N.S.	1	1.00	1.53	0.79	0.63	1.89	0.00	0.63	0.68
time (sec)	N/A	0.050	0.042	7.676	0.219	0.243	0.000	0.311	4.286

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	139	58	83	123	0	62	57
N.S.	1	1.00	3.48	1.45	2.08	3.08	0.00	1.55	1.42
time (sec)	N/A	0.181	0.044	131.157	0.207	0.254	0.000	0.330	5.001

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	41	28	24	54	0	24	27
N.S.	1	1.00	1.11	0.76	0.65	1.46	0.00	0.65	0.73
time (sec)	N/A	0.061	0.041	0.274	0.186	0.244	0.000	0.302	4.410

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	103	80	91	97	0	10412	121
N.S.	1	1.00	1.02	0.79	0.90	0.96	0.00	103.09	1.20
time (sec)	N/A	0.100	0.092	3.127	0.205	0.270	0.000	1.730	5.219

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	82	73	87	74	0	976	155
N.S.	1	1.00	0.99	0.88	1.05	0.89	0.00	11.76	1.87
time (sec)	N/A	0.169	0.095	1.734	0.296	0.275	0.000	0.535	4.446

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	71	60	70	74	0	4486	87
N.S.	1	1.00	1.03	0.87	1.01	1.07	0.00	65.01	1.26
time (sec)	N/A	0.094	0.028	1.351	0.225	0.274	0.000	1.005	4.715

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	56	52	52	47	0	373	50
N.S.	1	1.00	1.14	1.06	1.06	0.96	0.00	7.61	1.02
time (sec)	N/A	0.108	0.052	0.586	0.292	0.266	0.000	0.376	4.281

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	48	40	46	49	0	1020	53
N.S.	1	1.00	1.30	1.08	1.24	1.32	0.00	27.57	1.43
time (sec)	N/A	0.051	0.021	0.385	0.220	0.269	0.000	0.424	4.218

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	52	40	46	58	0	49	86
N.S.	1	1.00	2.00	1.54	1.77	2.23	0.00	1.88	3.31
time (sec)	N/A	0.037	0.021	0.454	0.203	0.276	0.000	0.346	4.515

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	38	26	25	62	0	35	25
N.S.	1	1.00	1.52	1.04	1.00	2.48	0.00	1.40	1.00
time (sec)	N/A	0.089	0.029	0.426	0.209	0.258	0.000	0.356	4.174

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	107	68	83	142	0	118	149
N.S.	1	1.00	1.78	1.13	1.38	2.37	0.00	1.97	2.48
time (sec)	N/A	0.082	0.030	1.352	0.204	0.282	0.000	0.366	4.883

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	78	46	50	122	0	62	49
N.S.	1	1.00	1.37	0.81	0.88	2.14	0.00	1.09	0.86
time (sec)	N/A	0.110	0.032	1.965	0.205	0.265	0.000	0.367	4.180

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	151	90	123	213	0	177	211
N.S.	1	1.00	1.54	0.92	1.26	2.17	0.00	1.81	2.15
time (sec)	N/A	0.112	0.096	3.683	0.207	0.278	0.000	0.392	4.631

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	116	66	72	174	0	84	70
N.S.	1	1.00	1.33	0.76	0.83	2.00	0.00	0.97	0.80
time (sec)	N/A	0.128	0.040	5.391	0.202	0.267	0.000	0.390	4.566

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	250	140	128	137	0	5604	127
N.S.	1	1.00	2.21	1.24	1.13	1.21	0.00	49.59	1.12
time (sec)	N/A	0.210	3.122	1.872	0.297	0.274	0.000	2.742	4.151

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	152	113	104	126	0	32694	174
N.S.	1	1.00	1.25	0.93	0.85	1.03	0.00	267.98	1.43
time (sec)	N/A	0.147	1.639	2.102	0.198	0.272	0.000	16.946	7.452

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	162	109	82	101	0	981	75
N.S.	1	1.00	2.13	1.43	1.08	1.33	0.00	12.91	0.99
time (sec)	N/A	0.133	2.701	1.171	0.293	0.264	0.000	0.619	4.624

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	111	83	67	90	0	2405	93
N.S.	1	1.00	1.63	1.22	0.99	1.32	0.00	35.37	1.37
time (sec)	N/A	0.084	0.855	1.121	0.259	0.277	0.000	1.076	4.795

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	97	56	60	102	0	74	125
N.S.	1	1.00	2.26	1.30	1.40	2.37	0.00	1.72	2.91
time (sec)	N/A	0.067	1.152	0.592	0.298	0.287	0.000	0.447	4.387

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	91	38	39	96	0	51	44
N.S.	1	1.00	2.17	0.90	0.93	2.29	0.00	1.21	1.05
time (sec)	N/A	0.064	1.396	1.908	0.356	0.265	0.000	0.458	4.562

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	250	101	122	230	0	172	292
N.S.	1	1.00	2.63	1.06	1.28	2.42	0.00	1.81	3.07
time (sec)	N/A	0.122	2.783	1.808	0.228	0.309	0.000	0.508	4.160

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	127	80	69	174	0	91	72
N.S.	1	1.00	1.61	1.01	0.87	2.20	0.00	1.15	0.91
time (sec)	N/A	0.083	2.754	3.493	0.213	0.271	0.000	0.491	4.665

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	994	145	187	333	0	269	378
N.S.	1	1.00	6.02	0.88	1.13	2.02	0.00	1.63	2.29
time (sec)	N/A	0.183	7.247	6.330	0.214	0.301	0.000	0.505	4.219

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	114	119	104	240	0	131	107
N.S.	1	1.00	0.93	0.98	0.85	1.97	0.00	1.07	0.88
time (sec)	N/A	0.115	2.820	12.763	0.232	0.273	0.000	0.502	4.951

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	771	184	173	188	0	66584	291
N.S.	1	1.00	3.76	0.90	0.84	0.92	0.00	324.80	1.42
time (sec)	N/A	0.227	7.288	2.718	0.217	0.276	0.000	147.410	7.567

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	203	163	113	149	0	2370	151
N.S.	1	1.00	1.97	1.58	1.10	1.45	0.00	23.01	1.47
time (sec)	N/A	0.161	4.787	1.759	0.518	0.272	0.000	1.178	4.679

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	637	144	128	144	0	12476	193
N.S.	1	1.00	4.79	1.08	0.96	1.08	0.00	93.80	1.45
time (sec)	N/A	0.136	6.788	1.467	0.361	0.273	0.000	8.347	6.429

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	241	107	111	148	0	144	278
N.S.	1	1.00	2.80	1.24	1.29	1.72	0.00	1.67	3.23
time (sec)	N/A	0.099	3.873	0.551	0.280	0.300	0.000	0.679	5.636

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	126	55	56	127	0	70	62
N.S.	1	1.00	1.97	0.86	0.88	1.98	0.00	1.09	0.97
time (sec)	N/A	0.066	2.520	2.518	0.202	0.276	0.000	0.708	4.165

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	897	140	171	299	0	304	581
N.S.	1	1.00	6.36	0.99	1.21	2.12	0.00	2.16	4.12
time (sec)	N/A	0.160	7.383	4.121	0.218	0.319	0.000	0.747	5.204

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	212	106	98	237	0	133	103
N.S.	1	1.00	1.88	0.94	0.87	2.10	0.00	1.18	0.91
time (sec)	N/A	0.099	4.193	6.966	0.213	0.281	0.000	0.695	4.287

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	1229	198	250	427	0	373	698
N.S.	1	1.00	5.37	0.86	1.09	1.86	0.00	1.63	3.05
time (sec)	N/A	0.252	7.635	10.945	0.223	0.363	0.000	0.758	5.465

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	515	165	142	343	0	189	146
N.S.	1	1.00	3.08	0.99	0.85	2.05	0.00	1.13	0.87
time (sec)	N/A	0.156	3.550	19.516	0.403	0.283	0.000	0.767	4.318

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	1017	267	218	224	0	0	319
N.S.	1	1.00	3.70	0.97	0.79	0.81	0.00	0.00	1.16
time (sec)	N/A	0.286	7.867	4.709	0.313	0.286	0.000	0.000	8.568

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	263	250	154	186	0	3651	161
N.S.	1	1.00	1.89	1.80	1.11	1.34	0.00	26.27	1.16
time (sec)	N/A	0.179	6.311	3.530	0.701	0.269	0.000	2.567	4.979

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	383	217	166	176	0	19074	268
N.S.	1	1.00	2.13	1.21	0.92	0.98	0.00	105.97	1.49
time (sec)	N/A	0.178	6.406	3.035	0.211	0.298	0.000	11.223	8.146

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	352	170	139	175	0	193	496
N.S.	1	1.00	2.98	1.44	1.18	1.48	0.00	1.64	4.20
time (sec)	N/A	0.141	6.852	1.374	0.209	0.323	0.000	1.035	6.075

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	162	79	72	159	0	86	81
N.S.	1	1.00	1.95	0.95	0.87	1.92	0.00	1.04	0.98
time (sec)	N/A	0.067	3.302	3.400	0.224	0.279	0.000	1.119	4.119

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	1128	157	188	346	0	300	670
N.S.	1	1.00	7.01	0.98	1.17	2.15	0.00	1.86	4.16
time (sec)	N/A	0.183	8.039	6.386	0.213	0.335	0.000	1.065	5.099

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	188	133	120	267	0	161	132
N.S.	1	1.00	1.37	0.97	0.88	1.95	0.00	1.18	0.96
time (sec)	N/A	0.124	5.981	11.767	0.364	0.279	0.000	1.159	4.780

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	1491	241	304	547	0	479	857
N.S.	1	1.00	5.44	0.88	1.11	2.00	0.00	1.75	3.13
time (sec)	N/A	0.302	8.071	17.346	0.410	0.398	0.000	1.093	4.898

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	233	218	171	386	0	235	181
N.S.	1	1.00	1.20	1.12	0.88	1.99	0.00	1.21	0.93
time (sec)	N/A	0.190	6.102	25.775	0.564	0.294	0.000	1.175	4.620

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	402	402	660	327	387	697	0	647	990
N.S.	1	1.00	1.64	0.81	0.96	1.73	0.00	1.61	2.46
time (sec)	N/A	0.455	8.994	50.856	0.236	0.416	0.000	1.153	4.700

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	289	358	658	370	0	464	683
N.S.	1	1.00	1.05	1.31	2.40	1.35	0.00	1.69	2.49
time (sec)	N/A	0.614	3.796	9.862	0.311	0.307	0.000	0.452	8.297

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	249	208	280	216	0	334	313
N.S.	1	1.00	1.58	1.32	1.77	1.37	0.00	2.11	1.98
time (sec)	N/A	0.483	4.171	4.691	0.301	0.284	0.000	0.404	5.202

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	139	202	364	261	0	241	324
N.S.	1	1.00	0.83	1.20	2.17	1.55	0.00	1.43	1.93
time (sec)	N/A	0.397	1.798	2.096	0.295	0.293	0.000	0.465	7.206

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	170	122	144	122	0	184	147
N.S.	1	1.00	1.81	1.30	1.53	1.30	0.00	1.96	1.56
time (sec)	N/A	0.235	0.873	1.153	0.300	0.270	0.000	0.389	5.008

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	79	101	141	185	0	118	110
N.S.	1	1.00	0.88	1.12	1.57	2.06	0.00	1.31	1.22
time (sec)	N/A	0.139	0.460	0.698	0.479	0.285	0.000	0.430	4.450

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	75	63	107	183	0	94	174
N.S.	1	1.00	1.14	0.95	1.62	2.77	0.00	1.42	2.64
time (sec)	N/A	0.165	0.352	0.750	0.516	0.309	0.000	0.432	4.913

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	47	48	47	95	0	60	39
N.S.	1	1.00	0.94	0.96	0.94	1.90	0.00	1.20	0.78
time (sec)	N/A	0.074	0.534	0.787	0.206	0.273	0.000	0.422	4.231

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	179	140	215	270	0	209	764
N.S.	1	1.00	1.47	1.15	1.76	2.21	0.00	1.71	6.26
time (sec)	N/A	0.372	1.306	1.507	0.305	0.313	0.000	0.497	5.554

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	95	96	97	208	0	144	102
N.S.	1	1.00	0.88	0.89	0.90	1.93	0.00	1.33	0.94
time (sec)	N/A	0.120	2.944	1.425	0.229	0.277	0.000	0.399	4.362

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	150	165	168	385	0	251	167
N.S.	1	1.00	0.89	0.98	0.99	2.28	0.00	1.49	0.99
time (sec)	N/A	0.188	6.166	4.075	0.210	0.287	0.000	0.437	5.689

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	664	383	799	619	0	735	757
N.S.	1	1.00	2.24	1.29	2.69	2.08	0.00	2.47	2.55
time (sec)	N/A	1.269	6.724	33.908	0.316	0.340	0.000	0.545	6.106

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	392	269	507	444	0	513	481
N.S.	1	1.00	1.81	1.24	2.34	2.05	0.00	2.36	2.22
time (sec)	N/A	0.731	4.469	9.209	0.657	0.312	0.000	0.534	5.821

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	246	171	293	292	0	263	255
N.S.	1	1.00	1.66	1.16	1.98	1.97	0.00	1.78	1.72
time (sec)	N/A	0.357	3.747	2.489	0.526	0.279	0.000	0.510	4.788

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	109	67	74	293	0	74	79
N.S.	1	1.00	1.51	0.93	1.03	4.07	0.00	1.03	1.10
time (sec)	N/A	0.081	0.873	0.964	0.203	0.283	0.000	0.454	5.084

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	244	127	144	442	0	203	150
N.S.	1	1.00	1.74	0.91	1.03	3.16	0.00	1.45	1.07
time (sec)	N/A	0.144	4.103	1.725	0.204	0.290	0.000	0.505	4.314

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	589	205	225	787	0	332	237
N.S.	1	1.00	2.69	0.94	1.03	3.59	0.00	1.52	1.08
time (sec)	N/A	0.235	7.713	2.484	0.223	0.317	0.000	0.494	5.993

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	382	382	746	466	1088	932	0	923	1068
N.S.	1	1.00	1.95	1.22	2.85	2.44	0.00	2.42	2.80
time (sec)	N/A	2.118	6.717	70.944	0.341	0.377	0.000	0.686	6.858

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	521	331	744	705	0	588	717
N.S.	1	1.00	1.83	1.16	2.61	2.47	0.00	2.06	2.52
time (sec)	N/A	1.237	6.503	20.582	0.547	0.338	0.000	0.682	6.090

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	316	236	463	526	0	482	433
N.S.	1	1.00	1.53	1.15	2.25	2.55	0.00	2.34	2.10
time (sec)	N/A	0.535	4.212	5.726	0.446	0.301	0.000	0.636	5.518

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	241	85	108	565	0	113	99
N.S.	1	1.00	2.54	0.89	1.14	5.95	0.00	1.19	1.04
time (sec)	N/A	0.090	4.194	1.467	0.568	0.280	0.000	0.568	4.867

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	456	158	192	811	0	237	200
N.S.	1	1.00	2.56	0.89	1.08	4.56	0.00	1.33	1.12
time (sec)	N/A	0.188	6.671	2.467	0.233	0.298	0.000	0.600	5.051

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	494	246	281	1018	0	382	297
N.S.	1	1.00	1.86	0.93	1.06	3.84	0.00	1.44	1.12
time (sec)	N/A	0.290	6.951	4.388	0.252	0.306	0.000	0.687	5.836

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	366	366	590	425	997	1053	0	902	962
N.S.	1	1.00	1.61	1.16	2.72	2.88	0.00	2.46	2.63
time (sec)	N/A	2.434	5.757	55.089	0.421	0.374	0.000	0.908	7.612

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	395	288	662	802	0	642	597
N.S.	1	1.00	1.50	1.09	2.51	3.04	0.00	2.43	2.26
time (sec)	N/A	0.771	3.952	17.837	0.342	0.329	0.000	0.828	5.610

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	259	103	140	874	0	129	131
N.S.	1	1.00	2.23	0.89	1.21	7.53	0.00	1.11	1.13
time (sec)	N/A	0.109	3.276	2.236	0.392	0.297	0.000	0.674	5.324

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	528	184	228	1235	0	222	232
N.S.	1	1.00	2.58	0.90	1.11	6.02	0.00	1.08	1.13
time (sec)	N/A	0.211	2.879	4.263	0.403	0.317	0.000	0.702	5.642

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	673	280	325	1536	0	428	337
N.S.	1	1.00	2.24	0.93	1.08	5.12	0.00	1.43	1.12
time (sec)	N/A	0.330	3.101	7.964	0.650	0.366	0.000	0.761	6.834

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	41	26	50	55	0	44	38
N.S.	1	1.00	1.58	1.00	1.92	2.12	0.00	1.69	1.46
time (sec)	N/A	0.085	0.120	0.373	0.293	0.259	0.000	0.364	4.558

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	229	229	205	0	0	0	0	0	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.559	2.756	0.000	0.000	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	179	179	166	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.315	1.274	0.000	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	109	109	109	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.149	0.337	0.000	0.000	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	765	765	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	6.241	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	23	23	20	23	23
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.95	1.10	1.10
time (sec)	N/A	3.152	6.546	1.297	2.514	0.298	54.374	0.717	8.005

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	435	435	910	0	0	0	0	0	0
N.S.	1	1.00	2.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.282	6.810	0.000	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	276	276	270	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.492	1.329	0.000	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.077	6.136	0.000	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	140	140	78	0	0	0	0	0	0
N.S.	1	1.00	0.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.138	7.965	0.000	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	23	32	0	23	23
N.S.	1	1.00	1.10	1.00	1.10	1.52	0.00	1.10	1.10
time (sec)	N/A	2.322	4.963	3.144	4.326	0.243	0.000	1.344	7.380

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	19	21	21	19	21	21
N.S.	1	1.00	1.11	1.00	1.11	1.11	1.00	1.11	1.11
time (sec)	N/A	1.003	2.767	0.568	1.577	0.236	4.717	1.209	5.479

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	19	21	21	19	21	23
N.S.	1	1.00	1.11	1.00	1.11	1.11	1.00	1.11	1.21
time (sec)	N/A	0.562	4.425	1.531	1.487	0.238	3.695	0.857	4.562

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	23	23	20	23	23
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.95	1.10	1.10
time (sec)	N/A	1.982	23.126	1.567	2.959	0.246	34.947	1.238	6.322

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [5] had the largest ratio of [.6363999999999999966]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	4	1.00	13	0.308
2	A	9	7	1.00	13	0.538
3	A	5	4	1.00	13	0.308
4	A	8	6	1.00	11	0.546
5	A	8	7	1.00	11	0.636
6	A	3	2	1.00	13	0.154
7	A	8	7	1.00	13	0.538
8	A	4	3	1.00	13	0.231
9	A	9	8	1.00	13	0.615
10	A	4	3	1.00	13	0.231
11	A	8	5	1.00	19	0.263
12	A	6	4	1.00	19	0.210
13	A	8	5	1.00	19	0.263
14	A	5	4	1.00	19	0.210
15	A	6	5	1.00	17	0.294
16	A	4	2	1.00	17	0.118
17	A	3	1	1.00	19	0.053
18	A	7	6	1.00	19	0.316
19	A	3	1	1.00	19	0.053
20	A	9	6	1.00	19	0.316
21	A	3	1	1.00	19	0.053
22	A	8	6	1.00	21	0.286
23	A	11	7	1.00	21	0.333
24	A	6	6	1.00	21	0.286

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	9	7	1.00	19	0.368
26	A	6	4	1.00	19	0.210
27	A	3	2	1.00	21	0.095
28	A	10	7	1.00	21	0.333
29	A	3	2	1.00	21	0.095
30	A	13	9	1.00	21	0.429
31	A	3	2	1.00	21	0.095
32	A	16	8	1.00	21	0.381
33	A	7	6	1.00	21	0.286
34	A	13	8	1.00	19	0.421
35	A	8	5	1.00	19	0.263
36	A	3	2	1.00	21	0.095
37	A	12	7	1.00	21	0.333
38	A	3	2	1.00	21	0.095
39	A	17	9	1.00	21	0.429
40	A	3	2	1.00	21	0.095
41	A	19	8	1.00	21	0.381
42	A	7	6	1.00	21	0.286
43	A	16	9	1.00	19	0.474
44	A	10	5	1.00	19	0.263
45	A	3	2	1.00	21	0.095
46	A	14	9	1.00	21	0.429
47	A	3	2	1.00	21	0.095
48	A	21	9	1.00	21	0.429
49	A	3	2	1.00	21	0.095
50	A	25	9	1.00	21	0.429
51	A	13	9	1.00	21	0.429
52	A	8	6	1.00	21	0.286
53	A	10	9	1.00	21	0.429
54	A	7	6	1.00	21	0.286
55	A	6	6	1.00	19	0.316
56	A	6	5	1.00	19	0.263
57	A	3	2	1.00	21	0.095
58	A	15	11	1.00	21	0.524
59	A	3	2	1.00	21	0.095

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	3	2	1.00	21	0.095
61	A	9	6	1.00	21	0.286
62	A	8	6	1.00	21	0.286
63	A	7	6	1.00	21	0.286
64	A	3	2	1.00	21	0.095
65	A	3	2	1.00	21	0.095
66	A	3	2	1.00	21	0.095
67	A	9	6	1.00	21	0.286
68	A	8	6	1.00	21	0.286
69	A	7	6	1.00	21	0.286
70	A	3	2	1.00	21	0.095
71	A	3	2	1.00	21	0.095
72	A	3	2	1.00	21	0.095
73	A	8	6	1.00	21	0.286
74	A	7	6	1.00	21	0.286
75	A	3	2	1.00	21	0.095
76	A	3	2	1.00	21	0.095
77	A	3	2	1.00	21	0.095
78	A	6	5	1.00	9	0.556
79	A	8	5	1.00	21	0.238
80	A	6	5	1.00	21	0.238
81	A	5	4	1.00	19	0.210
82	A	14	6	1.00	21	0.286
83	N/A	0	0	1.00	21	0.000
84	A	7	4	1.00	21	0.190
85	A	6	4	1.00	21	0.190
86	A	2	2	1.00	21	0.095
87	A	4	4	1.00	21	0.190
88	N/A	0	0	1.00	21	0.000
89	N/A	0	0	1.00	19	0.000
90	N/A	0	0	1.00	19	0.000
91	N/A	0	0	1.00	21	0.000

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{\sin^4(x)}{i+\tan(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 78

$$\int \frac{\sin^4(x)}{i+\tan(x)}\,dx = -\frac{ix}{16} - \frac{1}{32(i-\tan(x))^2} - \frac{i}{8(i-\tan(x))} + \frac{i}{24(i+\tan(x))^3} - \frac{5}{32(i+\tan(x))^2} - \frac{3i}{16(i+\tan(x))}$$

```
[Out] -1/16*I*x-1/32/(I-tan(x))^2-1/8*I/(I-tan(x))+1/24*I/(I+tan(x))^3-5/32/(I+ta
n(x))^2-3/16*I/(I+tan(x))
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3597, 862, 90, 209}

$$\int \frac{\sin^4(x)}{i+\tan(x)}\,dx = -\frac{ix}{16} - \frac{i}{8(-\tan(x)+i)} - \frac{3i}{16(\tan(x)+i)} - \frac{1}{32(-\tan(x)+i)^2} - \frac{5}{32(\tan(x)+i)^2} + \frac{i}{24(\tan(x)+i)^3}$$

```
[In] Int[Sin[x]^4/(I + Tan[x]),x]
```

```
[Out] (-1/16*I)*x - 1/(32*(I - Tan[x])^2) - (I/8)/(I - Tan[x]) + (I/24)/(I + Tan[
x])^3 - 5/(32*(I + Tan[x])^2) - ((3*I)/16)/(I + Tan[x])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

```
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{x^4}{(i+x)\,(1+x^2)^3}\,dx, x, \tan(x)\right) \\
&= \text{Subst}\left(\int \frac{x^4}{(-i+x)^3(i+x)^4}\,dx, x, \tan(x)\right) \\
&= \text{Subst}\left(\int \left(\frac{1}{16(-i+x)^3} - \frac{i}{8(-i+x)^2} - \frac{i}{8(i+x)^4} + \frac{5}{16(i+x)^3} + \frac{3i}{16(i+x)^2} - \frac{i}{16\,(1+x^2)}\right) dx, x, \tan(x)\right) \\
&= -\frac{1}{32(i-\tan(x))^2} - \frac{i}{8(i-\tan(x))} + \frac{i}{24(i+\tan(x))^3} - \frac{5}{32(i+\tan(x))^2} \\
&\quad - \frac{3i}{16(i+\tan(x))} - \frac{1}{16} i\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right) \\
&= -\frac{ix}{16} - \frac{1}{32(i-\tan(x))^2} - \frac{i}{8(i-\tan(x))} \\
&\quad + \frac{i}{24(i+\tan(x))^3} - \frac{5}{32(i+\tan(x))^2} - \frac{3i}{16(i+\tan(x))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.86

$$\int \frac{\sin^4(x)}{i+\tan(x)}\,dx$$
$$=\frac{\sec(x)(-56i\cos(x)-9i\cos(3x)+i\cos(5x)+24\arctan(\tan(x))(\cos(x)-i\sin(x))-32\sin(x)-27\sin(3x}{384(i+\tan(x))}$$

```
[In] Integrate[Sin[x]^4/(I + Tan[x]),x]
```

```
[Out] (Sec[x]*((-56*I)*Cos[x] - (9*I)*Cos[3*x] + I*Cos[5*x] + 24*ArcTan[Tan[x]]*(
Cos[x] - I*Sin[x]) - 32*Sin[x] - 27*Sin[3*x] + 5*Sin[5*x]))/(384*(I + Tan[x
]))
```

Maple [A] (verified)

Time = 35.65 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.50

method	result
risch	$-\frac{ix}{16}-\frac{e^{6ix}}{192}+\frac{\cos(4x)}{32}+\frac{i\sin(4x)}{64}-\frac{5\cos(2x)}{64}+\frac{i\sin(2x)}{64}$
parallelrisch	$-\frac{7}{480}-\frac{ix}{12}+\ln\left(\frac{1}{(i+\tan(x))^{\frac{1}{48}}}\right)+\ln\left(\left(\sec^2(x)\right)^{\frac{1}{96}}\right)+\frac{i\sin(2x)}{64}-\frac{i\sin(6x)}{192}+\frac{i\sin(4x)}{64}-\frac{\cos(6x)}{192}+\frac{\cos(}{32}$
default	$\frac{i}{8\tan(x)-8i}-\frac{1}{32(\tan(x)-i)^2}-\frac{\ln(\tan(x)-i)}{32}+\frac{i}{24(i+\tan(x))^3}-\frac{3i}{16(i+\tan(x))}-\frac{5}{32(i+\tan(x))^2}+\frac{\ln(i+\tan(x))}{32}$

```
[In] int(sin(x)^4/(I+tan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*I*x-1/192*exp(6*I*x)+1/32*cos(4*x)+1/64*I*sin(4*x)-5/64*cos(2*x)+1/64
*I*sin(2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.50

$$\int \frac{\sin^4(x)}{i+\tan(x)}\,dx=\frac{1}{384}\left(-24i\,xe^{(4i\,x)}-2\,e^{(10i\,x)}+9\,e^{(8i\,x)}-12\,e^{(6i\,x)}-18\,e^{(2i\,x)}+3\right)e^{(-4i\,x)}$$

```
[In] integrate(sin(x)^4/(I+tan(x)),x, algorithm="fricas")
```

```
[Out] 1/384*(-24*I*x*e^(4*I*x) - 2*e^(10*I*x) + 9*e^(8*I*x) - 12*e^(6*I*x) - 18*e
^(2*I*x) + 3)*e^(-4*I*x)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.65

$$\int \frac{\sin^4(x)}{i+\tan(x)}\,dx = -\frac{ix}{16} - \frac{e^{6ix}}{192} + \frac{3e^{4ix}}{128} - \frac{e^{2ix}}{32} - \frac{3e^{-2ix}}{64} + \frac{e^{-4ix}}{128}$$

```
[In] integrate(sin(x)**4/(I+tan(x)),x)
```

```
[Out] -I*x/16 - exp(6*I*x)/192 + 3*exp(4*I*x)/128 - exp(2*I*x)/32 - 3*exp(-2*I*x)
/64 + exp(-4*I*x)/128
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^4(x)}{i+\tan(x)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(sin(x)^4/(I+tan(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.68

$$\int \frac{\sin^4(x)}{i+\tan(x)}\,dx = -\frac{3i\,\tan(x)^4 + 21\,\tan(x)^3 + 13i\,\tan(x)^2 + 11\,\tan(x) + 8i}{48\,(\tan(x)+i)^3(\tan(x)-i)^2} + \frac{1}{32}\,\log(\tan(x)+i) - \frac{1}{32}\,\log(\tan(x)-i)$$

```
[In] integrate(sin(x)^4/(I+tan(x)),x, algorithm="giac")
```

```
[Out] -1/48*(3*I*tan(x)^4 + 21*tan(x)^3 + 13*I*tan(x)^2 + 11*tan(x) + 8*I)/((tan(
x) + I)^3*(tan(x) - I)^2) + 1/32*log(tan(x) + I) - 1/32*log(tan(x) - I)
```

Mupad [B] (verification not implemented)

Time = 4.71 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.63

$$\int \frac{\sin^4(x)}{i+\tan(x)}\,dx = -\frac{x\,1\mathrm{i}}{16} + \frac{\frac{\tan(x)^4\,1\mathrm{i}}{16} + \frac{7\tan(x)^3}{16} + \frac{\tan(x)^2\,13\mathrm{i}}{48} + \frac{11\tan(x)}{48} + \frac{1}{6}\mathrm{i}}{\left(\tan(x)+1\mathrm{i}\right)^3\left(1+\tan(x)\ 1\mathrm{i}\right)^2}$$

```
[In] int(sin(x)^4/(tan(x) + 1i),x)
[Out] ((11*tan(x))/48 + (tan(x)^2*13i)/48 + (7*tan(x)^3)/16 + (tan(x)^4*1i)/16 +
1i/6)/((tan(x) + 1i)^3*(tan(x)*1i + 1)^2) - (x*1i)/16
```

3.2 $\int \frac{\sin^3(x)}{i+\tan(x)} dx$

Optimal result

Integrand size = 13, antiderivative size = 29

$$\int \frac{\sin^3(x)}{i+\tan(x)} dx = \frac{1}{3} i \cos^3(x) - \frac{1}{5} i \cos^5(x) + \frac{\sin^5(x)}{5}$$

```
[Out] 1/3*I*cos(x)^3-1/5*I*cos(x)^5+1/5*sin(x)^5
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3599, 3187, 3186, 2645, 14, 2644, 30}

$$\int \frac{\sin^3(x)}{i+\tan(x)} dx = \frac{\sin^5(x)}{5} - \frac{1}{5} i \cos^5(x) + \frac{1}{3} i \cos^3(x)$$

```
[In] Int[Sin[x]^3/(I + Tan[x]),x]
[Out] (I/3)*Cos[x]^3 - (I/5)*Cos[x]^5 + Sin[x]^5/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 3186

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 3187

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\cos(x)\sin^3(x)}{i\cos(x)+\sin(x)}\,dx \\
&= -\left(i\int \cos(x)(\cos(x)+i\sin(x))\sin^3(x)\,dx\right) \\
&= -\left(i\int \left(\cos^2(x)\sin^3(x)+i\cos(x)\sin^4(x)\right)\,dx\right)
\end{aligned}$$

$$= -\left(i \int \cos^2(x) \sin^3(x)\, dx\right) + \int \cos(x) \sin^4(x)\, dx$$

$$= i\text{Subst}\left(\int x^2 \left(1 - x^2\right)\, dx, x, \cos(x)\right) + \text{Subst}\left(\int x^4\, dx, x, \sin(x)\right)$$

$$= \frac{\sin^5(x)}{5} + i\text{Subst}\left(\int \left(x^2 - x^4\right)\, dx, x, \cos(x)\right)$$

$$= \frac{1}{3} i \cos^3(x) - \frac{1}{5} i \cos^5(x) + \frac{\sin^5(x)}{5}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.76

$$\int \frac{\sin^3(x)}{i + \tan(x)}\, dx = \frac{1}{8} i \cos(x) + \frac{1}{48} i \cos(3x) - \frac{1}{80} i \cos(5x) + \frac{\sin(x)}{8} - \frac{1}{16} \sin(3x) + \frac{1}{80} \sin(5x)$$

```
[In] Integrate[Sin[x]^3/(I + Tan[x]),x]
[Out] (I/8)*Cos[x] + (I/48)*Cos[3*x] - (I/80)*Cos[5*x] + Sin[x]/8 - Sin[3*x]/16 +
 Sin[5*x]/80
```

Maple [A] (verified)

Time = 4.87 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

method	result
risch	$-\frac{i e^{5ix}}{80} + \frac{i e^{-ix}}{8} + \frac{i \cos(3x)}{48} - \frac{\sin(3x)}{16}$
parallelrisch	$\frac{2i}{15} + \frac{i \cos(3x)}{48} + \frac{i \cos(x)}{8} - \frac{i \cos(5x)}{80} + \frac{\sin(5x)}{80} - \frac{\sin(3x)}{16} + \frac{\sin(x)}{8}$
default	$\frac{i}{\left(\tan\left(\frac{x}{2}\right)+i\right)^4} + \frac{2}{5\left(\tan\left(\frac{x}{2}\right)+i\right)^5} - \frac{2}{3\left(\tan\left(\frac{x}{2}\right)+i\right)^3} - \frac{1}{8\left(\tan\left(\frac{x}{2}\right)+i\right)} - \frac{i}{4\left(\tan\left(\frac{x}{2}\right)-i\right)^2} + \frac{1}{6\left(\tan\left(\frac{x}{2}\right)-i\right)^3} + \frac{1}{8\tan\left(\frac{x}{2}\right)-8i}$

```
[In] int(sin(x)^3/(I+tan(x)),x,method=_RETURNVERBOSE)
[Out] -1/80*I*exp(5*I*x)+1/8*I*exp(-I*x)+1/48*I*cos(3*x)-1/16*sin(3*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{\sin^3(x)}{i+\tan(x)}\,dx = \frac{1}{240}\left(-3i\,e^{(8i\,x)}+10i\,e^{(6i\,x)}+30i\,e^{(2i\,x)}-5i\right)e^{(-3i\,x)}$$

```
[In] integrate(sin(x)^3/(I+tan(x)),x, algorithm="fricas")
[Out] 1/240*(-3*I*e^(8*I*x) + 10*I*e^(6*I*x) + 30*I*e^(2*I*x) - 5*I)*e^(-3*I*x)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.28

$$\int \frac{\sin^3(x)}{i+\tan(x)}\,dx = -\frac{ie^{5ix}}{80}+\frac{ie^{3ix}}{24}+\frac{ie^{-ix}}{8}-\frac{ie^{-3ix}}{48}$$

```
[In] integrate(sin(x)**3/(I+tan(x)),x)
[Out] -I*exp(5*I*x)/80 + I*exp(3*I*x)/24 + I*exp(-I*x)/8 - I*exp(-3*I*x)/48
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^3(x)}{i+\tan(x)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(sin(x)^3/(I+tan(x)),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(19) = 38.

Time = 0.33 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.45

$$\int \frac{\sin^3(x)}{i+\tan(x)}\,dx = -\frac{-3i\tan\left(\frac{1}{2}x\right)^2-12\tan\left(\frac{1}{2}x\right)+5i}{24\left(-i\tan\left(\frac{1}{2}x\right)-1\right)^3} - \frac{15\tan\left(\frac{1}{2}x\right)^4+60i\tan\left(\frac{1}{2}x\right)^3-10\tan\left(\frac{1}{2}x\right)^2-20i\tan\left(\frac{1}{2}x\right)+7}{120\left(\tan\left(\frac{1}{2}x\right)+i\right)^5}$$

```
[In] integrate(sin(x)^3/(I+tan(x)),x, algorithm="giac")
```

```
[Out] -1/24*(-3*I*tan(1/2*x)^2 - 12*tan(1/2*x) + 5*I)/(-I*tan(1/2*x) - 1)^3 - 1/1
20*(15*tan(1/2*x)^4 + 60*I*tan(1/2*x)^3 - 10*tan(1/2*x)^2 - 20*I*tan(1/2*x)
 + 7)/(tan(1/2*x) + I)^5
```

Mupad [B] (verification not implemented)

Time = 4.63 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.03

$$\int \frac{\sin^3(x)}{i+\tan(x)}\,dx = -\frac{4\left(-\tan\left(\frac{x}{2}\right)^4 15\mathrm{i}+6\tan\left(\frac{x}{2}\right)^3+\tan\left(\frac{x}{2}\right)^2 2\mathrm{i}+2\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)}{15\left(-1+\tan\left(\frac{x}{2}\right)\,1\mathrm{i}\right)^5\left(1+\tan\left(\frac{x}{2}\right)\,1\mathrm{i}\right)^3}$$

```
[In] int(sin(x)^3/(tan(x) + 1i),x)
```

```
[Out] -(4*(2*tan(x/2) + tan(x/2)^2*2i + 6*tan(x/2)^3 - tan(x/2)^4*15i + 1i))/(15*
(tan(x/2)*1i - 1)^5*(tan(x/2)*1i + 1)^3)
```

3.3 $\int \frac{\sin^2(x)}{i+\tan(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 50

$$\int \frac{\sin^2(x)}{i+\tan(x)}\,dx=-\frac{ix}{8}-\frac{i}{8(i-\tan(x))}-\frac{1}{8(i+\tan(x))^2}-\frac{i}{4(i+\tan(x))}$$

```
[Out] -1/8*I*x-1/8*I/(I-tan(x))-1/8/(I+tan(x))^2-1/4*I/(I+tan(x))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3597, 862, 90, 209}

$$\int \frac{\sin^2(x)}{i+\tan(x)}\,dx=-\frac{ix}{8}-\frac{i}{8(-\tan(x)+i)}-\frac{i}{4(\tan(x)+i)}-\frac{1}{8(\tan(x)+i)^2}$$

```
[In] Int[Sin[x]^2/(I + Tan[x]),x]
```

```
[Out] (-1/8*I)*x - (I/8)/(I - Tan[x]) - 1/(8*(I + Tan[x])^2) - (I/4)/(I + Tan[x])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{x^2}{(i+x)\left(1+x^2\right)^2}\,dx, x, \tan(x)\right)\\
&= \text{Subst}\left(\int \frac{x^2}{(-i+x)^2(i+x)^3}\,dx, x, \tan(x)\right)\\
&= \text{Subst}\left(\int \left(-\frac{i}{8(-i+x)^2}+\frac{1}{4(i+x)^3}+\frac{i}{4(i+x)^2}-\frac{i}{8\left(1+x^2\right)}\right)dx, x, \tan(x)\right)\\
&= -\frac{i}{8(i-\tan(x))}-\frac{1}{8(i+\tan(x))^2}-\frac{i}{4(i+\tan(x))}-\frac{1}{8}i\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)\\
&= -\frac{ix}{8}-\frac{i}{8(i-\tan(x))}-\frac{1}{8(i+\tan(x))^2}-\frac{i}{4(i+\tan(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int \frac{\sin^2(x)}{i+\tan(x)}\,dx = -\frac{i(3+\cos(2x)-3i\sin(2x)+2\arctan(\tan(x))(i+\tan(x)))}{16(i+\tan(x))}$$

```
[In] Integrate[Sin[x]^2/(I + Tan[x]),x]
[Out] ((-1/16*I)*(3 + Cos[2*x] - (3*I)*Sin[2*x] + 2*ArcTan[Tan[x]]*(I + Tan[x])))
/(I + Tan[x])
```

Maple [A] (verified)

Time = 2.68 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.38

method	result
risch	$-\frac{ix}{8}+\frac{\mathrm{e}^{4ix}}{32}-\frac{\cos(2x)}{8}$
parallelrisch	$-\frac{ix}{4}-\frac{7}{96}+\ln\left(\frac{1}{(i+\tan(x))^{\frac{1}{8}}}\right)+\ln\left(\left(\sec^{2}(x)\right)^{\frac{1}{16}}\right)+\frac{i\sin(4x)}{32}+\frac{\cos(4x)}{32}-\frac{\cos(2x)}{8}$
default	$\frac{i}{8\tan(x)-8i}-\frac{\ln(\tan(x)-i)}{16}-\frac{i}{4(i+\tan(x))}-\frac{1}{8(i+\tan(x))^{2}}+\frac{\ln(i+\tan(x))}{16}$
norman	$-\frac{1}{4}-\frac{\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{4}-\frac{\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{2}-\frac{ix}{8}+ix\tan(x)\tan\left(\frac{x}{2}\right)-\frac{3ix\left(\tan^{2}(x)\right)\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{8}-ix\tan(x)\left(\tan^{3}\left(\frac{x}{2}\right)\right)+\frac{ix\left(\tan^{2}(x)\right)\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{4}-\frac{x(}{}$

```
[In] int(sin(x)^2/(I+tan(x)),x,method=_RETURNVERBOSE)
[Out] -1/8*I*x+1/32*exp(4*I*x)-1/8*cos(2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.50

$$\int \frac{\sin^2(x)}{i+\tan(x)}\,dx = \frac{1}{32}\left(-4i\,xe^{(2i\,x)}+e^{(6i\,x)}-2\,e^{(4i\,x)}-2\right)e^{(-2i\,x)}$$

```
[In] integrate(sin(x)^2/(I+tan(x)),x, algorithm="fricas")
[Out] 1/32*(-4*I*x*e^(2*I*x) + e^(6*I*x) - 2*e^(4*I*x) - 2)*e^(-2*I*x)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.62

$$\int \frac{\sin^2(x)}{i+\tan(x)}\,dx = -\frac{ix}{8}+\frac{e^{4ix}}{32}-\frac{e^{2ix}}{16}-\frac{e^{-2ix}}{16}$$

```
[In] integrate(sin(x)**2/(I+tan(x)),x)
[Out] -I*x/8 + exp(4*I*x)/32 - exp(2*I*x)/16 - exp(-2*I*x)/16
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^2(x)}{i+\tan(x)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(sin(x)^2/(I+tan(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int \frac{\sin^2(x)}{i+\tan(x)}\,dx = -\frac{i\,\tan{(x)}^2+3\,\tan{(x)}+2i}{8\,(\tan{(x)}+i)^2(\tan{(x)}-i)}+\frac{1}{16}\,\log{(\tan{(x)}+i)}-\frac{1}{16}\,\log{(\tan{(x)}-i)}$$

```
[In] integrate(sin(x)^2/(I+tan(x)),x, algorithm="giac")
```

```
[Out] -1/8*(I*tan(x)^2 + 3*tan(x) + 2*I)/((tan(x) + I)^2*(tan(x) - I)) + 1/16*log
(tan(x) + I) - 1/16*log(tan(x) - I)
```

Mupad [B] (verification not implemented)

Time = 4.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.70

$$\int \frac{\sin^2(x)}{i+\tan(x)}\,dx = -\frac{x\,1\mathrm{i}}{8}+\frac{\frac{\tan(x)^2}{8}-\frac{\tan(x)\,3\mathrm{i}}{8}+\frac{1}{4}}{(\tan{(x)}+1\mathrm{i})^2\,\left(1+\tan{(x)}\,1\mathrm{i}\right)}$$

```
[In] int(sin(x)^2/(tan(x) + 1i),x)
```

```
[Out] (tan(x)^2/8 - (tan(x)*3i)/8 + 1/4)/((tan(x) + 1i)^2*(tan(x)*1i + 1)) - (x*1
i)/8
```

3.4 $\int \frac{\sin(x)}{i+\tan(x)} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{\sin(x)}{i+\tan(x)} \, dx = \frac{1}{3} i \cos^3(x) + \frac{\sin^3(x)}{3}$$

```
[Out] 1/3*I*cos(x)^3+1/3*sin(x)^3
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3599, 3187, 3186, 2645, 30, 2644}

$$\int \frac{\sin(x)}{i+\tan(x)} \, dx = \frac{\sin^3(x)}{3} + \frac{1}{3} i \cos^3(x)$$

```
[In] Int[Sin[x]/(I + Tan[x]),x]
```

```
[Out] (I/3)*Cos[x]^3 + Sin[x]^3/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 3186

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 3187

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cos(x)\sin(x)}{i\cos(x)+\sin(x)}\,dx \\
&= -(i\int \cos(x)(\cos(x)+i\sin(x))\sin(x)\,dx) \\
&= -\left(i\int \left(\cos^2(x)\sin(x)+i\cos(x)\sin^2(x)\right)\,dx\right) \\
&= -\left(i\int \cos^2(x)\sin(x)\,dx\right)+\int \cos(x)\sin^2(x)\,dx \\
&= i\text{Subst}\left(\int x^2\,dx,x,\cos(x)\right)+\text{Subst}\left(\int x^2\,dx,x,\sin(x)\right) \\
&= \frac{1}{3}i\cos^3(x)+\frac{\sin^3(x)}{3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int \frac{\sin(x)}{i+\tan(x)}\,dx = \frac{1}{4}i\cos(x)+\frac{1}{12}i\cos(3x)+\frac{\sin(x)}{4}-\frac{1}{12}\sin(3x)$$

```
[In] Integrate[Sin[x]/(I + Tan[x]),x]
[Out] (I/4)*Cos[x] + (I/12)*Cos[3*x] + Sin[x]/4 - Sin[3*x]/12
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
risch	$\frac{i\mathrm{e}^{3ix}}{12}+\frac{i\mathrm{e}^{-ix}}{4}$	18
parallelrisch	$\frac{2i}{3}+\frac{i\cos(3x)}{12}+\frac{i\cos(x)}{4}-\frac{\sin(3x)}{12}+\frac{\sin(x)}{4}$	26
default	$\frac{1}{2\tan\left(\frac{x}{2}\right)-2i}+\frac{i}{\left(\tan\left(\frac{x}{2}\right)+i\right)^{2}}+\frac{2}{3\left(\tan\left(\frac{x}{2}\right)+i\right)^{3}}-\frac{1}{2\left(\tan\left(\frac{x}{2}\right)+i\right)}$	47
norman	$\frac{\frac{i\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{3}+\frac{4i\left(\tan^{2}(x)\right)}{3}+\frac{2\left(\tan^{2}(x)\right)\tan\left(\frac{x}{2}\right)}{3}-\frac{\left(\tan^{2}\left(\frac{x}{2}\right)\right)\tan(x)}{3}-\frac{4i\tan(x)\tan\left(\frac{x}{2}\right)}{3}+\frac{\tan(x)}{3}-\frac{2\tan\left(\frac{x}{2}\right)}{3}+i}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)\left(\tan^{2}(x)+1\right)}$	78

```
[In] int(sin(x)/(I+tan(x)),x,method=_RETURNVERBOSE)
[Out] 1/12*I*exp(3*I*x)+1/4*I*exp(-I*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{\sin(x)}{i+\tan(x)}\,dx = \frac{1}{12}\left(i\,e^{(4i\,x)}+3i\right)e^{(-i\,x)}$$

```
[In] integrate(sin(x)/(I+tan(x)),x, algorithm="fricas")
[Out] 1/12*(I*e^(4*I*x) + 3*I)*e^(-I*x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\sin(x)}{i+\tan(x)}\,dx = \frac{ie^{3ix}}{12} + \frac{ie^{-ix}}{4}$$

```
[In] integrate(sin(x)/(I+tan(x)),x)
[Out] I*exp(3*I*x)/12 + I*exp(-I*x)/4
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin(x)}{i+\tan(x)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(sin(x)/(I+tan(x)),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(13) = 26.

Time = 0.31 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int \frac{\sin(x)}{i+\tan(x)}\,dx = -\frac{i}{2\left(-i\tan\left(\frac{1}{2}x\right)-1\right)} - \frac{3\tan\left(\frac{1}{2}x\right)^2-1}{6\left(\tan\left(\frac{1}{2}x\right)+i\right)^3}$$

```
[In] integrate(sin(x)/(I+tan(x)),x, algorithm="giac")
[Out] -1/2*I/(-I*tan(1/2*x) - 1) - 1/6*(3*tan(1/2*x)^2 - 1)/(tan(1/2*x) + I)^3
```

Mupad [B] (verification not implemented)

Time = 4.91 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.05

$$\int \frac{\sin(x)}{i+\tan(x)}\,dx = -\frac{2\left(3\tan\left(\frac{x}{2}\right)^2+\tan\left(\frac{x}{2}\right)\,2\mathrm{i}-1\right)}{3\,\left(1+\tan\left(\frac{x}{2}\right)\,1\mathrm{i}\right)\,\left(\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)^3}$$

```
[In] int(sin(x)/(tan(x) + 1i),x)
[Out] -(2*(tan(x/2)*2i + 3*tan(x/2)^2 - 1))/(3*(tan(x/2)*1i + 1)*(tan(x/2) + 1i)^
3)
```

3.5 $\int \frac{\csc(x)}{i+\tan(x)} dx$

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{\csc(x)}{i+\tan(x)} dx = i\text{arctanh}(\cos(x)) - i\cos(x) + \sin(x)$$

```
[Out] I*arctanh(cos(x))-I*cos(x)+sin(x)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {3599, 3187, 3186, 2717, 2672, 327, 212}

$$\int \frac{\csc(x)}{i+\tan(x)} dx = i\text{arctanh}(\cos(x)) + \sin(x) - i\cos(x)$$

```
[In] Int[Csc[x]/(I + Tan[x]),x]
```

```
[Out] I*ArcTanh[Cos[x]] - I*Cos[x] + Sin[x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3186

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 3187

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cot(x)}{i\cos(x)+\sin(x)}\,dx \\
&= -(i\int \cot(x)(\cos(x)+i\sin(x))\,dx)
\end{aligned}
$$

$$= -(i \int (i\cos(x) + \cos(x)\cot(x))\,dx)$$

$$= -(i \int \cos(x)\cot(x)\,dx) + \int \cos(x)\,dx$$

$$= \sin(x) + i\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \cos(x)\right)$$

$$= -i\cos(x) + \sin(x) + i\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(x)\right)$$

$$= i\text{arctanh}(\cos(x)) - i\cos(x) + \sin(x)$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.94

$$\int \frac{\csc(x)}{i+\tan(x)}\,dx = -i\cos(x) + i\log\left(\cos\left(\frac{x}{2}\right)\right) - i\log\left(\sin\left(\frac{x}{2}\right)\right) + \sin(x)$$

```
[In] Integrate[Csc[x]/(I + Tan[x]),x]
[Out] (-I)*Cos[x] + I*Log[Cos[x/2]] - I*Log[Sin[x/2]] + Sin[x]
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

method	result	size
default	$\frac{2}{\tan(\frac{x}{2})+i} - i\ln\left(\tan\left(\frac{x}{2}\right)\right)$	21
risch	$-ie^{ix} + i\ln\left(e^{ix}+1\right) - i\ln\left(e^{ix}-1\right)$	32

```
[In] int(csc(x)/(I+tan(x)),x,method=_RETURNVERBOSE)
[Out] 2/(tan(1/2*x)+I)-I*ln(tan(1/2*x))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(12) = 24$.

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.56

$$\int \frac{\csc(x)}{i+\tan(x)}\,dx = -i\,e^{(i\,x)} + i\,\log\left(e^{(i\,x)} + 1\right) - i\,\log\left(e^{(i\,x)} - 1\right)$$

```
[In] integrate(csc(x)/(I+tan(x)),x, algorithm="fricas")
[Out] -I*e^(I*x) + I*log(e^(I*x) + 1) - I*log(e^(I*x) - 1)
```

Sympy [F]

$$\int \frac{\csc(x)}{i+\tan(x)}\,dx = \int \frac{\csc(x)}{\tan(x)+i}\,dx$$

```
[In] integrate(csc(x)/(I+tan(x)),x)
[Out] Integral(csc(x)/(tan(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(12) = 24$.

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.75

$$\int \frac{\csc(x)}{i+\tan(x)}\,dx = \frac{2}{\frac{\sin(x)}{\cos(x)+1} + i} - i\,\log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

```
[In] integrate(csc(x)/(I+tan(x)),x, algorithm="maxima")
[Out] 2/(sin(x)/(cos(x) + 1) + I) - I*log(sin(x)/(cos(x) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\csc(x)}{i+\tan(x)}\,dx = -\frac{2i}{-i\tan\left(\frac{1}{2}x\right)+1} - i\log\left(\tan\left(\frac{1}{2}x\right)\right)$$

```
[In] integrate(csc(x)/(I+tan(x)),x, algorithm="giac")
[Out] -2*I/(-I*tan(1/2*x) + 1) - I*log(tan(1/2*x))
```

Mupad [B] (verification not implemented)

Time = 4.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\csc(x)}{i+\tan(x)}\,dx = -\ln\left(\tan\left(\frac{x}{2}\right)\right)\,1\mathrm{i} + \frac{2}{\tan\left(\frac{x}{2}\right)+1\mathrm{i}}$$

```
[In] int(1/(sin(x)*(tan(x) + 1i)),x)
[Out] 2/(tan(x/2) + 1i) - log(tan(x/2))*1i
```

3.6 $\int \frac{\csc^2(x)}{i+\tan(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 18

$$\int \frac{\csc^2(x)}{i+\tan(x)}\,dx = ix + i\cot(x) + \log(\cos(x)) + \log(\tan(x))$$

```
[Out] I*x+I*cot(x)+ln(cos(x))+ln(tan(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3597, 46}

$$\int \frac{\csc^2(x)}{i+\tan(x)}\,dx = ix + i\cot(x) + \log(\tan(x)) + \log(\cos(x))$$

```
[In] Int[Csc[x]^2/(I + Tan[x]),x]
```

```
[Out] I*x + I*Cot[x] + Log[Cos[x]] + Log[Tan[x]]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
```

```
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{x^2(i+x)}\,dx, x, \tan(x)\right)\\ &= \text{Subst}\left(\int \left(\frac{1}{-i-x} - \frac{i}{x^2} + \frac{1}{x}\right)dx, x, \tan(x)\right)\\ &= ix + i\cot(x) + \log(\cos(x)) + \log(\tan(x))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{\csc^2(x)}{i+\tan(x)}\,dx = ix + i\cot(x) + \log(\sin(x))$$

```
[In] Integrate[Csc[x]^2/(I + Tan[x]),x]
[Out] I*x + I*Cot[x] + Log[Sin[x]]
```

Maple [A] (verified)

Time = 2.97 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

method	result	size
default	$\ln(\tan(x)) + \frac{i}{\tan(x)} - \ln(i + \tan(x))$	20
risch	$-\frac{2}{e^{2ix}-1} + \ln(e^{2ix} - 1)$	21

```
[In] int(csc(x)^2/(I+tan(x)),x,method=_RETURNVERBOSE)
[Out] ln(tan(x))+I/tan(x)-ln(I+tan(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.39

$$\int \frac{\csc^2(x)}{i+\tan(x)}\,dx = \frac{\left(e^{(2i\,x)}-1\right)\log\left(e^{(2i\,x)}-1\right)-2}{e^{(2i\,x)}-1}$$

```
[In] integrate(csc(x)^2/(I+tan(x)),x, algorithm="fricas")
[Out] ((e^(2*I*x) - 1)*log(e^(2*I*x) - 1) - 2)/(e^(2*I*x) - 1)
```

Sympy [F]

$$\int \frac{\csc^2(x)}{i+\tan(x)}\,dx = \int \frac{\csc^2(x)}{\tan(x)+i}\,dx$$

```
[In] integrate(csc(x)**2/(I+tan(x)),x)
[Out] Integral(csc(x)**2/(tan(x) + I), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{\csc^2(x)}{i+\tan(x)}\,dx = \frac{i}{\tan(x)} - \log(\tan(x)+i) + \log(\tan(x))$$

```
[In] integrate(csc(x)^2/(I+tan(x)),x, algorithm="maxima")
[Out] I/tan(x) - log(tan(x) + I) + log(tan(x))
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\csc^2(x)}{i+\tan(x)}\,dx = \frac{i}{\tan(x)} - \log(\tan(x)+i) + \log(|\tan(x)|)$$

```
[In] integrate(csc(x)^2/(I+tan(x)),x, algorithm="giac")
[Out] I/tan(x) - log(tan(x) + I) + log(abs(tan(x)))
```

Mupad [B] (verification not implemented)

Time = 4.67 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\csc^2(x)}{i+\tan(x)}\,dx = \mathrm{atan}(2\tan(x)+1\mathrm{i})\ 2\mathrm{i} + \frac{1\mathrm{i}}{\tan{(x)}}$$

```
[In] int(1/(sin(x)^2*(tan(x) + 1i)),x)
[Out] atan(2*tan(x) + 1i)*2i + 1i/tan(x)
```

3.7 $\int \frac{\csc^3(x)}{i+\tan(x)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 24

$$\int \frac{\csc^3(x)}{i+\tan(x)} \, dx = -\frac{1}{2} i \operatorname{arctanh}(\cos(x)) - \csc(x) + \frac{1}{2} i \cot(x) \csc(x)$$

```
[Out] -1/2*I*arctanh(cos(x))-csc(x)+1/2*I*cot(x)*csc(x)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3599, 3187, 3186, 2686, 8, 2691, 3855}

$$\int \frac{\csc^3(x)}{i+\tan(x)} \, dx = -\frac{1}{2} i \operatorname{arctanh}(\cos(x)) - \csc(x) + \frac{1}{2} i \cot(x) \csc(x)$$

```
[In] Int[Csc[x]^3/(I + Tan[x]),x]
```

```
[Out] (-1/2*I)*ArcTanh[Cos[x]] - Csc[x] + (I/2)*Cot[x]*Csc[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3186

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 3187

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\cot(x)\csc^2(x)}{i\cos(x)+\sin(x)}\,dx \\
&= -\left(i\int \cot(x)\csc^2(x)(\cos(x)+i\sin(x))\,dx\right) \\
&= -\left(i\int \left(i\cot(x)\csc(x)+\cot^2(x)\csc(x)\right)\,dx\right) \\
&= -\left(i\int \cot^2(x)\csc(x)\,dx\right)+\int \cot(x)\csc(x)\,dx
\end{aligned}$$

$$= \frac{1}{2}i\cot(x)\csc(x) + \frac{1}{2}i\int \csc(x)\,dx - \text{Subst}(\int 1\,dx, x, \csc(x))$$
$$= -\frac{1}{2}i\text{arctanh}(\cos(x)) - \csc(x) + \frac{1}{2}i\cot(x)\csc(x)$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 75 vs. 2(24) = 48.

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 3.12

$$\int \frac{\csc^3(x)}{i+\tan(x)}\,dx = -\frac{1}{2}\cot\left(\frac{x}{2}\right) + \frac{1}{8}i\csc^2\left(\frac{x}{2}\right) - \frac{1}{2}i\log\left(\cos\left(\frac{x}{2}\right)\right) + \frac{1}{2}i\log\left(\sin\left(\frac{x}{2}\right)\right) - \frac{1}{8}i\sec^2\left(\frac{x}{2}\right) - \frac{1}{2}\tan\left(\frac{x}{2}\right)$$

```
[In] Integrate[Csc[x]^3/(I + Tan[x]),x]
```

```
[Out] -1/2*Cot[x/2] + (I/8)*Csc[x/2]^2 - (I/2)*Log[Cos[x/2]] + (I/2)*Log[Sin[x/2]
] - (I/8)*Sec[x/2]^2 - Tan[x/2]/2
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(18) = 36.

Time = 5.62 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.75

method	result	size
default	$-\frac{\tan\left(\frac{x}{2}\right)}{2}-\frac{i\left(\tan^2\left(\frac{x}{2}\right)\right)}{8}+\frac{i}{8\tan\left(\frac{x}{2}\right)^2}+\frac{i\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2}-\frac{1}{2\tan\left(\frac{x}{2}\right)}$	42
risch	$-\frac{i\left(3\,e^{3ix}-e^{ix}\right)}{\left(e^{2ix}-1\right)^2}-\frac{i\ln\left(e^{ix}+1\right)}{2}+\frac{i\ln\left(e^{ix}-1\right)}{2}$	51

```
[In] int(csc(x)^3/(I+tan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*tan(1/2*x)-1/8*I*tan(1/2*x)^2+1/8*I/tan(1/2*x)^2+1/2*I*ln(tan(1/2*x))-
1/2/tan(1/2*x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(16) = 32$.

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 3.04

$$\int \frac{\csc^3(x)}{i+\tan(x)}\,dx = \frac{\left(-i\,e^{(4i\,x)}+2i\,e^{(2i\,x)}-i\right)\log\left(e^{(i\,x)}+1\right)+\left(i\,e^{(4i\,x)}-2i\,e^{(2i\,x)}+i\right)\log\left(e^{(i\,x)}-1\right)-6i\,e^{(3i\,x)}+2i\,e^{(i\,x)}}{2\left(e^{(4i\,x)}-2\,e^{(2i\,x)}+1\right)}$$

```
[In] integrate(csc(x)^3/(I+tan(x)),x, algorithm="fricas")
```

```
[Out] 1/2*((-I*e^(4*I*x) + 2*I*e^(2*I*x) - I)*log(e^(I*x) + 1) + (I*e^(4*I*x) - 2
*I*e^(2*I*x) + I)*log(e^(I*x) - 1) - 6*I*e^(3*I*x) + 2*I*e^(I*x))/(e^(4*I*x
) - 2*e^(2*I*x) + 1)
```

Sympy [F]

$$\int \frac{\csc^3(x)}{i+\tan(x)}\,dx = \int \frac{\csc^3(x)}{\tan(x)+i}\,dx$$

```
[In] integrate(csc(x)**3/(I+tan(x)),x)
```

```
[Out] Integral(csc(x)**3/(tan(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(16) = 32$.

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.46

$$\int \frac{\csc^3(x)}{i+\tan(x)}\,dx = -\frac{\left(\frac{4\sin(x)}{\cos(x)+1}-i\right)(\cos(x)+1)^2}{8\sin(x)^2}-\frac{\sin(x)}{2(\cos(x)+1)}-\frac{i\sin(x)^2}{8(\cos(x)+1)^2}+\frac{1}{2}i\log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

```
[In] integrate(csc(x)^3/(I+tan(x)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*sin(x)/(cos(x) + 1) - I)*(cos(x) + 1)^2/sin(x)^2 - 1/2*sin(x)/(cos(
x) + 1) - 1/8*I*sin(x)^2/(cos(x) + 1)^2 + 1/2*I*log(sin(x)/(cos(x) + 1))
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(16) = 32$.

Time = 0.33 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.92

$$\int \frac{\csc^3(x)}{i+\tan(x)}\,dx = -\frac{1}{8}i\tan\left(\frac{1}{2}x\right)^2 - \frac{6i\tan\left(\frac{1}{2}x\right)^2 + 4\tan\left(\frac{1}{2}x\right) - i}{8\tan\left(\frac{1}{2}x\right)^2} + \frac{1}{2}i\log\left(\tan\left(\frac{1}{2}x\right)\right) - \frac{1}{2}\tan\left(\frac{1}{2}x\right)$$

```
[In] integrate(csc(x)^3/(I+tan(x)),x, algorithm="giac")
```

```
[Out] -1/8*I*tan(1/2*x)^2 - 1/8*(6*I*tan(1/2*x)^2 + 4*tan(1/2*x) - I)/tan(1/2*x)^
2 + 1/2*I*log(tan(1/2*x)) - 1/2*tan(1/2*x)
```

Mupad [B] (verification not implemented)

Time = 4.74 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.71

$$\int \frac{\csc^3(x)}{i+\tan(x)}\,dx = -\frac{\tan\left(\frac{x}{2}\right)}{2} + \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)\,1\mathrm{i}}{2} - \frac{2\tan\left(\frac{x}{2}\right) - \frac{1}{2}\mathrm{i}}{4\tan\left(\frac{x}{2}\right)^2} - \frac{\tan\left(\frac{x}{2}\right)^2\,1\mathrm{i}}{8}$$

```
[In] int(1/(sin(x)^3*(tan(x) + 1i)),x)
```

```
[Out] (log(tan(x/2))*1i)/2 - tan(x/2)/2 - (2*tan(x/2) - 1i/2)/(4*tan(x/2)^2) - (t
an(x/2)^2*1i)/8
```

3.8 $\int \frac{\csc^4(x)}{i+\tan(x)} dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{\csc^4(x)}{i+\tan(x)} dx = -\frac{1}{2}\cot^2(x) + \frac{1}{3}i\cot^3(x)$$

```
[Out] -1/2*cot(x)^2+1/3*I*cot(x)^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3597, 862, 45}

$$\int \frac{\csc^4(x)}{i+\tan(x)} dx = -\frac{\cot^2(x)}{2} + \frac{1}{3}i\cot^3(x)$$

```
[In] Int[Csc[x]^4/(I + Tan[x]),x]
```

```
[Out] -1/2*Cot[x]^2 + (I/3)*Cot[x]^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
```

```
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1+x^2}{x^4(i+x)}\,dx, x, \tan(x)\right)\\ &= \text{Subst}\left(\int \frac{-i+x}{x^4}\,dx, x, \tan(x)\right)\\ &= \text{Subst}\left(\int \left(-\frac{i}{x^4}+\frac{1}{x^3}\right)\,dx, x, \tan(x)\right)\\ &= -\frac{1}{2}\cot^2(x)+\frac{1}{3}i\cot^3(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.53

$$\int \frac{\csc^4(x)}{i+\tan(x)}\,dx = -\frac{1}{3}i\cot(x)-\frac{\csc^2(x)}{2}+\frac{1}{3}i\cot(x)\csc^2(x)$$

```
[In] Integrate[Csc[x]^4/(I + Tan[x]),x]
[Out] (-1/3*I)*Cot[x] - Csc[x]^2/2 + (I/3)*Cot[x]*Csc[x]^2
```

Maple [A] (verified)

Time = 7.68 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$-\frac{1}{2\tan(x)^2}+\frac{i}{3\tan(x)^3}$	15
default	$-\frac{1}{2\tan(x)^2}+\frac{i}{3\tan(x)^3}$	15
risch	$\frac{4\,e^{4ix}-2\,e^{2ix}+\frac{2}{3}}{\left(e^{2ix}-1\right)^3}$	28
parallelrisch	$\frac{(7-4i\cot(x))\cos(2x)+5-4i\cot(x)}{-12+12\cos(2x)}$	31

```
[In] int(csc(x)^4/(I+tan(x)),x,method=_RETURNVERBOSE)
[Out] -1/2/tan(x)^2+1/3*I/tan(x)^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(13) = 26.

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{\csc^4(x)}{i+\tan(x)}\,dx = \frac{2\left(6\,e^{(4i\,x)}-3\,e^{(2i\,x)}+1\right)}{3\left(e^{(6i\,x)}-3\,e^{(4i\,x)}+3\,e^{(2i\,x)}-1\right)}$$

```
[In] integrate(csc(x)^4/(I+tan(x)),x, algorithm="fricas")
[Out] 2/3*(6*e^(4*I*x) - 3*e^(2*I*x) + 1)/(e^(6*I*x) - 3*e^(4*I*x) + 3*e^(2*I*x)
- 1)
```

Sympy [F]

$$\int \frac{\csc^4(x)}{i+\tan(x)}\,dx = \int \frac{\csc^4(x)}{\tan(x)+i}\,dx$$

```
[In] integrate(csc(x)**4/(I+tan(x)),x)
[Out] Integral(csc(x)**4/(tan(x) + I), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{\csc^4(x)}{i+\tan(x)}\,dx = -\frac{i\,(-3i\,\tan(x)-2)}{6\,\tan(x)^3}$$

```
[In] integrate(csc(x)^4/(I+tan(x)),x, algorithm="maxima")
[Out] -1/6*I*(-3*I*tan(x) - 2)/tan(x)^3
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{\csc^4(x)}{i + \tan(x)}\,dx = -\frac{3\,\tan(x) - 2i}{6\,\tan(x)^3}$$

```
[In] integrate(csc(x)^4/(I+tan(x)),x, algorithm="giac")
[Out] -1/6*(3*tan(x) - 2*I)/tan(x)^3
```

Mupad [B] (verification not implemented)

Time = 4.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{\csc^4(x)}{i + \tan(x)}\,dx = \frac{\cot(x)^2\,(-3 + \cot(x)\,2\mathrm{i})}{6}$$

```
[In] int(1/(sin(x)^4*(tan(x) + 1i)),x)
[Out] (cot(x)^2*(cot(x)*2i - 3))/6
```

3.9 $\int \frac{\csc^5(x)}{i+\tan(x)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{\csc^5(x)}{i+\tan(x)} \, dx = -\frac{1}{8} i \operatorname{arctanh}(\cos(x)) - \frac{1}{8} i \cot(x) \csc(x) - \frac{\csc^3(x)}{3} + \frac{1}{4} i \cot(x) \csc^3(x)$$

```
[Out] -1/8*I*arctanh(cos(x))-1/8*I*cot(x)*csc(x)-1/3*csc(x)^3+1/4*I*cot(x)*csc(x)
^3
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {3599, 3187, 3186, 2686, 30, 2691, 3853, 3855}

$$\int \frac{\csc^5(x)}{i+\tan(x)} \, dx = -\frac{1}{8} i \operatorname{arctanh}(\cos(x)) - \frac{\csc^3(x)}{3} + \frac{1}{4} i \cot(x) \csc^3(x) - \frac{1}{8} i \cot(x) \csc(x)$$

```
[In] Int[Csc[x]^5/(I + Tan[x]),x]
```

```
[Out] (-1/8*I)*ArcTanh[Cos[x]] - (I/8)*Cot[x]*Csc[x] - Csc[x]^3/3 + (I/4)*Cot[x]*
Csc[x]^3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
```

```
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3186

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 3187

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cot(x)\csc^4(x)}{i\cos(x)+\sin(x)}\,dx \\
&= -\left(i\int \cot(x)\csc^4(x)(\cos(x)+i\sin(x))\,dx\right) \\
&= -\left(i\int \left(i\cot(x)\csc^3(x)+\cot^2(x)\csc^3(x)\right)\,dx\right) \\
&= -\left(i\int \cot^2(x)\csc^3(x)\,dx\right)+\int \cot(x)\csc^3(x)\,dx \\
&= \frac{1}{4}i\cot(x)\csc^3(x)+\frac{1}{4}i\int \csc^3(x)\,dx-\text{Subst}\left(\int x^2\,dx,x,\csc(x)\right) \\
&= -\frac{1}{8}i\cot(x)\csc(x)-\frac{\csc^3(x)}{3}+\frac{1}{4}i\cot(x)\csc^3(x)+\frac{1}{8}i\int \csc(x)\,dx \\
&= -\frac{1}{8}i\text{arctanh}(\cos(x))-\frac{1}{8}i\cot(x)\csc(x)-\frac{\csc^3(x)}{3}+\frac{1}{4}i\cot(x)\csc^3(x)
\end{aligned}
$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 139 vs. 2(40) = 80.

Time = 0.04 (sec) , antiderivative size = 139, normalized size of antiderivative = 3.48

$$
\begin{aligned}
\int \frac{\csc^5(x)}{i+\tan(x)}\,dx = &-\frac{1}{12}\cot\left(\frac{x}{2}\right)-\frac{1}{32}i\csc^2\left(\frac{x}{2}\right)-\frac{1}{24}\cot\left(\frac{x}{2}\right)\csc^2\left(\frac{x}{2}\right)+\frac{1}{64}i\csc^4\left(\frac{x}{2}\right) \\
&-\frac{1}{8}i\log\left(\cos\left(\frac{x}{2}\right)\right)+\frac{1}{8}i\log\left(\sin\left(\frac{x}{2}\right)\right)+\frac{1}{32}i\sec^2\left(\frac{x}{2}\right) \\
&-\frac{1}{64}i\sec^4\left(\frac{x}{2}\right)-\frac{1}{12}\tan\left(\frac{x}{2}\right)-\frac{1}{24}\sec^2\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}\right)
\end{aligned}
$$

```
[In] Integrate[Csc[x]^5/(I + Tan[x]),x]
```

```
[Out] -1/12*Cot[x/2] - (I/32)*Csc[x/2]^2 - (Cot[x/2]*Csc[x/2]^2)/24 + (I/64)*Csc[
x/2]^4 - (I/8)*Log[Cos[x/2]] + (I/8)*Log[Sin[x/2]] + (I/32)*Sec[x/2]^2 - (I
/64)*Sec[x/2]^4 - Tan[x/2]/12 - (Sec[x/2]^2*Tan[x/2])/24
```

Maple [A] (verified)

Time = 131.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

method	result	size
default	$-\frac{\tan\left(\frac{x}{2}\right)}{8}-\frac{i\left(\tan^4\left(\frac{x}{2}\right)\right)}{64}-\frac{\left(\tan^3\left(\frac{x}{2}\right)\right)}{24}+\frac{i}{64\tan\left(\frac{x}{2}\right)^4}-\frac{1}{24\tan\left(\frac{x}{2}\right)^3}+\frac{i\ln\left(\tan\left(\frac{x}{2}\right)\right)}{8}-\frac{1}{8\tan\left(\frac{x}{2}\right)}$	58
risch	$\frac{i\left(3\,\mathrm{e}^{7ix}+53\,\mathrm{e}^{5ix}-11\,\mathrm{e}^{3ix}+3\,\mathrm{e}^{ix}\right)}{12\left(\mathrm{e}^{2ix}-1\right)^4}+\frac{i\ln\left(\mathrm{e}^{ix}-1\right)}{8}-\frac{i\ln\left(\mathrm{e}^{ix}+1\right)}{8}$	65

```
[In] int(csc(x)^5/(I+tan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*tan(1/2*x)-1/64*I*tan(1/2*x)^4-1/24*tan(1/2*x)^3+1/64*I/tan(1/2*x)^4-1
/24/tan(1/2*x)^3+1/8*I*ln(tan(1/2*x))-1/8/tan(1/2*x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(26) = 52.

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 3.08

$$\int \frac{\csc^5(x)}{i+\tan(x)}\,dx = -\frac{3\left(i\,e^{(8i\,x)}-4i\,e^{(6i\,x)}+6i\,e^{(4i\,x)}-4i\,e^{(2i\,x)}+i\right)\log\left(e^{(i\,x)}+1\right)+3\left(-i\,e^{(8i\,x)}+4i\,e^{(6i\,x)}-6i\,e^{(4i\,x)}+4i\,e^{(2}\right.}{24\left(e^{(8i\,x)}-4\,e^{(6i\,x)}+6\,e^{(4i\,x)}-4\,e^{(2i\,x)}\right.}$$

```
[In] integrate(csc(x)^5/(I+tan(x)),x, algorithm="fricas")
```

```
[Out] -1/24*(3*(I*e^(8*I*x) - 4*I*e^(6*I*x) + 6*I*e^(4*I*x) - 4*I*e^(2*I*x) + I)*
log(e^(I*x) + 1) + 3*(-I*e^(8*I*x) + 4*I*e^(6*I*x) - 6*I*e^(4*I*x) + 4*I*e^
(2*I*x) - I)*log(e^(I*x) - 1) - 6*I*e^(7*I*x) - 106*I*e^(5*I*x) + 22*I*e^(3
*I*x) - 6*I*e^(I*x))/(e^(8*I*x) - 4*e^(6*I*x) + 6*e^(4*I*x) - 4*e^(2*I*x) +
 1)
```

Sympy [F]

$$\int \frac{\csc^5(x)}{i+\tan(x)}\,dx = \int \frac{\csc^5(x)}{\tan(x)+i}\,dx$$

```
[In] integrate(csc(x)**5/(I+tan(x)),x)
```

```
[Out] Integral(csc(x)**5/(tan(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(26) = 52$.

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.08

$$\int \frac{\csc^5(x)}{i+\tan(x)}\,dx = -\frac{\left(\frac{8\sin(x)}{\cos(x)+1}+\frac{24\sin(x)^3}{(\cos(x)+1)^3}-3i\right)(\cos(x)+1)^4}{192\sin(x)^4} - \frac{\sin(x)}{8(\cos(x)+1)} - \frac{\sin(x)^3}{24(\cos(x)+1)^3} - \frac{i\sin(x)^4}{64(\cos(x)+1)^4} + \frac{1}{8}i\log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

```
[In] integrate(csc(x)^5/(I+tan(x)),x, algorithm="maxima")
```

```
[Out] -1/192*(8*sin(x)/(cos(x) + 1) + 24*sin(x)^3/(cos(x) + 1)^3 - 3*I)*(cos(x) +
 1)^4/sin(x)^4 - 1/8*sin(x)/(cos(x) + 1) - 1/24*sin(x)^3/(cos(x) + 1)^3 - 1
/64*I*sin(x)^4/(cos(x) + 1)^4 + 1/8*I*log(sin(x)/(cos(x) + 1))
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(26) = 52$.

Time = 0.33 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.55

$$\int \frac{\csc^5(x)}{i+\tan(x)}\,dx = -\frac{1}{64}i\tan\left(\frac{1}{2}x\right)^4 - \frac{1}{24}\tan\left(\frac{1}{2}x\right)^3 - \frac{50i\tan\left(\frac{1}{2}x\right)^4+24\tan\left(\frac{1}{2}x\right)^3+8\tan\left(\frac{1}{2}x\right)-3i}{192\tan\left(\frac{1}{2}x\right)^4} + \frac{1}{8}i\log\left(\tan\left(\frac{1}{2}x\right)\right) - \frac{1}{8}\tan\left(\frac{1}{2}x\right)$$

```
[In] integrate(csc(x)^5/(I+tan(x)),x, algorithm="giac")
```

```
[Out] -1/64*I*tan(1/2*x)^4 - 1/24*tan(1/2*x)^3 - 1/192*(50*I*tan(1/2*x)^4 + 24*ta
n(1/2*x)^3 + 8*tan(1/2*x) - 3*I)/tan(1/2*x)^4 + 1/8*I*log(tan(1/2*x)) - 1/8
*tan(1/2*x)
```

Mupad [B] (verification not implemented)

Time = 5.00 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.42

$$\int \frac{\csc^5(x)}{i+\tan(x)}\,dx = -\frac{\tan\left(\frac{x}{2}\right)}{8} + \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)\ 1\mathrm{i}}{8} - \frac{2\tan\left(\frac{x}{2}\right)^3 + \frac{2\tan\left(\frac{x}{2}\right)}{3} - \frac{1}{4}\mathrm{i}}{16\tan\left(\frac{x}{2}\right)^4} - \frac{\tan\left(\frac{x}{2}\right)^3}{24} - \frac{\tan\left(\frac{x}{2}\right)^4\ 1\mathrm{i}}{64}$$

```
[In] int(1/(sin(x)^5*(tan(x) + 1i)),x)
```

```
[Out] (log(tan(x/2))*1i)/8 - tan(x/2)/8 - ((2*tan(x/2))/3 + 2*tan(x/2)^3 - 1i/4)/
(16*tan(x/2)^4) - tan(x/2)^3/24 - (tan(x/2)^4*1i)/64
```

3.10 $\int \frac{\csc^6(x)}{i+\tan(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 37

$$\int \frac{\csc^6(x)}{i+\tan(x)}\,dx = -\frac{1}{2}\cot^2(x)+\frac{1}{3}i\cot^3(x)-\frac{\cot^4(x)}{4}+\frac{1}{5}i\cot^5(x)$$

```
[Out] -1/2*cot(x)^2+1/3*I*cot(x)^3-1/4*cot(x)^4+1/5*I*cot(x)^5
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3597, 862, 76}

$$\int \frac{\csc^6(x)}{i+\tan(x)}\,dx = \frac{1}{5}i\cot^5(x)-\frac{\cot^4(x)}{4}+\frac{1}{3}i\cot^3(x)-\frac{\cot^2(x)}{2}$$

```
[In] Int[Csc[x]^6/(I + Tan[x]),x]
[Out] -1/2*Cot[x]^2 + (I/3)*Cot[x]^3 - Cot[x]^4/4 + (I/5)*Cot[x]^5
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
```

```
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{(1+x^2)^2}{x^6(i+x)}\,dx, x, \tan(x)\right)\\ &= \text{Subst}\left(\int \frac{(-i+x)^2(i+x)}{x^6}\,dx, x, \tan(x)\right)\\ &= \text{Subst}\left(\int \left(-\frac{i}{x^6}+\frac{1}{x^5}-\frac{i}{x^4}+\frac{1}{x^3}\right)dx, x, \tan(x)\right)\\ &= -\frac{1}{2}\cot^2(x)+\frac{1}{3}i\cot^3(x)-\frac{\cot^4(x)}{4}+\frac{1}{5}i\cot^5(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

$$\int \frac{\csc^6(x)}{i+\tan(x)}\,dx = -\frac{2}{15}i\cot(x)-\frac{1}{15}i\cot(x)\csc^2(x)-\frac{\csc^4(x)}{4}+\frac{1}{5}i\cot(x)\csc^4(x)$$

```
[In] Integrate[Csc[x]^6/(I + Tan[x]),x]
[Out] ((-2*I)/15)*Cot[x] - (I/15)*Cot[x]*Csc[x]^2 - Csc[x]^4/4 + (I/5)*Cot[x]*Csc
[x]^4
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

$$-\frac{1}{4\tan(x)^4}+\frac{i}{5\tan(x)^5}+\frac{i}{3\tan(x)^3}-\frac{1}{2\tan(x)^2}$$

```
[In] int(csc(x)^6/(I+tan(x)),x)
[Out] -1/4/tan(x)^4+1/5*I/tan(x)^5+1/3*I/tan(x)^3-1/2/tan(x)^2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(25) = 50$.

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.46

$$\int \frac{\csc^6(x)}{i+\tan(x)}\,dx = -\frac{4\left(30\,e^{(6i\,x)} - 10\,e^{(4i\,x)} + 5\,e^{(2i\,x)} - 1\right)}{15\,\left(e^{(10i\,x)} - 5\,e^{(8i\,x)} + 10\,e^{(6i\,x)} - 10\,e^{(4i\,x)} + 5\,e^{(2i\,x)} - 1\right)}$$

```
[In] integrate(csc(x)^6/(I+tan(x)),x, algorithm="fricas")
```

```
[Out] -4/15*(30*e^(6*I*x) - 10*e^(4*I*x) + 5*e^(2*I*x) - 1)/(e^(10*I*x) - 5*e^(8*
I*x) + 10*e^(6*I*x) - 10*e^(4*I*x) + 5*e^(2*I*x) - 1)
```

Sympy [F]

$$\int \frac{\csc^6(x)}{i+\tan(x)}\,dx = \int \frac{\csc^6(x)}{\tan(x)+i}\,dx$$

```
[In] integrate(csc(x)**6/(I+tan(x)),x)
```

```
[Out] Integral(csc(x)**6/(tan(x) + I), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.65

$$\int \frac{\csc^6(x)}{i+\tan(x)}\,dx = \frac{i\left(30i\,\tan(x)^3 + 20\,\tan(x)^2 + 15i\,\tan(x) + 12\right)}{60\,\tan(x)^5}$$

```
[In] integrate(csc(x)^6/(I+tan(x)),x, algorithm="maxima")
```

```
[Out] 1/60*I*(30*I*tan(x)^3 + 20*tan(x)^2 + 15*I*tan(x) + 12)/tan(x)^5
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.65

$$\int \frac{\csc^6(x)}{i+\tan(x)}\,dx = -\frac{30\,\tan\left(x\right)^{3} - 20i\,\tan\left(x\right)^{2} + 15\,\tan\left(x\right) - 12i}{60\,\tan\left(x\right)^{5}}$$

```
[In] integrate(csc(x)^6/(I+tan(x)),x, algorithm="giac")
[Out] -1/60*(30*tan(x)^3 - 20*I*tan(x)^2 + 15*tan(x) - 12*I)/tan(x)^5
```

Mupad [B] (verification not implemented)

Time = 4.41 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{\csc^6(x)}{i+\tan(x)}\,dx = \frac{\cot(x)^5\,1\mathrm{i}}{5} - \frac{\cot(x)^4}{4} + \frac{\cot(x)^3\,1\mathrm{i}}{3} - \frac{\cot(x)^2}{2}$$

```
[In] int(1/(sin(x)^6*(tan(x) + 1i)),x)
[Out] (cot(x)^3*1i)/3 - cot(x)^2/2 - cot(x)^4/4 + (cot(x)^5*1i)/5
```

3.11 $\int \sin^5(c+dx)(a+b\tan(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 101

$$\int \sin^5(c+dx)(a+b\tan(c+dx))\,dx = \frac{b\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{a\cos(c+dx)}{d} + \frac{2a\cos^3(c+dx)}{3d} - \frac{a\cos^5(c+dx)}{5d} - \frac{b\sin(c+dx)}{d} - \frac{b\sin^3(c+dx)}{3d} - \frac{b\sin^5(c+dx)}{5d}$$

```
[Out] b*arctanh(sin(d*x+c))/d-a*cos(d*x+c)/d+2/3*a*cos(d*x+c)^3/d-1/5*a*cos(d*x+c
)^5/d-b*sin(d*x+c)/d-1/3*b*sin(d*x+c)^3/d-1/5*b*sin(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3598, 2713, 2672, 308, 212}

$$\int \sin^5(c+dx)(a+b\tan(c+dx))\,dx = -\frac{a\cos^5(c+dx)}{5d} + \frac{2a\cos^3(c+dx)}{3d} - \frac{a\cos(c+dx)}{d} + \frac{b\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{b\sin^5(c+dx)}{5d} - \frac{b\sin^3(c+dx)}{3d} - \frac{b\sin(c+dx)}{d}$$

```
[In] Int[Sin[c + d*x]^5*(a + b*Tan[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Cos[c + d*x])/d + (2*a*Cos[c + d*x]^3)/(3*
d) - (a*Cos[c + d*x]^5)/(5*d) - (b*Sin[c + d*x])/d - (b*Sin[c + d*x]^3)/(3*
d) - (b*Sin[c + d*x]^5)/(5*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(a \sin^5(c+dx) + b \sin^5(c+dx) \tan(c+dx)\right) \, dx$$

$$= a \int \sin^5(c+dx) \, dx + b \int \sin^5(c+dx) \tan(c+dx) \, dx$$

$$= -\frac{a\text{Subst}\left(\int \left(1-2x^2+x^4\right) \, dx, x, \cos(c+dx)\right)}{d} + \frac{b\text{Subst}\left(\int \frac{x^6}{1-x^2} \, dx, x, \sin(c+dx)\right)}{d}$$

$$= -\frac{a\cos(c+dx)}{d} + \frac{2a\cos^3(c+dx)}{3d} - \frac{a\cos^5(c+dx)}{5d} + \frac{b\text{Subst}\left(\int \left(-1-x^2-x^4+\frac{1}{1-x^2}\right) \, dx, x, \sin(c+dx)\right)}{d}$$

$$
\begin{aligned}
&= -\frac{a\cos(c+dx)}{d}+\frac{2a\cos^3(c+dx)}{3d}-\frac{a\cos^5(c+dx)}{5d}-\frac{b\sin(c+dx)}{d} \\
&\quad -\frac{b\sin^3(c+dx)}{3d}-\frac{b\sin^5(c+dx)}{5d}+\frac{b\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\sin(c+dx)\right)}{d} \\
&= \frac{b\text{arctanh}(\sin(c+dx))}{d}-\frac{a\cos(c+dx)}{d}+\frac{2a\cos^3(c+dx)}{3d} \\
&\quad -\frac{a\cos^5(c+dx)}{5d}-\frac{b\sin(c+dx)}{d}-\frac{b\sin^3(c+dx)}{3d}-\frac{b\sin^5(c+dx)}{5d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.02

$$
\begin{aligned}
\int \sin^5(c+dx)(a+b\tan(c+dx))\,dx &= \frac{b\text{arctanh}(\sin(c+dx))}{d}-\frac{5a\cos(c+dx)}{8d} \\
&\quad +\frac{5a\cos(3(c+dx))}{48d}-\frac{a\cos(5(c+dx))}{80d} \\
&\quad -\frac{b\sin(c+dx)}{d}-\frac{b\sin^3(c+dx)}{3d}-\frac{b\sin^5(c+dx)}{5d}
\end{aligned}
$$

```
[In] Integrate[Sin[c + d*x]^5*(a + b*Tan[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (5*a*Cos[c + d*x])/(8*d) + (5*a*Cos[3*(c + d*
x)])/(48*d) - (a*Cos[5*(c + d*x)])/(80*d) - (b*Sin[c + d*x])/d - (b*Sin[c +
 d*x]^3)/(3*d) - (b*Sin[c + d*x]^5)/(5*d)
```

Maple [A] (verified)

Time = 3.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{-\frac{a\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}+b\left(-\frac{\left(\sin^5(dx+c)\right)}{5}-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)}{d}$
default	$\frac{-\frac{a\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}+b\left(-\frac{\left(\sin^5(dx+c)\right)}{5}-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)}{d}$
risch	$\frac{11i\,e^{i(dx+c)}b}{16d}-\frac{5\,e^{i(dx+c)}a}{16d}-\frac{11i\,e^{-i(dx+c)}b}{16d}-\frac{5\,e^{-i(dx+c)}a}{16d}+\frac{b\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{b\ln\left(e^{i(dx+c)}-i\right)}{d}-\frac{a\cos(5dx}{80d}$

```
[In] int(sin(d*x+c)^5*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/5*a*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)+b*(-1/5*sin(d*x+
c)^5-1/3*sin(d*x+c)^3-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96

$$\int \sin^5(c+dx)(a+b\tan(c+dx))\,dx = -\frac{6\,a\cos(dx+c)^5 - 20\,a\cos(dx+c)^3 + 30\,a\cos(dx+c) - 15\,b\log(\sin(dx+c)+1) + 15\,b\log(-\sin(d\ldots}{30\,d}$$

```
[In] integrate(sin(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/30*(6*a*cos(d*x + c)^5 - 20*a*cos(d*x + c)^3 + 30*a*cos(d*x + c) - 15*b*
log(sin(d*x + c) + 1) + 15*b*log(-sin(d*x + c) + 1) + 2*(3*b*cos(d*x + c)^4
 - 11*b*cos(d*x + c)^2 + 23*b)*sin(d*x + c))/d
```

Sympy [F]

$$\int \sin^5(c+dx)(a+b\tan(c+dx))\,dx = \int (a+b\tan(c+dx))\sin^5(c+dx)\,dx$$

```
[In] integrate(sin(d*x+c)**5*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*sin(c + d*x)**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.90

$$\int \sin^5(c+dx)(a+b\tan(c+dx))\,dx = -\frac{2\left(3\cos(dx+c)^5 - 10\cos(dx+c)^3 + 15\cos(dx+c)\right)a + \left(6\sin(dx+c)^5 + 10\sin(dx+c)^3 - 15\log\ldots}{30\,d}$$

```
[In] integrate(sin(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/30*(2*(3*cos(d*x + c)^5 - 10*cos(d*x + c)^3 + 15*cos(d*x + c))*a + (6*si
n(d*x + c)^5 + 10*sin(d*x + c)^3 - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*
x + c) - 1) + 30*sin(d*x + c))*b)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10412 vs. 2(93) = 186.

Time = 1.73 (sec) , antiderivative size = 10412, normalized size of antiderivative = 103.09

$$\int \sin^5(c+dx)(a+b\tan(c+dx))\,dx = \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/30*(15*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c
) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2
*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + t
an(1/2*c)^2 + 1))*tan(1/2*d*x)^10*tan(1/2*c)^10 - 15*b*log(2*(tan(1/2*d*x)^
2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2
+ tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1
/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^1
0*tan(1/2*c)^10 + 16*a*tan(1/2*d*x)^10*tan(1/2*c)^10 + 75*b*log(2*(tan(1/2*
d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*
c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(
tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d
*x)^10*tan(1/2*c)^8 - 75*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d
*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c
)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan
(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^10*tan(1/2*c)^8 - 60*b*tan(1/
2*d*x)^10*tan(1/2*c)^9 + 75*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/
2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/
2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 +
tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^10 - 75*b*log
(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d
*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1
/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1
))*tan(1/2*d*x)^8*tan(1/2*c)^10 - 60*b*tan(1/2*d*x)^9*tan(1/2*c)^10 + 80*a*
tan(1/2*d*x)^10*tan(1/2*c)^8 + 80*a*tan(1/2*d*x)^8*tan(1/2*c)^10 + 150*b*lo
g(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*
d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(
1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 +
1))*tan(1/2*d*x)^10*tan(1/2*c)^6 - 150*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2
 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)
^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(
1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^10*tan(1/2*c)^6
 - 320*b*tan(1/2*d*x)^10*tan(1/2*c)^7 + 375*b*log(2*(tan(1/2*d*x)^2*tan(1/2
*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2
*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2
*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*
```

```
c)^8 - 375*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*
c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/
2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 - 420*b*tan(1/2*d*x)^9*tan(1
/2*c)^8 - 420*b*tan(1/2*d*x)^8*tan(1/2*c)^9 + 150*b*log(2*(tan(1/2*d*x)^2*t
an(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + t
an(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*
d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*ta
n(1/2*c)^10 - 150*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*t
an(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2
*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*
x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^10 - 320*b*tan(1/2*d*x)
^7*tan(1/2*c)^10 + 160*a*tan(1/2*d*x)^10*tan(1/2*c)^6 + 400*a*tan(1/2*d*x)^
8*tan(1/2*c)^8 + 160*a*tan(1/2*d*x)^6*tan(1/2*c)^10 + 150*b*log(2*(tan(1/2*
d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*
c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(
tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d
*x)^10*tan(1/2*c)^4 - 150*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*
d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*
c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + ta
n(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^10*tan(1/2*c)^4 - 712*b*tan(
1/2*d*x)^10*tan(1/2*c)^5 + 750*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan
(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan
(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2
 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^6 - 750*b*
log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/
2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*ta
n(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2
+ 1))*tan(1/2*d*x)^8*tan(1/2*c)^6 - 1320*b*tan(1/2*d*x)^9*tan(1/2*c)^6 - 25
60*b*tan(1/2*d*x)^8*tan(1/2*c)^7 + 750*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2
 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)
^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(
1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^8
- 750*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) -
2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x
) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1
/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^8 - 2560*b*tan(1/2*d*x)^7*tan(1/2*c
)^8 - 1320*b*tan(1/2*d*x)^6*tan(1/2*c)^9 + 150*b*log(2*(tan(1/2*d*x)^2*tan(
1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(
1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x
)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1
/2*c)^10 - 150*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(
1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*ta
n(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^
2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^10 - 712*b*tan(1/2*d*x)^5*
```

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tan(1/2*c)^10 - 160*a*tan(1/2*d*x)^10*tan(1/2*c)^4 - 1920*a*tan(1/2*d*x)^9*
tan(1/2*c)^5 - 4000*a*tan(1/2*d*x)^8*tan(1/2*c)^6 - 6400*a*tan(1/2*d*x)^7*t
an(1/2*c)^7 - 4000*a*tan(1/2*d*x)^6*tan(1/2*c)^8 - 1920*a*tan(1/2*d*x)^5*ta
n(1/2*c)^9 - 160*a*tan(1/2*d*x)^4*tan(1/2*c)^10 + 75*b*log(2*(tan(1/2*d*x)^
2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2
+ tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1
/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^1
0*tan(1/2*c)^2 - 75*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2
*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 +
 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*
d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^10*tan(1/2*c)^2 - 320*b*tan(1/2*d*
x)^10*tan(1/2*c)^3 + 750*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d
*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c
)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan
(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^4 - 750*b*log(2*
(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)
*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*
c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*
tan(1/2*d*x)^8*tan(1/2*c)^4 + 1320*b*tan(1/2*d*x)^9*tan(1/2*c)^4 + 4360*b*t
an(1/2*d*x)^8*tan(1/2*c)^5 + 1500*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*
tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 +
tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c
)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 - 150
0*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*ta
n(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) +
2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c
)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 + 9280*b*tan(1/2*d*x)^7*tan(1/2*c)^6
+ 9280*b*tan(1/2*d*x)^6*tan(1/2*c)^7 + 750*b*log(2*(tan(1/2*d*x)^2*tan(1/2*
c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*
d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*
tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c
)^8 - 750*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c
) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2
*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + t
an(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^8 + 4360*b*tan(1/2*d*x)^5*tan(1
/2*c)^8 + 1320*b*tan(1/2*d*x)^4*tan(1/2*c)^9 + 75*b*log(2*(tan(1/2*d*x)^2*t
an(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + t
an(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*
d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*ta
n(1/2*c)^10 - 75*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*ta
n(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*
tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x
)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^10 - 320*b*tan(1/2*d*x)^
3*tan(1/2*c)^10 - 80*a*tan(1/2*d*x)^10*tan(1/2*c)^2 + 4000*a*tan(1/2*d*x)^8
*tan(1/2*c)^4 + 10240*a*tan(1/2*d*x)^7*tan(1/2*c)^5 + 16000*a*tan(1/2*d*x)^
```

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6*tan(1/2*c)^6 + 10240*a*tan(1/2*d*x)^5*tan(1/2*c)^7 + 4000*a*tan(1/2*d*x)^
4*tan(1/2*c)^8 - 80*a*tan(1/2*d*x)^2*tan(1/2*c)^10 + 15*b*log(2*(tan(1/2*d*
x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)
^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(ta
n(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x
)^10 - 15*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c
) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2
*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + t
an(1/2*c)^2 + 1))*tan(1/2*d*x)^10 - 60*b*tan(1/2*d*x)^10*tan(1/2*c) + 375*b
*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1
/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*t
an(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2
 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^2 - 375*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)
^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*
x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*ta
n(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^
2 + 420*b*tan(1/2*d*x)^9*tan(1/2*c)^2 - 2560*b*tan(1/2*d*x)^8*tan(1/2*c)^3
+ 1500*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) +
 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*
x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(
1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 - 1500*b*log(2*(tan(1/2*d*x)^2*t
an(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + t
an(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*
d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*ta
n(1/2*c)^4 - 9280*b*tan(1/2*d*x)^7*tan(1/2*c)^4 - 19120*b*tan(1/2*d*x)^6*ta
n(1/2*c)^5 + 1500*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*t
an(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2
*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*
x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^6 - 1500*b*log(2*(tan(1
/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1
/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1
)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/
2*d*x)^4*tan(1/2*c)^6 - 19120*b*tan(1/2*d*x)^5*tan(1/2*c)^6 - 9280*b*tan(1/
2*d*x)^4*tan(1/2*c)^7 + 375*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/
2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/
2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 +
tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^8 - 375*b*log
(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d
*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1
/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1
))*tan(1/2*d*x)^2*tan(1/2*c)^8 - 2560*b*tan(1/2*d*x)^3*tan(1/2*c)^8 + 420*b
*tan(1/2*d*x)^2*tan(1/2*c)^9 + 15*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*
tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 +
tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c
)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^10 - 15*b*log(2*(tan(1
```

```
/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1
/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1
)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/
2*c)^10 - 60*b*tan(1/2*d*x)*tan(1/2*c)^10 - 16*a*tan(1/2*d*x)^10 - 400*a*ta
n(1/2*d*x)^8*tan(1/2*c)^2 - 6400*a*tan(1/2*d*x)^7*tan(1/2*c)^3 - 16000*a*ta
n(1/2*d*x)^6*tan(1/2*c)^4 - 24064*a*tan(1/2*d*x)^5*tan(1/2*c)^5 - 16000*a*t
an(1/2*d*x)^4*tan(1/2*c)^6 - 6400*a*tan(1/2*d*x)^3*tan(1/2*c)^7 - 400*a*tan
(1/2*d*x)^2*tan(1/2*c)^8 - 16*a*tan(1/2*c)^10 + 75*b*log(2*(tan(1/2*d*x)^2*
tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 +
tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2
*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8 -
 75*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*
tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x)
 + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2
*c)^2 + 1))*tan(1/2*d*x)^8 + 60*b*tan(1/2*d*x)^9 - 420*b*tan(1/2*d*x)^8*tan
(1/2*c) + 750*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1
/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan
(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2
 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 - 750*b*log(2*(tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)
^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(ta
n(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x
)^6*tan(1/2*c)^2 + 2560*b*tan(1/2*d*x)^7*tan(1/2*c)^2 + 9280*b*tan(1/2*d*x)
^6*tan(1/2*c)^3 + 1500*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x
)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^
2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1
/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 - 1500*b*log(2*(
tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*
tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c
) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*t
an(1/2*d*x)^4*tan(1/2*c)^4 + 19120*b*tan(1/2*d*x)^5*tan(1/2*c)^4 + 19120*b*
tan(1/2*d*x)^4*tan(1/2*c)^5 + 750*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*
tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 +
tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c
)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 - 750
*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan
(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2
*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)
^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 + 9280*b*tan(1/2*d*x)^3*tan(1/2*c)^6 +
 2560*b*tan(1/2*d*x)^2*tan(1/2*c)^7 + 75*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)
^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*
x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*ta
n(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^8 - 75*b*log(2*
(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)
*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*
```

```
c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*
tan(1/2*c)^8 - 420*b*tan(1/2*d*x)*tan(1/2*c)^8 + 60*b*tan(1/2*c)^9 - 80*a*t
an(1/2*d*x)^8 + 4000*a*tan(1/2*d*x)^6*tan(1/2*c)^2 + 10240*a*tan(1/2*d*x)^5
*tan(1/2*c)^3 + 16000*a*tan(1/2*d*x)^4*tan(1/2*c)^4 + 10240*a*tan(1/2*d*x)^
3*tan(1/2*c)^5 + 4000*a*tan(1/2*d*x)^2*tan(1/2*c)^6 - 80*a*tan(1/2*c)^8 + 1
50*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*t
an(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) -
 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*
c)^2 + 1))*tan(1/2*d*x)^6 - 150*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*ta
n(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + ta
n(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^
2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6 + 320*b*tan(1/2*d*x)
^7 - 1320*b*tan(1/2*d*x)^6*tan(1/2*c) + 750*b*log(2*(tan(1/2*d*x)^2*tan(1/2
*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2
*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2
*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*
c)^2 - 750*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*
c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/
2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^2 - 4360*b*tan(1/2*d*x)^5*tan(
1/2*c)^2 - 9280*b*tan(1/2*d*x)^4*tan(1/2*c)^3 + 750*b*log(2*(tan(1/2*d*x)^2
*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 +
 tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/
2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*
tan(1/2*c)^4 - 750*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*
tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 +
2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d
*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^4 - 9280*b*tan(1/2*d*x
)^3*tan(1/2*c)^4 - 4360*b*tan(1/2*d*x)^2*tan(1/2*c)^5 + 150*b*log(2*(tan(1/
2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/
2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)
/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2
*c)^6 - 150*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2
*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1
/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 +
 tan(1/2*c)^2 + 1))*tan(1/2*c)^6 - 1320*b*tan(1/2*d*x)*tan(1/2*c)^6 + 320*b
*tan(1/2*c)^7 - 160*a*tan(1/2*d*x)^6 - 1920*a*tan(1/2*d*x)^5*tan(1/2*c) - 4
000*a*tan(1/2*d*x)^4*tan(1/2*c)^2 - 6400*a*tan(1/2*d*x)^3*tan(1/2*c)^3 - 40
00*a*tan(1/2*d*x)^2*tan(1/2*c)^4 - 1920*a*tan(1/2*d*x)*tan(1/2*c)^5 - 160*a
*tan(1/2*c)^6 + 150*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2
*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 -
 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*
d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4 - 150*b*log(2*(tan(1/2*d*x)^2*ta
n(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + ta
n(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d
```

```
*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4 + 7
12*b*tan(1/2*d*x)^5 + 1320*b*tan(1/2*d*x)^4*tan(1/2*c) + 375*b*log(2*(tan(1
/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1
/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1
)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/
2*d*x)^2*tan(1/2*c)^2 - 375*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/
2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/
2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 +
tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2560*b*ta
n(1/2*d*x)^3*tan(1/2*c)^2 + 2560*b*tan(1/2*d*x)^2*tan(1/2*c)^3 + 150*b*log(
2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*
x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/
2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1)
)*tan(1/2*c)^4 - 150*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^
2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2
+ 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2
*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^4 + 1320*b*tan(1/2*d*x)*tan(1/2*c)^
4 + 712*b*tan(1/2*c)^5 + 160*a*tan(1/2*d*x)^4 + 400*a*tan(1/2*d*x)^2*tan(1/
2*c)^2 + 160*a*tan(1/2*c)^4 + 75*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*t
an(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + t
an(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)
^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2 - 75*b*log(2*(tan(1
/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1
/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1
)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/
2*d*x)^2 + 320*b*tan(1/2*d*x)^3 + 420*b*tan(1/2*d*x)^2*tan(1/2*c) + 75*b*lo
g(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*
d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(
1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 +
1))*tan(1/2*c)^2 - 75*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)
^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2
 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/
2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^2 + 420*b*tan(1/2*d*x)*tan(1/2*c)^
2 + 320*b*tan(1/2*c)^3 + 80*a*tan(1/2*d*x)^2 + 80*a*tan(1/2*c)^2 + 15*b*log
(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d
*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1
/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1
)) - 15*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c)
- 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d
*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan
(1/2*c)^2 + 1)) + 60*b*tan(1/2*d*x) + 60*b*tan(1/2*c) + 16*a)/(d*tan(1/2*d*
x)^10*tan(1/2*c)^10 + 5*d*tan(1/2*d*x)^10*tan(1/2*c)^8 + 5*d*tan(1/2*d*x)^8
*tan(1/2*c)^10 + 10*d*tan(1/2*d*x)^10*tan(1/2*c)^6 + 25*d*tan(1/2*d*x)^8*ta
n(1/2*c)^8 + 10*d*tan(1/2*d*x)^6*tan(1/2*c)^10 + 10*d*tan(1/2*d*x)^10*tan(1
/2*c)^4 + 50*d*tan(1/2*d*x)^8*tan(1/2*c)^6 + 50*d*tan(1/2*d*x)^6*tan(1/2*c)
```

```
^8 + 10*d*tan(1/2*d*x)^4*tan(1/2*c)^10 + 5*d*tan(1/2*d*x)^10*tan(1/2*c)^2 +
 50*d*tan(1/2*d*x)^8*tan(1/2*c)^4 + 100*d*tan(1/2*d*x)^6*tan(1/2*c)^6 + 50*
d*tan(1/2*d*x)^4*tan(1/2*c)^8 + 5*d*tan(1/2*d*x)^2*tan(1/2*c)^10 + d*tan(1/
2*d*x)^10 + 25*d*tan(1/2*d*x)^8*tan(1/2*c)^2 + 100*d*tan(1/2*d*x)^6*tan(1/2
*c)^4 + 100*d*tan(1/2*d*x)^4*tan(1/2*c)^6 + 25*d*tan(1/2*d*x)^2*tan(1/2*c)^
8 + d*tan(1/2*c)^10 + 5*d*tan(1/2*d*x)^8 + 50*d*tan(1/2*d*x)^6*tan(1/2*c)^2
 + 100*d*tan(1/2*d*x)^4*tan(1/2*c)^4 + 50*d*tan(1/2*d*x)^2*tan(1/2*c)^6 + 5
*d*tan(1/2*c)^8 + 10*d*tan(1/2*d*x)^6 + 50*d*tan(1/2*d*x)^4*tan(1/2*c)^2 +
50*d*tan(1/2*d*x)^2*tan(1/2*c)^4 + 10*d*tan(1/2*c)^6 + 10*d*tan(1/2*d*x)^4
+ 25*d*tan(1/2*d*x)^2*tan(1/2*c)^2 + 10*d*tan(1/2*c)^4 + 5*d*tan(1/2*d*x)^2
 + 5*d*tan(1/2*c)^2 + d)
```

Mupad [B] (verification not implemented)

Time = 5.22 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.20

$$\int \sin^5(c+dx)(a+b\tan(c+dx))\,dx = \frac{2\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,a\cos\left(c+d\,x\right)^3}{3\,d}-\frac{a\cos\left(c+d\,x\right)^5}{5\,d}-\frac{a\,\cos\left(c+d\,x\right)}{d}-\frac{23\,b\,\sin\left(c+d\,x\right)}{15\,d}+\frac{11\,b\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{15\,d}-\frac{b\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)}{5\,d}$$

```
[In] int(sin(c + d*x)^5*(a + b*tan(c + d*x)),x)
```

```
[Out] (2*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*a*cos(c + d*x)^3)
/(3*d) - (a*cos(c + d*x)^5)/(5*d) - (a*cos(c + d*x))/d - (23*b*sin(c + d*x)
)/(15*d) + (11*b*cos(c + d*x)^2*sin(c + d*x))/(15*d) - (b*cos(c + d*x)^4*si
n(c + d*x))/(5*d)
```

3.12 $\int \sin^4(c+dx)(a+b\tan(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 83

$$\int \sin^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{3ax}{8}-\frac{b\log(\cos(c+dx))}{d}-\frac{\cos(c+dx)\sin^3(c+dx)(a+b\tan(c+dx))}{4d}-\frac{\cos(c+dx)\sin(c+dx)(3a+4b\tan(c+dx))}{8d}$$

```
[Out] 3/8*a*x-b*ln(cos(d*x+c))/d-1/4*cos(d*x+c)*sin(d*x+c)^3*(a+b*tan(d*x+c))/d-1
/8*cos(d*x+c)*sin(d*x+c)*(3*a+4*b*tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {833, 649, 209, 266}

$$\int \sin^4(c+dx)(a+b\tan(c+dx))\,dx = -\frac{\sin^3(c+dx)\cos(c+dx)(a+b\tan(c+dx))}{4d}-\frac{\sin(c+dx)\cos(c+dx)(3a+4b\tan(c+dx))}{8d}+\frac{3ax}{8}-\frac{b\log(\cos(c+dx))}{d}$$

```
[In] Int[Sin[c + d*x]^4*(a + b*Tan[c + d*x]),x]
```

```
[Out] (3*a*x)/8 - (b*Log[Cos[c + d*x]])/d - (Cos[c + d*x]*Sin[c + d*x]^3*(a + b*T
an[c + d*x]))/(4*d) - (Cos[c + d*x]*Sin[c + d*x]*(3*a + 4*b*Tan[c + d*x]))/
(8*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^4(a+bx)}{(1+x^2)^3}\,dx, x, \tan(c+dx)\right)}{d} \\
&= -\frac{\cos(c+dx)\sin^3(c+dx)(a+b\tan(c+dx))}{4d} + \frac{\text{Subst}\left(\int \frac{x^2(3a+4bx)}{(1+x^2)^2}\,dx, x, \tan(c+dx)\right)}{4d} \\
&= -\frac{\cos(c+dx)\sin^3(c+dx)(a+b\tan(c+dx))}{4d} \\
&\quad - \frac{\cos(c+dx)\sin(c+dx)(3a+4b\tan(c+dx))}{8d} \\
&\quad + \frac{\text{Subst}\left(\int \frac{3a+8bx}{1+x^2}\,dx, x, \tan(c+dx)\right)}{8d}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{\cos(c+dx)\sin^3(c+dx)(a+b\tan(c+dx))}{4d} \\
&\quad -\frac{\cos(c+dx)\sin(c+dx)(3a+4b\tan(c+dx))}{8d} \\
&\quad +\frac{(3a)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(c+dx)\right)}{8d}+\frac{b\text{Subst}\left(\int \frac{x}{1+x^2}\,dx,x,\tan(c+dx)\right)}{d} \\
&= \frac{3ax}{8}-\frac{b\log(\cos(c+dx))}{d}-\frac{\cos(c+dx)\sin^3(c+dx)(a+b\tan(c+dx))}{4d} \\
&\quad -\frac{\cos(c+dx)\sin(c+dx)(3a+4b\tan(c+dx))}{8d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99

$$
\begin{aligned}
&\int \sin^4(c+dx)(a+b\tan(c+dx))\,dx \\
&= \frac{3a(c+dx)}{8d}-\frac{b\left(-\cos^2(c+dx)+\frac{1}{4}\cos^4(c+dx)+\log(\cos(c+dx))\right)}{d} \\
&\quad -\frac{a\sin(2(c+dx))}{4d}+\frac{a\sin(4(c+dx))}{32d}
\end{aligned}
$$

```
[In] Integrate[Sin[c + d*x]^4*(a + b*Tan[c + d*x]),x]
[Out] (3*a*(c + d*x))/(8*d) - (b*(-Cos[c + d*x]^2 + Cos[c + d*x]^4/4 + Log[Cos[c
+ d*x]]))/d - (a*Sin[2*(c + d*x)])/(4*d) + (a*Sin[4*(c + d*x)])/(32*d)
```

Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{a\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(-\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)}{d}$
default	$\frac{a\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(-\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)}{d}$
risch	$ibx+\frac{3ax}{8}+\frac{3\,\mathrm{e}^{2i(dx+c)}b}{16d}+\frac{i\mathrm{e}^{2i(dx+c)}a}{8d}+\frac{3\,\mathrm{e}^{-2i(dx+c)}b}{16d}-\frac{i\mathrm{e}^{-2i(dx+c)}a}{8d}+\frac{2ibc}{d}-\frac{b\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{d}-\frac{b\cos}{}$

```
[In] int(sin(d*x+c)^4*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)+b*(-1/
4*sin(d*x+c)^4-1/2*sin(d*x+c)^2-ln(cos(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.89

$$\int \sin^4(c+dx)(a+b\tan(c+dx))\,dx =$$
$$-\frac{2\,b\cos\left(dx+c\right)^4-3\,adx-8\,b\cos\left(dx+c\right)^2+8\,b\log\left(-\cos\left(dx+c\right)\right)-\left(2\,a\cos\left(dx+c\right)^3-5\,a\cos\left(dx\right.\right.}{8\,d}$$

```
[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/8*(2*b*cos(d*x + c)^4 - 3*a*d*x - 8*b*cos(d*x + c)^2 + 8*b*log(-cos(d*x
+ c)) - (2*a*cos(d*x + c)^3 - 5*a*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F]

$$\int \sin^4(c+dx)(a+b\tan(c+dx))\,dx = \int \left(a+b\tan\left(c+dx\right)\right)\sin^4\left(c+dx\right)\,dx$$

```
[In] integrate(sin(d*x+c)**4*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*sin(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

$$\int \sin^4(c+dx)(a+b\tan(c+dx))\,dx$$
$$=\frac{3\left(dx+c\right)a+4\,b\log\left(\tan\left(dx+c\right)^2+1\right)-\frac{5\,a\tan(dx+c)^3-8\,b\tan(dx+c)^2+3\,a\tan(dx+c)-6\,b}{\tan(dx+c)^4+2\,\tan(dx+c)^2+1}}{8\,d}$$

```
[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/8*(3*(d*x + c)*a + 4*b*log(tan(d*x + c)^2 + 1) - (5*a*tan(d*x + c)^3 - 8*
b*tan(d*x + c)^2 + 3*a*tan(d*x + c) - 6*b)/(tan(d*x + c)^4 + 2*tan(d*x + c)
^2 + 1))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 976 vs. $2(77) = 154$.

Time = 0.54 (sec) , antiderivative size = 976, normalized size of antiderivative = 11.76

$$\int \sin^4(c+dx)(a+b\tan(c+dx))\,dx = \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*(12*a*d*x*tan(d*x)^4*tan(c)^4 - 16*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*ta
n(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d
*x)^4*tan(c)^4 + 24*a*d*x*tan(d*x)^4*tan(c)^2 + 24*a*d*x*tan(d*x)^2*tan(c)^
4 + 11*b*tan(d*x)^4*tan(c)^4 - 32*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)
*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^4*
tan(c)^2 + 12*a*tan(d*x)^4*tan(c)^3 - 32*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*t
an(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(
d*x)^2*tan(c)^4 + 12*a*tan(d*x)^3*tan(c)^4 + 12*a*d*x*tan(d*x)^4 + 48*a*d*x
*tan(d*x)^2*tan(c)^2 + 6*b*tan(d*x)^4*tan(c)^2 - 32*b*tan(d*x)^3*tan(c)^3 +
 12*a*d*x*tan(c)^4 + 6*b*tan(d*x)^2*tan(c)^4 - 16*b*log(4*(tan(d*x)^2*tan(c
)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 +
 1))*tan(d*x)^4 + 20*a*tan(d*x)^4*tan(c) - 64*b*log(4*(tan(d*x)^2*tan(c)^2
- 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))
*tan(d*x)^2*tan(c)^2 + 24*a*tan(d*x)^3*tan(c)^2 + 24*a*tan(d*x)^2*tan(c)^3
- 16*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(
c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(c)^4 + 20*a*tan(d*x)*tan(c)^4 + 24*a
*d*x*tan(d*x)^2 - 13*b*tan(d*x)^4 - 64*b*tan(d*x)^3*tan(c) + 24*a*d*x*tan(c
)^2 - 36*b*tan(d*x)^2*tan(c)^2 - 64*b*tan(d*x)*tan(c)^3 - 13*b*tan(c)^4 - 3
2*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^
2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^2 - 20*a*tan(d*x)^3 - 24*a*tan(d*x
)^2*tan(c) - 32*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(
d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(c)^2 - 24*a*tan(d*x)*tan(
c)^2 - 20*a*tan(c)^3 + 12*a*d*x + 6*b*tan(d*x)^2 - 32*b*tan(d*x)*tan(c) + 6
*b*tan(c)^2 - 16*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan
(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1)) - 12*a*tan(d*x) - 12*a*tan(c
) + 11*b)/(d*tan(d*x)^4*tan(c)^4 + 2*d*tan(d*x)^4*tan(c)^2 + 2*d*tan(d*x)^2
*tan(c)^4 + d*tan(d*x)^4 + 4*d*tan(d*x)^2*tan(c)^2 + d*tan(c)^4 + 2*d*tan(d
*x)^2 + 2*d*tan(c)^2 + d)
```

Mupad [B] (verification not implemented)

Time = 4.45 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.87

$$\int \sin^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{3\,a\,x}{8} + \frac{b\,\ln\left(\tan(c+d\,x)^2+1\right)}{2\,d} + \frac{3\,b}{4\,d\,\left(\tan\left(c+d\,x\right)^4+2\tan\left(c+d\,x\right)^2+1\right)} - \frac{5\,a\tan(c+d\,x)^3}{8\,d\,\left(\tan\left(c+d\,x\right)^4+2\tan\left(c+d\,x\right)^2+1\right)} + \frac{b\tan(c+d\,x)^2}{d\,\left(\tan\left(c+d\,x\right)^4+2\tan\left(c+d\,x\right)^2+1\right)} - \frac{3\,a\tan(c+d\,x)}{8\,d\,\left(\tan\left(c+d\,x\right)^4+2\tan\left(c+d\,x\right)^2+1\right)}$$

```
[In] int(sin(c + d*x)^4*(a + b*tan(c + d*x)),x)
```

```
[Out] (3*a*x)/8 + (b*log(tan(c + d*x)^2 + 1))/(2*d) + (3*b)/(4*d*(2*tan(c + d*x)^
2 + tan(c + d*x)^4 + 1)) - (5*a*tan(c + d*x)^3)/(8*d*(2*tan(c + d*x)^2 + ta
n(c + d*x)^4 + 1)) + (b*tan(c + d*x)^2)/(d*(2*tan(c + d*x)^2 + tan(c + d*x)
^4 + 1)) - (3*a*tan(c + d*x))/(8*d*(2*tan(c + d*x)^2 + tan(c + d*x)^4 + 1))
```

3.13 $\int \sin^3(c+dx)(a+b\tan(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 69

$$\int \sin^3(c+dx)(a+b\tan(c+dx))\,dx = \frac{b\text{arctanh}(\sin(c+dx))}{d} - \frac{a\cos(c+dx)}{d} + \frac{a\cos^3(c+dx)}{3d} - \frac{b\sin(c+dx)}{d} - \frac{b\sin^3(c+dx)}{3d}$$

```
[Out] b*arctanh(sin(d*x+c))/d-a*cos(d*x+c)/d+1/3*a*cos(d*x+c)^3/d-b*sin(d*x+c)/d-
1/3*b*sin(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3598, 2713, 2672, 308, 212}

$$\int \sin^3(c+dx)(a+b\tan(c+dx))\,dx = \frac{a\cos^3(c+dx)}{3d} - \frac{a\cos(c+dx)}{d} + \frac{b\text{arctanh}(\sin(c+dx))}{d} - \frac{b\sin^3(c+dx)}{3d} - \frac{b\sin(c+dx)}{d}$$

```
[In] Int[Sin[c + d*x]^3*(a + b*Tan[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Cos[c + d*x])/d + (a*Cos[c + d*x]^3)/(3*d)
 - (b*Sin[c + d*x])/d - (b*Sin[c + d*x]^3)/(3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a\sin^3(c+dx)+b\sin^3(c+dx)\tan(c+dx)\right)\,dx\\
&= a\int \sin^3(c+dx)\,dx+b\int \sin^3(c+dx)\tan(c+dx)\,dx\\
&= -\frac{a\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,\cos(c+dx)\right)}{d}+\frac{b\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx,x,\sin(c+dx)\right)}{d}\\
&= -\frac{a\cos(c+dx)}{d}+\frac{a\cos^3(c+dx)}{3d}+\frac{b\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)\,dx,x,\sin(c+dx)\right)}{d}\\
&= -\frac{a\cos(c+dx)}{d}+\frac{a\cos^3(c+dx)}{3d}-\frac{b\sin(c+dx)}{d}\\
&\quad -\frac{b\sin^3(c+dx)}{3d}+\frac{b\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sin(c+dx)\right)}{d}
\end{aligned}$$

$$=\frac{b\operatorname{arctanh}(\sin(c+dx))}{d}-\frac{a\cos(c+dx)}{d}+\frac{a\cos^3(c+dx)}{3d}-\frac{b\sin(c+dx)}{d}-\frac{b\sin^3(c+dx)}{3d}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03

$$\int \sin^3(c+dx)(a+b\tan(c+dx))\,dx=\frac{b\operatorname{arctanh}(\sin(c+dx))}{d}-\frac{3a\cos(c+dx)}{4d}+\frac{a\cos(3(c+dx))}{12d}-\frac{b\sin(c+dx)}{d}-\frac{b\sin^3(c+dx)}{3d}$$

```
[In] Integrate[Sin[c + d*x]^3*(a + b*Tan[c + d*x]),x]
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (3*a*Cos[c + d*x])/(4*d) + (a*Cos[3*(c + d*x)
])/(12*d) - (b*Sin[c + d*x])/d - (b*Sin[c + d*x]^3)/(3*d)
```

Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{-\frac{a\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+b\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)}{d}$
default	$\frac{-\frac{a\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+b\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)}{d}$
risch	$\frac{5i e^{i(dx+c)}b}{8d}-\frac{3\,e^{i(dx+c)}a}{8d}-\frac{5i e^{-i(dx+c)}b}{8d}-\frac{3\,e^{-i(dx+c)}a}{8d}+\frac{b\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{b\ln\left(e^{i(dx+c)}-i\right)}{d}+\frac{a\cos(3dx+3}{12d}$

```
[In] int(sin(d*x+c)^3*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/3*a*(2+sin(d*x+c)^2)*cos(d*x+c)+b*(-1/3*sin(d*x+c)^3-sin(d*x+c)+ln(
sec(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07

$$\int \sin^3(c+dx)(a+b\tan(c+dx))\,dx$$
$$=\frac{2\,a\cos\left(dx+c\right)^3-6\,a\cos\left(dx+c\right)+3\,b\log\left(\sin\left(dx+c\right)+1\right)-3\,b\log\left(-\sin\left(dx+c\right)+1\right)+2\left(b\cos\left(dx+\right.\right.}{6\,d}$$

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*cos(d*x + c)^3 - 6*a*cos(d*x + c) + 3*b*log(sin(d*x + c) + 1) - 3*
b*log(-sin(d*x + c) + 1) + 2*(b*cos(d*x + c)^2 - 4*b)*sin(d*x + c))/d
```

Sympy [F]

$$\int \sin^3(c+dx)(a+b\tan(c+dx))\,dx = \int (a+b\tan(c+dx))\sin^3(c+dx)\,dx$$

```
[In] integrate(sin(d*x+c)**3*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*sin(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int \sin^3(c+dx)(a+b\tan(c+dx))\,dx = \frac{2\left(\cos(dx+c)^3 - 3\cos(dx+c)\right)a - \left(2\sin(dx+c)^3 - 3\log(\sin(dx+c)+1) + 3\log(\sin(dx+c)-1)\right.}{6d}$$

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*(cos(d*x + c)^3 - 3*cos(d*x + c))*a - (2*sin(d*x + c)^3 - 3*log(sin(
d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin(d*x + c))*b)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4486 vs. $2(65) = 130$.

Time = 1.00 (sec) , antiderivative size = 4486, normalized size of antiderivative = 65.01

$$\int \sin^3(c+dx)(a+b\tan(c+dx))\,dx = \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c)
+ 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d
*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan
(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 - 3*b*log(2*(tan(1/2*d*x)^2*tan
(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan
```

(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*
x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(
1/2*c)^6 + 4*a*tan(1/2*d*x)^6*tan(1/2*c)^6 + 9*b*log(2*(tan(1/2*d*x)^2*tan(
1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(
1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x
)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1
/2*c)^4 - 9*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2
*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1
/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 - 12*b*tan(1/2*d*x)^6*tan(1
/2*c)^5 + 9*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2
*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1
/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^6 - 9*b*log(2*(tan(1/2*d*x)^2
*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 +
tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/
2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*
tan(1/2*c)^6 - 12*b*tan(1/2*d*x)^5*tan(1/2*c)^6 + 12*a*tan(1/2*d*x)^6*tan(1
/2*c)^4 + 12*a*tan(1/2*d*x)^4*tan(1/2*c)^6 + 9*b*log(2*(tan(1/2*d*x)^2*tan(
1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(
1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x
)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1
/2*c)^2 - 9*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2
*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1
/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 - 40*b*tan(1/2*d*x)^6*tan(1
/2*c)^3 + 27*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/
2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(
1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2
+ tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 - 27*b*log(2*(tan(1/2*d*x)
^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2
+ tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(
1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^
4*tan(1/2*c)^4 - 60*b*tan(1/2*d*x)^5*tan(1/2*c)^4 - 60*b*tan(1/2*d*x)^4*tan
(1/2*c)^5 + 9*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1
/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan
(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2
+ tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 - 9*b*log(2*(tan(1/2*d*x)
^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2
+ tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(
1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^
2*tan(1/2*c)^6 - 40*b*tan(1/2*d*x)^3*tan(1/2*c)^6 - 12*a*tan(1/2*d*x)^6*tan
(1/2*c)^2 - 96*a*tan(1/2*d*x)^5*tan(1/2*c)^3 - 108*a*tan(1/2*d*x)^4*tan(1/2
*c)^4 - 96*a*tan(1/2*d*x)^3*tan(1/2*c)^5 - 12*a*tan(1/2*d*x)^2*tan(1/2*c)^6
+ 3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2

*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x)
 - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/
2*c)^2 + 1))*tan(1/2*d*x)^6 - 3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*ta
n(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + ta
n(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^
2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6 - 12*b*tan(1/2*d*x)^
6*tan(1/2*c) + 27*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*t
an(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2
*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*
x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^2 - 27*b*log(2*(tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2
*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/
(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*
d*x)^4*tan(1/2*c)^2 + 60*b*tan(1/2*d*x)^5*tan(1/2*c)^2 + 120*b*tan(1/2*d*x)
^4*tan(1/2*c)^3 + 27*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^
2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2
- 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2
*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^4 - 27*b*log(2*(tan(
1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(
1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) +
1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1
/2*d*x)^2*tan(1/2*c)^4 + 120*b*tan(1/2*d*x)^3*tan(1/2*c)^4 + 60*b*tan(1/2*d
*x)^2*tan(1/2*c)^5 + 3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x
)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^
2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1
/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^6 - 3*b*log(2*(tan(1/2*d*x)^2*tan
(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan
(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*
x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^6 - 12*b
*tan(1/2*d*x)*tan(1/2*c)^6 - 4*a*tan(1/2*d*x)^6 + 108*a*tan(1/2*d*x)^4*tan(
1/2*c)^2 + 128*a*tan(1/2*d*x)^3*tan(1/2*c)^3 + 108*a*tan(1/2*d*x)^2*tan(1/2
*c)^4 - 4*a*tan(1/2*c)^6 + 9*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1
/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1
/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 +
 tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4 - 9*b*log(2*(tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)
^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(ta
n(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x
)^4 + 12*b*tan(1/2*d*x)^5 - 60*b*tan(1/2*d*x)^4*tan(1/2*c) + 27*b*log(2*(ta
n(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*ta
n(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c)
+ 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan
(1/2*d*x)^2*tan(1/2*c)^2 - 27*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(
1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(
1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2

```
+ tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 120*b*t
an(1/2*d*x)^3*tan(1/2*c)^2 - 120*b*tan(1/2*d*x)^2*tan(1/2*c)^3 + 9*b*log(2*
(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)
*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*
c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*
tan(1/2*c)^4 - 9*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*ta
n(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*
tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x
)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^4 - 60*b*tan(1/2*d*x)*tan(1/2*c)^4 + 12
*b*tan(1/2*c)^5 - 12*a*tan(1/2*d*x)^4 - 96*a*tan(1/2*d*x)^3*tan(1/2*c) - 10
8*a*tan(1/2*d*x)^2*tan(1/2*c)^2 - 96*a*tan(1/2*d*x)*tan(1/2*c)^3 - 12*a*tan
(1/2*c)^4 + 9*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1
/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan
(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2
 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2 - 9*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)
^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*
x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*ta
n(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2 + 40*b*tan(
1/2*d*x)^3 + 60*b*tan(1/2*d*x)^2*tan(1/2*c) + 9*b*log(2*(tan(1/2*d*x)^2*tan
(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan
(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*
x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^2 - 9*b*
log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/
2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*ta
n(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2
+ 1))*tan(1/2*c)^2 + 60*b*tan(1/2*d*x)*tan(1/2*c)^2 + 40*b*tan(1/2*c)^3 + 1
2*a*tan(1/2*d*x)^2 + 12*a*tan(1/2*c)^2 + 3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*
c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*
d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*
tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1)) - 3*b*log(2*(tan(1/2*d*x
)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^
2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan
(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1)) + 12*b*tan(1
/2*d*x) + 12*b*tan(1/2*c) + 4*a)/(d*tan(1/2*d*x)^6*tan(1/2*c)^6 + 3*d*tan(1
/2*d*x)^6*tan(1/2*c)^4 + 3*d*tan(1/2*d*x)^4*tan(1/2*c)^6 + 3*d*tan(1/2*d*x)
^6*tan(1/2*c)^2 + 9*d*tan(1/2*d*x)^4*tan(1/2*c)^4 + 3*d*tan(1/2*d*x)^2*tan(
1/2*c)^6 + d*tan(1/2*d*x)^6 + 9*d*tan(1/2*d*x)^4*tan(1/2*c)^2 + 9*d*tan(1/2
*d*x)^2*tan(1/2*c)^4 + d*tan(1/2*c)^6 + 3*d*tan(1/2*d*x)^4 + 9*d*tan(1/2*d*
x)^2*tan(1/2*c)^2 + 3*d*tan(1/2*c)^4 + 3*d*tan(1/2*d*x)^2 + 3*d*tan(1/2*c)^
2 + d)
```

Mupad [B] (verification not implemented)

Time = 4.71 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.26

$$\int \sin^3(c+dx)(a+b\tan(c+dx))\,dx = \frac{2\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{a\cos\left(c+d\,x\right)^3}{3\,d}-\frac{a\,\cos\left(c+d\,x\right)}{d}-\frac{4\,b\,\sin\left(c+d\,x\right)}{3\,d}+\frac{b\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}$$

```
[In] int(sin(c + d*x)^3*(a + b*tan(c + d*x)),x)
```

```
[Out] (2*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (a*cos(c + d*x)^3)/(
3*d) - (a*cos(c + d*x))/d - (4*b*sin(c + d*x))/(3*d) + (b*cos(c + d*x)^2*si
n(c + d*x))/(3*d)
```

3.14 $\int \sin^2(c+dx)(a+b\tan(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 49

$$\int \sin^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{ax}{2} - \frac{b\log(\cos(c+dx))}{d} - \frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))}{2d}$$

```
[Out] 1/2*a*x-b*ln(cos(d*x+c))/d-1/2*cos(d*x+c)*sin(d*x+c)*(a+b*tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {833, 649, 209, 266}

$$\int \sin^2(c+dx)(a+b\tan(c+dx))\,dx = -\frac{\sin(c+dx)\cos(c+dx)(a+b\tan(c+dx))}{2d} + \frac{ax}{2} - \frac{b\log(\cos(c+dx))}{d}$$

```
[In] Int[Sin[c + d*x]^2*(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*x)/2 - (b*Log[Cos[c + d*x]])/d - (Cos[c + d*x]*Sin[c + d*x]*(a + b*Tan[c
 + d*x]))/(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2(a+bx)}{(1+x^2)^2}\,dx, x, \tan(c+dx)\right)}{d} \\
&= -\frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))}{2d} + \frac{\text{Subst}\left(\int \frac{a+2bx}{1+x^2}\,dx, x, \tan(c+dx)\right)}{2d} \\
&= -\frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))}{2d} \\
&\quad + \frac{a\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+dx)\right)}{2d} + \frac{b\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d} \\
&= \frac{ax}{2} - \frac{b\log(\cos(c+dx))}{d} - \frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.14

$$\int \sin^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{a(c+dx)}{2d} - \frac{b\left(-\frac{1}{2}\cos^2(c+dx)+\log(\cos(c+dx))\right)}{d} - \frac{a\sin(2(c+dx))}{4d}$$

```
[In] Integrate[Sin[c + d*x]^2*(a + b*Tan[c + d*x]),x]
[Out] (a*(c + d*x))/(2*d) - (b*(-1/2*Cos[c + d*x]^2 + Log[Cos[c + d*x]]))/d - (a*
Sin[2*(c + d*x)])/(4*d)
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{a\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+b\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)}{d}$	52
default	$\frac{a\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+b\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)}{d}$	52
risch	$ibx+\frac{ax}{2}+\frac{e^{2i(dx+c)}b}{8d}+\frac{ie^{2i(dx+c)}a}{8d}+\frac{e^{-2i(dx+c)}b}{8d}-\frac{ie^{-2i(dx+c)}a}{8d}+\frac{2ibc}{d}-\frac{b\ln\left(e^{2i(dx+c)}+1\right)}{d}$	99

```
[In] int(sin(d*x+c)^2*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*(-1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+b*(-1/2*sin(d*x+c)^2-ln(c
os(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

$$\int \sin^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{adx+b\cos(dx+c)^2-a\cos(dx+c)\sin(dx+c)-2\,b\log(-\cos(dx+c))}{2\,d}$$

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="fricas")
[Out] 1/2*(a*d*x + b*cos(d*x + c)^2 - a*cos(d*x + c)*sin(d*x + c) - 2*b*log(-cos(
d*x + c)))/d
```

Sympy [F]

$$\int \sin^2(c+dx)(a+b\tan(c+dx))\,dx = \int (a+b\tan(c+dx))\sin^2(c+dx)\,dx$$

```
[In] integrate(sin(d*x+c)**2*(a+b*tan(d*x+c)),x)
[Out] Integral((a + b*tan(c + d*x))*sin(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

$$\int \sin^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{(dx+c)a + b\log\left(\tan(dx+c)^2+1\right) - \frac{a\tan(dx+c)-b}{\tan(dx+c)^2+1}}{2\,d}$$

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="maxima")
[Out] 1/2*((d*x + c)*a + b*log(tan(d*x + c)^2 + 1) - (a*tan(d*x + c) - b)/(tan(d*
x + c)^2 + 1))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. 2(45) = 90.

Time = 0.38 (sec) , antiderivative size = 373, normalized size of antiderivative = 7.61

$$\int \sin^2(c+dx)(a+b\tan(c+dx))\,dx$$

$$= \frac{2\,adx\tan(dx)^2\tan(c)^2 - 2\,b\log\left(\frac{4\left(\tan(dx)^2\tan(c)^2-2\tan(dx)\tan(c)+1\right)}{\tan(dx)^2\tan(c)^2+\tan(dx)^2+\tan(c)^2+1}\right)\tan(dx)^2\tan(c)^2 + 2\,adx\tan(dx)^2 +}{}$$

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="giac")
[Out] 1/4*(2*a*d*x*tan(d*x)^2*tan(c)^2 - 2*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d
*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)
^2*tan(c)^2 + 2*a*d*x*tan(d*x)^2 + 2*a*d*x*tan(c)^2 + b*tan(d*x)^2*tan(c)^2
 - 2*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(
c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^2 + 2*a*tan(d*x)^2*tan(c) - 2*b
*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 +
 tan(d*x)^2 + tan(c)^2 + 1))*tan(c)^2 + 2*a*tan(d*x)*tan(c)^2 + 2*a*d*x - b
*tan(d*x)^2 - 4*b*tan(d*x)*tan(c) - b*tan(c)^2 - 2*b*log(4*(tan(d*x)^2*tan(
c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2
+ 1)) - 2*a*tan(d*x) - 2*a*tan(c) + b)/(d*tan(d*x)^2*tan(c)^2 + d*tan(d*x)^
2 + d*tan(c)^2 + d)
```

Mupad [B] (verification not implemented)

Time = 4.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

$$\int \sin^2(c+dx)(a+b\tan(c+dx))\,dx$$
$$=\frac{\frac{b\cos(c+d\,x)^2}{2}-\frac{a\,\sin(c+d\,x)\,\cos(c+d\,x)}{2}+\frac{b\,\ln\left(\tan(c+d\,x)^2+1\right)}{2}+\frac{a\,d\,x}{2}}{d}$$

```
[In] int(sin(c + d*x)^2*(a + b*tan(c + d*x)),x)
```

```
[Out] ((b*log(tan(c + d*x)^2 + 1))/2 + (b*cos(c + d*x)^2)/2 - (a*cos(c + d*x)*sin
(c + d*x))/2 + (a*d*x)/2)/d
```

3.15 $\int \sin(c+dx)(a+b\tan(c+dx))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 37

$$\int \sin(c+dx)(a+b\tan(c+dx))\,dx=\frac{b\,\text{arctanh}(\sin(c+dx))}{d}-\frac{a\cos(c+dx)}{d}-\frac{b\sin(c+dx)}{d}$$

```
[Out] b*arctanh(sin(d*x+c))/d-a*cos(d*x+c)/d-b*sin(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3598, 2718, 2672, 327, 212}

$$\int \sin(c+dx)(a+b\tan(c+dx))\,dx=-\frac{a\cos(c+dx)}{d}+\frac{b\,\text{arctanh}(\sin(c+dx))}{d}-\frac{b\sin(c+dx)}{d}$$

```
[In] Int[Sin[c + d*x]*(a + b*Tan[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Cos[c + d*x])/d - (b*Sin[c + d*x])/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int (a\sin(c+dx)+b\sin(c+dx)\tan(c+dx))\,dx \\
&= a\int \sin(c+dx)\,dx+b\int \sin(c+dx)\tan(c+dx)\,dx \\
&= -\frac{a\cos(c+dx)}{d}+\frac{b\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx,x,\sin(c+dx)\right)}{d} \\
&= -\frac{a\cos(c+dx)}{d}-\frac{b\sin(c+dx)}{d}+\frac{b\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sin(c+dx)\right)}{d} \\
&= \frac{b\text{arctanh}(\sin(c+dx))}{d}-\frac{a\cos(c+dx)}{d}-\frac{b\sin(c+dx)}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.30

$$\int \sin(c+dx)(a+b\tan(c+dx))\,dx = \frac{b\text{arctanh}(\sin(c+dx))}{d} - \frac{a\cos(c)\cos(dx)}{d} + \frac{a\sin(c)\sin(dx)}{d} - \frac{b\sin(c+dx)}{d}$$

```
[In] Integrate[Sin[c + d*x]*(a + b*Tan[c + d*x]),x]
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Cos[c]*Cos[d*x])/d + (a*Sin[c]*Sin[d*x])/d
 - (b*Sin[c + d*x])/d
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$\frac{-a\cos(dx+c)+b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))}{d}$	40
default	$\frac{-a\cos(dx+c)+b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))}{d}$	40
risch	$\frac{i\mathrm{e}^{i(dx+c)}b}{2d} - \frac{\mathrm{e}^{i(dx+c)}a}{2d} - \frac{i\mathrm{e}^{-i(dx+c)}b}{2d} - \frac{\mathrm{e}^{-i(dx+c)}a}{2d} - \frac{b\ln(\mathrm{e}^{i(dx+c)}-i)}{d} + \frac{b\ln(\mathrm{e}^{i(dx+c)}+i)}{d}$	101

```
[In] int(sin(d*x+c)*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-a*cos(d*x+c)+b*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.32

$$\int \sin(c+dx)(a+b\tan(c+dx))\,dx = -\frac{2\,a\cos(dx+c) - b\log(\sin(dx+c)+1) + b\log(-\sin(dx+c)+1) + 2\,b\sin(dx+c)}{2\,d}$$

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="fricas")
[Out] -1/2*(2*a*cos(d*x + c) - b*log(sin(d*x + c) + 1) + b*log(-sin(d*x + c) + 1)
 + 2*b*sin(d*x + c))/d
```

Sympy [F]

$$\int \sin(c+dx)(a+b\tan(c+dx))\,dx = \int (a+b\tan{(c+dx)})\sin{(c+dx)}\,dx$$

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c)),x)
[Out] Integral((a + b*tan(c + d*x))*sin(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.24

$$\int \sin(c+dx)(a+b\tan(c+dx))\,dx = \frac{b(\log{(\sin{(dx+c)}+1)} - \log{(\sin{(dx+c)}-1)} - 2\sin{(dx+c)}) - 2\,a\cos{(dx+c)}}{2\,d}$$

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="maxima")
[Out] 1/2*(b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)) - 2
*a*cos(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1020 vs. 2(37) = 74.

Time = 0.42 (sec) , antiderivative size = 1020, normalized size of antiderivative = 27.57

$$\int \sin(c+dx)(a+b\tan(c+dx))\,dx = \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="giac")
[Out] -1/2*(b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) +
2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x
) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1
/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 - b*log(2*(tan(1/2*d*x)^2*tan(1/2
*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2
*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2
*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*
c)^2 + 2*a*tan(1/2*d*x)^2*tan(1/2*c)^2 + b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)
^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*
x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*ta
```

```
n(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2 - b*log(2*(
tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*
tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c
) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*t
an(1/2*d*x)^2 - 4*b*tan(1/2*d*x)^2*tan(1/2*c) + b*log(2*(tan(1/2*d*x)^2*tan
(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan
(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*
x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^2 - b*lo
g(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*
d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(
1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 +
1))*tan(1/2*c)^2 - 4*b*tan(1/2*d*x)*tan(1/2*c)^2 - 2*a*tan(1/2*d*x)^2 - 8*a
*tan(1/2*d*x)*tan(1/2*c) - 2*a*tan(1/2*c)^2 + b*log(2*(tan(1/2*d*x)^2*tan(1
/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1
/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)
^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1)) - b*log(2*(tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)
^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(ta
n(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1)) + 4*b*tan(1
/2*d*x) + 4*b*tan(1/2*c) + 2*a)/(d*tan(1/2*d*x)^2*tan(1/2*c)^2 + d*tan(1/2*
d*x)^2 + d*tan(1/2*c)^2 + d)
```

Mupad [B] (verification not implemented)

Time = 4.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.43

$$\int \sin(c+dx)(a+b\tan(c+dx))\,dx=\frac{2\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{2\,a+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

```
[In] int(sin(c + d*x)*(a + b*tan(c + d*x)),x)
```

```
[Out] (2*b*atanh(tan(c/2 + (d*x)/2)))/d - (2*a + 2*b*tan(c/2 + (d*x)/2))/(d*(tan(
c/2 + (d*x)/2)^2 + 1))
```

3.16 $\int \csc(c+dx)(a+b\tan(c+dx))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 26

$$\int \csc(c+dx)(a+b\tan(c+dx))\,dx = -\frac{a\text{arctanh}(\cos(c+dx))}{d}+\frac{b\text{arctanh}(\sin(c+dx))}{d}$$

```
[Out] -a*arctanh(cos(d*x+c))/d+b*arctanh(sin(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3598, 3855}

$$\int \csc(c+dx)(a+b\tan(c+dx))\,dx = \frac{b\text{arctanh}(\sin(c+dx))}{d}-\frac{a\text{arctanh}(\cos(c+dx))}{d}$$

```
[In] Int[Csc[c + d*x]*(a + b*Tan[c + d*x]),x]
```

```
[Out] -((a*ArcTanh[Cos[c + d*x]])/d) + (b*ArcTanh[Sin[c + d*x]])/d
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int (a\csc(c+dx)+b\sec(c+dx))\,dx\\ &= a\int \csc(c+dx)\,dx + b\int \sec(c+dx)\,dx\\ &= -\frac{a\text{arctanh}(\cos(c+dx))}{d}+\frac{b\text{arctanh}(\sin(c+dx))}{d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.00

$$\int \csc(c+dx)(a+b\tan(c+dx))\,dx = \frac{b\text{arctanh}(\sin(c+dx))}{d} - \frac{a\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{a\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

```
[In] Integrate[Csc[c + d*x]*(a + b*Tan[c + d*x]),x]
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Log[Cos[c/2 + (d*x)/2]])/d + (a*Log[Sin[c/
2 + (d*x)/2]])/d
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.54

method	result	size
derivativedivides	$\frac{b\ln(\sec(dx+c)+\tan(dx+c))+a\ln(\csc(dx+c)-\cot(dx+c))}{d}$	40
default	$\frac{b\ln(\sec(dx+c)+\tan(dx+c))+a\ln(\csc(dx+c)-\cot(dx+c))}{d}$	40
risch	$-\frac{a\ln\left(e^{i(dx+c)}+1\right)}{d}+\frac{a\ln\left(e^{i(dx+c)}-1\right)}{d}+\frac{b\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{b\ln\left(e^{i(dx+c)}-i\right)}{d}$	74

```
[In] int(csc(d*x+c)*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(b*ln(sec(d*x+c)+tan(d*x+c))+a*ln(csc(d*x+c)-cot(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(26) = 52.

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.23

$$\int \csc(c+dx)(a+b\tan(c+dx))\,dx = -\frac{a\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-a\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-b\log(\sin(dx+c)+1)+b\log(-\sin(dx+c)}{2\,d}$$

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(a*log(1/2*cos(d*x + c) + 1/2) - a*log(-1/2*cos(d*x + c) + 1/2) - b*lo
g(sin(d*x + c) + 1) + b*log(-sin(d*x + c) + 1))/d
```

Sympy [F]

$$\int \csc(c+dx)(a+b\tan(c+dx))\,dx = \int (a+b\tan(c+dx))\csc(c+dx)\,dx$$

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*csc(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.77

$$\int \csc(c+dx)(a+b\tan(c+dx))\,dx = \frac{b(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))-2\,a\log(\cot(dx+c)+\csc(dx+c))}{2\,d}$$

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) - 2*a*log(cot(d*x +
c) + csc(d*x + c)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.88

$$\int \csc(c+dx)(a+b\tan(c+dx))\,dx = \frac{b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{d}$$

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] (b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - b*log(abs(tan(1/2*d*x + 1/2*c) - 1)
) + a*log(abs(tan(1/2*d*x + 1/2*c))))/d
```

Mupad [B] (verification not implemented)

Time = 4.51 (sec) , antiderivative size = 86, normalized size of antiderivative = 3.31

$$\int \csc(c+dx)(a+b\tan(c+dx))\,dx = \frac{a\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{2\,b\,\mathrm{atanh}\left(\frac{b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

```
[In] int((a + b*tan(c + d*x))/sin(c + d*x),x)
```

```
[Out] (a*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (2*b*atanh((b*cos(c/2 +
(d*x)/2) - a*sin(c/2 + (d*x)/2))/(a*cos(c/2 + (d*x)/2) - b*sin(c/2 + (d*x)/
2))))/d
```

3.17 $\int \csc^2(c+dx)(a+b\tan(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \csc^2(c+dx)(a+b\tan(c+dx))\,dx = -\frac{a\cot(c+dx)}{d}+\frac{b\log(\tan(c+dx))}{d}$$

```
[Out] -a*cot(d*x+c)/d+b*ln(tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {45}

$$\int \csc^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{b\log(\tan(c+dx))}{d}-\frac{a\cot(c+dx)}{d}$$

```
[In] Int[Csc[c + d*x]^2*(a + b*Tan[c + d*x]),x]
```

```
[Out] -((a*Cot[c + d*x])/d) + (b*Log[Tan[c + d*x]])/d
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{a+bx}{x^2}\,dx, x, \tan(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{a}{x^2}+\frac{b}{x}\right)\,dx, x, \tan(c+dx)\right)}{d}\\
&= -\frac{a\cot(c+dx)}{d}+\frac{b\log(\tan(c+dx))}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.52

$$\int \csc^2(c+dx)(a+b\tan(c+dx))\,dx = -\frac{a\cot(c+dx)}{d}-\frac{b\log(\cos(c+dx))}{d}+\frac{b\log(\sin(c+dx))}{d}$$

```
[In] Integrate[Csc[c + d*x]^2*(a + b*Tan[c + d*x]),x]
[Out] -((a*Cot[c + d*x])/d) - (b*Log[Cos[c + d*x]])/d + (b*Log[Sin[c + d*x]])/d
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{-\frac{a}{\tan(dx+c)}+b\ln(\tan(dx+c))}{d}$	26
default	$\frac{-\frac{a}{\tan(dx+c)}+b\ln(\tan(dx+c))}{d}$	26
risch	$-\frac{2ia}{d\left(e^{2i(dx+c)}-1\right)}+\frac{b\ln\left(e^{2i(dx+c)}-1\right)}{d}-\frac{b\ln\left(e^{2i(dx+c)}+1\right)}{d}$	57

```
[In] int(csc(d*x+c)^2*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/tan(d*x+c)*a+b*ln(tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(25) = 50$.

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.48

$$\int \csc^2(c+dx)(a+b\tan(c+dx))\,dx = -\frac{b\log\left(\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right) - b\log\left(-\frac{1}{4}\cos\left(dx+c\right)^2+\frac{1}{4}\right)\sin\left(dx+c\right) + 2\,a\cos\left(dx+c\right)}{2\,d\sin\left(dx+c\right)}$$

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="fricas")
[Out] -1/2*(b*log(cos(d*x + c)^2)*sin(d*x + c) - b*log(-1/4*cos(d*x + c)^2 + 1/4)
*sin(d*x + c) + 2*a*cos(d*x + c))/(d*sin(d*x + c))
```

Sympy [F]

$$\int \csc^2(c+dx)(a+b\tan(c+dx))\,dx = \int (a+b\tan\left(c+dx\right))\csc^2\left(c+dx\right)\,dx$$

```
[In] integrate(csc(d*x+c)**2*(a+b*tan(d*x+c)),x)
[Out] Integral((a + b*tan(c + d*x))*csc(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \csc^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{b\log\left(\tan\left(dx+c\right)\right) - \frac{a}{\tan(dx+c)}}{d}$$

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="maxima")
[Out] (b*log(tan(d*x + c)) - a/tan(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.40

$$\int \csc^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{b\log\left(|\tan\left(dx+c\right)|\right) - \frac{b\tan(dx+c)+a}{\tan(dx+c)}}{d}$$

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="giac")
[Out] (b*log(abs(tan(d*x + c))) - (b*tan(d*x + c) + a)/tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 4.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \csc^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{b\,\ln\left(\tan(c+d\,x)\right)}{d} - \frac{a\cot(c+d\,x)}{d}$$

```
[In] int((a + b*tan(c + d*x))/sin(c + d*x)^2,x)
[Out] (b*log(tan(c + d*x)))/d - (a*cot(c + d*x))/d
```

3.18 $\int \csc^3(c+dx)(a+b\tan(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 60

$$\int \csc^3(c+dx)(a+b\tan(c+dx))\,dx = -\frac{a\text{arctanh}(\cos(c+dx))}{2d}+\frac{b\text{arctanh}(\sin(c+dx))}{d}-\frac{b\csc(c+dx)}{d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}$$

```
[Out] -1/2*a*arctanh(cos(d*x+c))/d+b*arctanh(sin(d*x+c))/d-b*csc(d*x+c)/d-1/2*a*c
ot(d*x+c)*csc(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3598, 3853, 3855, 2701, 327, 213}

$$\int \csc^3(c+dx)(a+b\tan(c+dx))\,dx = -\frac{a\text{arctanh}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}+\frac{b\text{arctanh}(\sin(c+dx))}{d}-\frac{b\csc(c+dx)}{d}$$

```
[In] Int[Csc[c + d*x]^3*(a + b*Tan[c + d*x]),x]
```

```
[Out] -1/2*(a*ArcTanh[Cos[c + d*x]])/d + (b*ArcTanh[Sin[c + d*x]])/d - (b*Csc[c +
 d*x])/d - (a*Cot[c + d*x]*Csc[c + d*x])/(2*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \int \left(a\csc^3(c+dx) + b\csc^2(c+dx)\sec(c+dx)\right)\,dx$$

$$= a\int \csc^3(c+dx)\,dx + b\int \csc^2(c+dx)\sec(c+dx)\,dx$$

$$= -\frac{a\cot(c+dx)\csc(c+dx)}{2d} + \frac{1}{2}a\int \csc(c+dx)\,dx - \frac{b\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d}$$

$$= -\frac{a\text{arctanh}(\cos(c+dx))}{2d} - \frac{b\csc(c+dx)}{d} - \frac{a\cot(c+dx)\csc(c+dx)}{2d} - \frac{b\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d}$$

$$= -\frac{a\text{arctanh}(\cos(c+dx))}{2d} + \frac{b\text{arctanh}(\sin(c+dx))}{d} - \frac{b\csc(c+dx)}{d} - \frac{a\cot(c+dx)\csc(c+dx)}{2d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.78

$$\int \csc^3(c+dx)(a+b\tan(c+dx))\,dx$$
$$= -\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{b\csc(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\sin^2(c+dx)\right)}{d} - \frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}$$

```
[In] Integrate[Csc[c + d*x]^3*(a + b*Tan[c + d*x]),x]
```

```
[Out] -1/8*(a*Csc[(c + d*x)/2]^2)/d - (b*Csc[c + d*x]*Hypergeometric2F1[-1/2, 1,
1/2, Sin[c + d*x]^2])/d - (a*Log[Cos[(c + d*x)/2]])/(2*d) + (a*Log[Sin[(c +
 d*x)/2]])/(2*d) + (a*Sec[(c + d*x)/2]^2)/(8*d)
```

Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{b\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)}{d}$
default	$\frac{b\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)}{d}$
risch	$-\frac{i\mathrm{e}^{i(dx+c)}\left(ia\,\mathrm{e}^{2i(dx+c)}+2b\,\mathrm{e}^{2i(dx+c)}+ia-2b\right)}{d\left(\mathrm{e}^{2i(dx+c)}-1\right)^2}+\frac{a\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{2d}-\frac{a\ln\left(\mathrm{e}^{i(dx+c)}+1\right)}{2d}-\frac{b\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{d}+\frac{b\ln(}{}$

```
[In] int(csc(d*x+c)^3*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*(-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a*(-1/2*csc(d*x+c)*cot(d*x
+c)+1/2*ln(csc(d*x+c)-cot(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(56) = 112$.

Time = 0.28 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.37

$$\int \csc^3(c+dx)(a+b\tan(c+dx))\,dx$$
$$= \frac{2\,a\cos\left(dx+c\right) - \left(a\cos\left(dx+c\right)^2 - a\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right) + \left(a\cos\left(dx+c\right)^2 - a\right)\log\left(-\frac{1}{2}\cos\left(dx+\right.\right.}{4\,(\ldots}$$

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*cos(d*x + c) - (a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x + c) + 1/2)
+ (a*cos(d*x + c)^2 - a)*log(-1/2*cos(d*x + c) + 1/2) + 2*(b*cos(d*x + c)^2
 - b)*log(sin(d*x + c) + 1) - 2*(b*cos(d*x + c)^2 - b)*log(-sin(d*x + c) +
1) + 4*b*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

Sympy [F]

$$\int \csc^3(c+dx)(a+b\tan(c+dx))\,dx = \int \left(a+b\tan\left(c+dx\right)\right)\csc^3\left(c+dx\right)dx$$

```
[In] integrate(csc(d*x+c)**3*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*csc(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.38

$$\int \csc^3(c+dx)(a+b\tan(c+dx))\,dx$$
$$= \frac{a\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1} - \log\left(\cos\left(dx+c\right)+1\right) + \log\left(\cos\left(dx+c\right)-1\right)\right) - 2\,b\left(\frac{2}{\sin(dx+c)} - \log\left(\sin\left(dx+c\right)+1\right) + \right.}{4\,d}$$

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(a*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - log(cos(d*x + c) + 1) + log(c
os(d*x + c) - 1)) - 2*b*(2/sin(d*x + c) - log(sin(d*x + c) + 1) + log(sin(d
*x + c) - 1)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(56) = 112.

Time = 0.37 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.97

$$\int \csc^3(c+dx)(a+b\tan(c+dx))\,dx$$
$$= \frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+8\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-8\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+4\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx\right.\right.\right.}{8\,d}$$

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(a*tan(1/2*d*x + 1/2*c)^2 + 8*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 8*
b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 4*a*log(abs(tan(1/2*d*x + 1/2*c))) -
 4*b*tan(1/2*d*x + 1/2*c) - (6*a*tan(1/2*d*x + 1/2*c)^2 + 4*b*tan(1/2*d*x +
 1/2*c) + a)/tan(1/2*d*x + 1/2*c)^2)/d
```

Mupad [B] (verification not implemented)

Time = 4.88 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.48

$$\int \csc^3(c+dx)(a+b\tan(c+dx))\,dx = \frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d} - \frac{\frac{a}{2}+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,d\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2} - \frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d} - \frac{2\,b\,\mathrm{atanh}\left(\frac{4\,b^2}{2\,a\,b-4\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{2\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a\,b-4\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}$$

```
[In] int((a + b*tan(c + d*x))/sin(c + d*x)^3,x)
```

```
[Out] (a*tan(c/2 + (d*x)/2)^2)/(8*d) - (a/2 + 2*b*tan(c/2 + (d*x)/2))/(4*d*tan(c/
2 + (d*x)/2)^2) - (b*tan(c/2 + (d*x)/2))/(2*d) - (2*b*atanh((4*b^2)/(2*a*b
- 4*b^2*tan(c/2 + (d*x)/2)) - (2*a*b*tan(c/2 + (d*x)/2))/(2*a*b - 4*b^2*tan
(c/2 + (d*x)/2))))/d + (a*log(tan(c/2 + (d*x)/2)))/(2*d)
```

3.19 $\int \csc^4(c+dx)(a+b\tan(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \csc^4(c+dx)(a+b\tan(c+dx))\,dx = -\frac{a\cot(c+dx)}{d}-\frac{b\cot^2(c+dx)}{2d}-\frac{a\cot^3(c+dx)}{3d}+\frac{b\log(\tan(c+dx))}{d}$$

```
[Out] -a*cot(d*x+c)/d-1/2*b*cot(d*x+c)^2/d-1/3*a*cot(d*x+c)^3/d+b*ln(tan(d*x+c))/
d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {780}

$$\int \csc^4(c+dx)(a+b\tan(c+dx))\,dx = -\frac{a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{b\cot^2(c+dx)}{2d}+\frac{b\log(\tan(c+dx))}{d}$$

```
[In] Int[Csc[c + d*x]^4*(a + b*Tan[c + d*x]),x]
```

```
[Out] -((a*Cot[c + d*x])/d) - (b*Cot[c + d*x]^2)/(2*d) - (a*Cot[c + d*x]^3)/(3*d)
 + (b*Log[Tan[c + d*x]])/d
```

Rule 780

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+bx)(1+x^2)}{x^4}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a}{x^4}+\frac{b}{x^3}+\frac{a}{x^2}+\frac{b}{x}\right)\,dx, x, \tan(c+dx)\right)}{d}$$
$$= -\frac{a\cot(c+dx)}{d}-\frac{b\cot^2(c+dx)}{2d}-\frac{a\cot^3(c+dx)}{3d}+\frac{b\log(\tan(c+dx))}{d}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.37

$$\int \csc^4(c+dx)(a+b\tan(c+dx))\,dx = -\frac{2a\cot(c+dx)}{3d}-\frac{b\csc^2(c+dx)}{2d}-\frac{a\cot(c+dx)\csc^2(c+dx)}{3d}-\frac{b\log(\cos(c+dx))}{d}+\frac{b\log(\sin(c+dx))}{d}$$

```
[In] Integrate[Csc[c + d*x]^4*(a + b*Tan[c + d*x]),x]
```

```
[Out] (-2*a*Cot[c + d*x])/(3*d) - (b*Csc[c + d*x]^2)/(2*d) - (a*Cot[c + d*x]*Csc[
c + d*x]^2)/(3*d) - (b*Log[Cos[c + d*x]])/d + (b*Log[Sin[c + d*x]])/d
```

Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{b\left(-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a\left(-\frac{2}{3}-\frac{\left(\csc^2(dx+c)\right)}{3}\right)\cot(dx+c)}{d}$	46
default	$\frac{b\left(-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a\left(-\frac{2}{3}-\frac{\left(\csc^2(dx+c)\right)}{3}\right)\cot(dx+c)}{d}$	46
risch	$\frac{2b\,e^{4i(dx+c)}+4ia\,e^{2i(dx+c)}-2b\,e^{2i(dx+c)}-\frac{4ia}{3}}{d\left(e^{2i(dx+c)}-1\right)^3}+\frac{b\ln\left(e^{2i(dx+c)}-1\right)}{d}-\frac{b\ln\left(e^{2i(dx+c)}+1\right)}{d}$	97

```
[In] int(csc(d*x+c)^4*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*(-1/2/sin(d*x+c)^2+ln(tan(d*x+c)))+a*(-2/3-1/3*csc(d*x+c)^2)*cot(d*x
+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(53) = 106.

Time = 0.26 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.14

$$\int \csc^4(c+dx)(a+b\tan(c+dx))\,dx = -\frac{4\,a\cos\left(dx+c\right)^3 + 3\left(b\cos\left(dx+c\right)^2 - b\right)\log\left(\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right) - 3\left(b\cos\left(dx+c\right)^2 - b\right)\log\left(-\right.}{6\left(d\cos\left(dx+c\right)^2 - d\right)\sin\left(dx+c\right.}$$

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(4*a*cos(d*x + c)^3 + 3*(b*cos(d*x + c)^2 - b)*log(cos(d*x + c)^2)*sin
(d*x + c) - 3*(b*cos(d*x + c)^2 - b)*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x
 + c) - 6*a*cos(d*x + c) - 3*b*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*
x + c))
```

Sympy [F]

$$\int \csc^4(c+dx)(a+b\tan(c+dx))\,dx = \int \left(a+b\tan\left(c+dx\right)\right)\csc^4\left(c+dx\right)dx$$

```
[In] integrate(csc(d*x+c)**4*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*csc(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

$$\int \csc^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{6\,b\log\left(\tan\left(dx+c\right)\right) - \frac{6\,a\tan\left(dx+c\right)^2 + 3\,b\tan\left(dx+c\right) + 2\,a}{\tan\left(dx+c\right)^3}}{6\,d}$$

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(6*b*log(tan(d*x + c)) - (6*a*tan(d*x + c)^2 + 3*b*tan(d*x + c) + 2*a)/
tan(d*x + c)^3)/d
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09

$$\int \csc^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{6\,b\log\left(\left|\tan\left(dx+c\right)\right|\right) - \frac{11\,b\tan(dx+c)^3+6\,a\tan(dx+c)^2+3\,b\tan(dx+c)+2\,a}{\tan(dx+c)^3}}{6\,d}$$

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*b*log(abs(tan(d*x + c))) - (11*b*tan(d*x + c)^3 + 6*a*tan(d*x + c)^2
 + 3*b*tan(d*x + c) + 2*a)/tan(d*x + c)^3)/d
```

Mupad [B] (verification not implemented)

Time = 4.18 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \csc^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{b\,\ln\left(\tan(c+d\,x)\right)}{d} - \frac{a\tan(c+d\,x)^2 + \frac{b\tan(c+d\,x)}{2} + \frac{a}{3}}{d\tan\left(c+d\,x\right)^3}$$

```
[In] int((a + b*tan(c + d*x))/sin(c + d*x)^4,x)
```

```
[Out] (b*log(tan(c + d*x)))/d - (a/3 + (b*tan(c + d*x))/2 + a*tan(c + d*x)^2)/(d*
tan(c + d*x)^3)
```

3.20 $\int \csc^5(c+dx)(a+b\tan(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 98

$$\int \csc^5(c+dx)(a+b\tan(c+dx))\,dx = -\frac{3a\operatorname{arctanh}(\cos(c+dx))}{8d}+\frac{b\operatorname{arctanh}(\sin(c+dx))}{d}-\frac{b\csc(c+dx)}{d}-\frac{3a\cot(c+dx)\csc(c+dx)}{8d}-\frac{b\csc^3(c+dx)}{3d}-\frac{a\cot(c+dx)\csc^3(c+dx)}{4d}$$

```
[Out] -3/8*a*arctanh(cos(d*x+c))/d+b*arctanh(sin(d*x+c))/d-b*csc(d*x+c)/d-3/8*a*c
ot(d*x+c)*csc(d*x+c)/d-1/3*b*csc(d*x+c)^3/d-1/4*a*cot(d*x+c)*csc(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3598, 3853, 3855, 2701, 308, 213}

$$\int \csc^5(c+dx)(a+b\tan(c+dx))\,dx = -\frac{3a\operatorname{arctanh}(\cos(c+dx))}{8d}-\frac{a\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3a\cot(c+dx)\csc(c+dx)}{8d}+\frac{b\operatorname{arctanh}(\sin(c+dx))}{d}-\frac{b\csc^3(c+dx)}{3d}-\frac{b\csc(c+dx)}{d}$$

```
[In] Int[Csc[c + d*x]^5*(a + b*Tan[c + d*x]),x]
```

```
[Out] (-3*a*ArcTanh[Cos[c + d*x]])/(8*d) + (b*ArcTanh[Sin[c + d*x]])/d - (b*Csc[c
 + d*x])/d - (3*a*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (b*Csc[c + d*x]^3)/(3*
d) - (a*Cot[c + d*x]*Csc[c + d*x]^3)/(4*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \int \left(a\csc^5(c+dx) + b\csc^4(c+dx)\sec(c+dx)\right)\,dx$$

$$= a\int \csc^5(c+dx)\,dx + b\int \csc^4(c+dx)\sec(c+dx)\,dx$$

$$= -\frac{a\cot(c+dx)\csc^3(c+dx)}{4d} + \frac{1}{4}(3a)\int \csc^3(c+dx)\,dx - \frac{b\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d}$$

$$= -\frac{3a\cot(c+dx)\csc(c+dx)}{8d} - \frac{a\cot(c+dx)\csc^3(c+dx)}{4d} + \frac{1}{8}(3a)\int \csc(c+dx)\,dx - \frac{b\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)\,dx, x, \csc(c+dx)\right)}{d}$$

$$= -\frac{3a\text{arctanh}(\cos(c+dx))}{8d} - \frac{b\csc(c+dx)}{d} - \frac{3a\cot(c+dx)\csc(c+dx)}{8d} - \frac{b\csc^3(c+dx)}{3d} - \frac{a\cot(c+dx)\csc^3(c+dx)}{4d} - \frac{b\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d}$$

$$= -\frac{3a\text{arctanh}(\cos(c+dx))}{8d} + \frac{b\text{arctanh}(\sin(c+dx))}{d} - \frac{b\csc(c+dx)}{d} - \frac{3a\cot(c+dx)\csc(c+dx)}{8d} - \frac{b\csc^3(c+dx)}{3d} - \frac{a\cot(c+dx)\csc^3(c+dx)}{4d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.54

$$\int \csc^5(c+dx)(a+b\tan(c+dx))\,dx$$
$$= -\frac{3a\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d} - \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{b\csc^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\sin^2(c+dx)\right)}{3d} - \frac{3a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8d} + \frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d} + \frac{3a\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}$$

```
[In] Integrate[Csc[c + d*x]^5*(a + b*Tan[c + d*x]),x]
```

```
[Out] (-3*a*Csc[(c + d*x)/2]^2)/(32*d) - (a*Csc[(c + d*x)/2]^4)/(64*d) - (b*Csc[c
 + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Sin[c + d*x]^2])/(3*d) - (3*a*Lo
g[Cos[(c + d*x)/2]])/(8*d) + (3*a*Log[Sin[(c + d*x)/2]])/(8*d) + (3*a*Sec[(
c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/2]^4)/(64*d)
```

Maple [A] (verified)

Time = 3.68 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{b\left(-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(\left(-\frac{\left(\csc^3(dx+c)\right)}{4}-\frac{3\csc(dx+c)}{8}\right)\cot(dx+c)+\frac{3\ln(\csc(dx+c)-}{8}\right.}{d}$
default	$\frac{b\left(-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(\left(-\frac{\left(\csc^3(dx+c)\right)}{4}-\frac{3\csc(dx+c)}{8}\right)\cot(dx+c)+\frac{3\ln(\csc(dx+c)-}{8}\right.}{d}$
risch	$-\frac{ie^{i(dx+c)}\left(9ia\,e^{6i(dx+c)}+24b\,e^{6i(dx+c)}-33ia\,e^{4i(dx+c)}-104b\,e^{4i(dx+c)}-33ia\,e^{2i(dx+c)}+104b\,e^{2i(dx+c)}+9ia-24b\right)}{12d\left(e^{2i(dx+c)}-1\right)^4}-3$

```
[In] int(csc(d*x+c)^5*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*(-1/3/sin(d*x+c)^3-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a*((-1/4*
csc(d*x+c)^3-3/8*csc(d*x+c))*cot(d*x+c)+3/8*ln(csc(d*x+c)-cot(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. 2(90) = 180.

Time = 0.28 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.17

$$\int \csc^5(c+dx)(a+b\tan(c+dx))\,dx$$
$$=\frac{18\,a\cos\left(dx+c\right)^3-30\,a\cos\left(dx+c\right)-9\left(a\cos\left(dx+c\right)^4-2\,a\cos\left(dx+c\right)^2+a\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-}{}$$

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/48*(18*a*cos(d*x + c)^3 - 30*a*cos(d*x + c) - 9*(a*cos(d*x + c)^4 - 2*a*c
os(d*x + c)^2 + a)*log(1/2*cos(d*x + c) + 1/2) + 9*(a*cos(d*x + c)^4 - 2*a*
cos(d*x + c)^2 + a)*log(-1/2*cos(d*x + c) + 1/2) + 24*(b*cos(d*x + c)^4 - 2
*b*cos(d*x + c)^2 + b)*log(sin(d*x + c) + 1) - 24*(b*cos(d*x + c)^4 - 2*b*c
os(d*x + c)^2 + b)*log(-sin(d*x + c) + 1) + 16*(3*b*cos(d*x + c)^2 - 4*b)*s
in(d*x + c))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F]

$$\int \csc^5(c+dx)(a+b\tan(c+dx))\,dx = \int (a+b\tan{(c+dx)})\csc^5{(c+dx)}\,dx$$

```
[In] integrate(csc(d*x+c)**5*(a+b*tan(d*x+c)),x)
[Out] Integral((a + b*tan(c + d*x))*csc(c + d*x)**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.26

$$\int \csc^5(c+dx)(a+b\tan(c+dx))\,dx = \frac{3\,a\left(\frac{2\left(3\,\cos(dx+c)^3-5\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1} - 3\,\log\left(\cos\left(dx+c\right)+1\right) + 3\,\log\left(\cos\left(dx+c\right)-1\right)\right) - 8\,b\left(\frac{2\left(3\,\sin(dx+c)^2+1\right)}{\sin(dx+c)^3}\right.}{48\,d}$$

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="maxima")
[Out] 1/48*(3*a*(2*(3*cos(d*x + c)^3 - 5*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*
x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 8*b*(2
*(3*sin(d*x + c)^2 + 1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(si
n(d*x + c) - 1)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.81

$$\int \csc^5(c+dx)(a+b\tan(c+dx))\,dx = \frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 - 8\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 24\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 + 192\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) -}{}$$

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="giac")
[Out] 1/192*(3*a*tan(1/2*d*x + 1/2*c)^4 - 8*b*tan(1/2*d*x + 1/2*c)^3 + 24*a*tan(1
/2*d*x + 1/2*c)^2 + 192*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 192*b*log(ab
s(tan(1/2*d*x + 1/2*c) - 1)) + 72*a*log(abs(tan(1/2*d*x + 1/2*c))) - 120*b*
tan(1/2*d*x + 1/2*c) - (150*a*tan(1/2*d*x + 1/2*c)^4 + 120*b*tan(1/2*d*x +
1/2*c)^3 + 24*a*tan(1/2*d*x + 1/2*c)^2 + 8*b*tan(1/2*d*x + 1/2*c) + 3*a)/ta
n(1/2*d*x + 1/2*c)^4)/d
```

Mupad [B] (verification not implemented)

Time = 4.63 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.15

$$\int \csc^5(c+dx)(a+b\tan(c+dx))\,dx$$
$$= \frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8\,d} - \frac{5\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8\,d} - \frac{2\,b\,\mathrm{atanh}\left(\frac{4\,b^2}{\frac{3\,a\,b}{2}-4\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)} - \frac{3\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\left(\frac{3\,a\,b}{2}-4\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}\right)}{d} + \frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{64\,d} - \frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24\,d} + \frac{3\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{8\,d} - \frac{10\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 + 2\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 + \frac{2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3} + \frac{a}{4}}{16\,d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}$$

```
[In] int((a + b*tan(c + d*x))/sin(c + d*x)^5,x)
```

```
[Out] (a*tan(c/2 + (d*x)/2)^2)/(8*d) - (5*b*tan(c/2 + (d*x)/2))/(8*d) - (2*b*atan
h((4*b^2)/((3*a*b)/2 - 4*b^2*tan(c/2 + (d*x)/2)) - (3*a*b*tan(c/2 + (d*x)/2
))/(2*((3*a*b)/2 - 4*b^2*tan(c/2 + (d*x)/2)))))/d + (a*tan(c/2 + (d*x)/2)^4
)/(64*d) - (b*tan(c/2 + (d*x)/2)^3)/(24*d) + (3*a*log(tan(c/2 + (d*x)/2)))/
(8*d) - (a/4 + (2*b*tan(c/2 + (d*x)/2))/3 + 2*a*tan(c/2 + (d*x)/2)^2 + 10*b
*tan(c/2 + (d*x)/2)^3)/(16*d*tan(c/2 + (d*x)/2)^4)
```

3.21 $\int \csc^6(c+dx)(a+b\tan(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 87

$$\int \csc^6(c+dx)(a+b\tan(c+dx))\,dx = -\frac{a\cot(c+dx)}{d}-\frac{b\cot^2(c+dx)}{d} -\frac{2a\cot^3(c+dx)}{3d}-\frac{b\cot^4(c+dx)}{4d} -\frac{a\cot^5(c+dx)}{5d}+\frac{b\log(\tan(c+dx))}{d}$$

```
[Out] -a*cot(d*x+c)/d-b*cot(d*x+c)^2/d-2/3*a*cot(d*x+c)^3/d-1/4*b*cot(d*x+c)^4/d-
1/5*a*cot(d*x+c)^5/d+b*ln(tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {780}

$$\int \csc^6(c+dx)(a+b\tan(c+dx))\,dx = -\frac{a\cot^5(c+dx)}{5d}-\frac{2a\cot^3(c+dx)}{3d} -\frac{a\cot(c+dx)}{d}-\frac{b\cot^4(c+dx)}{4d} -\frac{b\cot^2(c+dx)}{d}+\frac{b\log(\tan(c+dx))}{d}$$

```
[In] Int[Csc[c + d*x]^6*(a + b*Tan[c + d*x]),x]
```

```
[Out] -((a*Cot[c + d*x])/d) - (b*Cot[c + d*x]^2)/d - (2*a*Cot[c + d*x]^3)/(3*d) -
 (b*Cot[c + d*x]^4)/(4*d) - (a*Cot[c + d*x]^5)/(5*d) + (b*Log[Tan[c + d*x]]
)/d
```

Rule 780

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+bx)(1+x^2)^2}{x^6}\,dx, x, \tan(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(\frac{a}{x^6}+\frac{b}{x^5}+\frac{2a}{x^4}+\frac{2b}{x^3}+\frac{a}{x^2}+\frac{b}{x}\right)\,dx, x, \tan(c+dx)\right)}{d} \\
&= -\frac{a\cot(c+dx)}{d}-\frac{b\cot^2(c+dx)}{d}-\frac{2a\cot^3(c+dx)}{3d} \\
&\quad -\frac{b\cot^4(c+dx)}{4d}-\frac{a\cot^5(c+dx)}{5d}+\frac{b\log(\tan(c+dx))}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.33

$$\begin{aligned}
\int \csc^6(c+dx)(a+b\tan(c+dx))\,dx &= -\frac{8a\cot(c+dx)}{15d}-\frac{b\csc^2(c+dx)}{2d} \\
&\quad -\frac{4a\cot(c+dx)\csc^2(c+dx)}{15d} \\
&\quad -\frac{b\csc^4(c+dx)}{4d}-\frac{a\cot(c+dx)\csc^4(c+dx)}{5d} \\
&\quad -\frac{b\log(\cos(c+dx))}{d}+\frac{b\log(\sin(c+dx))}{d}
\end{aligned}$$

```
[In] Integrate[Csc[c + d*x]^6*(a + b*Tan[c + d*x]),x]
```

```
[Out] (-8*a*Cot[c + d*x])/(15*d) - (b*Csc[c + d*x]^2)/(2*d) - (4*a*Cot[c + d*x]*C
sc[c + d*x]^2)/(15*d) - (b*Csc[c + d*x]^4)/(4*d) - (a*Cot[c + d*x]*Csc[c +
d*x]^4)/(5*d) - (b*Log[Cos[c + d*x]])/d + (b*Log[Sin[c + d*x]])/d
```

Maple [A] (verified)

Time = 5.39 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{b\left(-\frac{1}{4\sin(dx+c)^4}-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a\left(-\frac{8}{15}-\frac{\left(\csc^4(dx+c)\right)}{5}-\frac{4\left(\csc^2(dx+c)\right)}{15}\right)\cot(dx+c)}{d}$
default	$\frac{b\left(-\frac{1}{4\sin(dx+c)^4}-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a\left(-\frac{8}{15}-\frac{\left(\csc^4(dx+c)\right)}{5}-\frac{4\left(\csc^2(dx+c)\right)}{15}\right)\cot(dx+c)}{d}$
risch	$\frac{2b\,e^{8i(dx+c)}-10b\,e^{6i(dx+c)}-\frac{32ia\,e^{4i(dx+c)}}{3}+10b\,e^{4i(dx+c)}+\frac{16ia\,e^{2i(dx+c)}}{3}-2b\,e^{2i(dx+c)}-\frac{16ia}{15}}{d\left(e^{2i(dx+c)}-1\right)^5}+\frac{b\ln\left(e^{2i(dx+c)}-1\right)}{d}-$

[In] int(csc(d*x+c)^6*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(b*(-1/4/sin(d*x+c)^4-1/2/sin(d*x+c)^2+ln(tan(d*x+c)))+a*(-8/15-1/5*csc (d*x+c)^4-4/15*csc(d*x+c)^2)*cot(d*x+c))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(81) = 162.

Time = 0.27 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.00

$$\int \csc^6(c+dx)(a+b\tan(c+dx))\,dx = -\frac{32\,a\cos\left(dx+c\right)^5-80\,a\cos\left(dx+c\right)^3+30\left(b\cos\left(dx+c\right)^4-2\,b\cos\left(dx+c\right)^2+b\right)\log\left(\cos\left(dx+c\right)^2\right)\text{s}}{6}$$

[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c)),x, algorithm="fricas")

[Out] -1/60*(32*a*cos(d*x + c)^5 - 80*a*cos(d*x + c)^3 + 30*(b*cos(d*x + c)^4 - 2 *b*cos(d*x + c)^2 + b)*log(cos(d*x + c)^2)*sin(d*x + c) - 30*(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + b)*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) + 60*a*cos(d*x + c) - 15*(2*b*cos(d*x + c)^2 - 3*b)*sin(d*x + c))/((d*cos(d* x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))

Sympy [F]

$$\int \csc^6(c+dx)(a+b\tan(c+dx))\,dx = \int (a+b\tan{(c+dx)})\csc^6{(c+dx)}\,dx$$

```
[In] integrate(csc(d*x+c)**6*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*csc(c + d*x)**6, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.83

$$\int \csc^6(c+dx)(a+b\tan(c+dx))\,dx = \frac{60\,b\log{(\tan{(dx+c)})} - \frac{60\,a\tan(dx+c)^4+60\,b\tan(dx+c)^3+40\,a\tan(dx+c)^2+15\,b\tan(dx+c)+12\,a}{\tan(dx+c)^5}}{60\,d}$$

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/60*(60*b*log(tan(d*x + c)) - (60*a*tan(d*x + c)^4 + 60*b*tan(d*x + c)^3 +
 40*a*tan(d*x + c)^2 + 15*b*tan(d*x + c) + 12*a)/tan(d*x + c)^5)/d
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.97

$$\int \csc^6(c+dx)(a+b\tan(c+dx))\,dx = \frac{60\,b\log{(|\tan{(dx+c)}|)} - \frac{137\,b\tan(dx+c)^5+60\,a\tan(dx+c)^4+60\,b\tan(dx+c)^3+40\,a\tan(dx+c)^2+15\,b\tan(dx+c)+12\,a}{\tan(dx+c)^5}}{60\,d}$$

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/60*(60*b*log(abs(tan(d*x + c))) - (137*b*tan(d*x + c)^5 + 60*a*tan(d*x +
c)^4 + 60*b*tan(d*x + c)^3 + 40*a*tan(d*x + c)^2 + 15*b*tan(d*x + c) + 12*a
)/tan(d*x + c)^5)/d
```

Mupad [B] (verification not implemented)

Time = 4.57 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

$$\int \csc^6(c+dx)(a+b\tan(c+dx))\,dx$$
$$= \frac{b\ln(\tan(c+dx))}{d} - \frac{a\tan(c+dx)^4 + b\tan(c+dx)^3 + \frac{2a\tan(c+dx)^2}{3} + \frac{b\tan(c+dx)}{4} + \frac{a}{5}}{d\tan(c+dx)^5}$$

```
[In] int((a + b*tan(c + d*x))/sin(c + d*x)^6,x)
```

```
[Out] (b*log(tan(c + d*x)))/d - (a/5 + (b*tan(c + d*x))/4 + (2*a*tan(c + d*x)^2)/
3 + a*tan(c + d*x)^4 + b*tan(c + d*x)^3)/(d*tan(c + d*x)^5)
```

3.22 $\int \sin^4(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 113

$$\int \sin^4(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{3}{8}\left(a^2-5b^2\right)x-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}+\frac{\cos^2(c+dx)(7b-5a\tan(c+dx))(a+b\tan(c+dx))}{8d}+\frac{\cos^3(c+dx)\sin(c+dx)(a+b\tan(c+dx))^2}{4d}$$

```
[Out] 3/8*(a^2-5*b^2)*x-2*a*b*ln(cos(d*x+c))/d+b^2*tan(d*x+c)/d+1/8*cos(d*x+c)^2*
(7*b-5*a*tan(d*x+c))*(a+b*tan(d*x+c))/d+1/4*cos(d*x+c)^3*sin(d*x+c)*(a+b*ta
n(d*x+c))^2/d
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3597, 1659, 1824, 649, 209, 266}

$$\int \sin^4(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{3}{8}x\left(a^2-5b^2\right)+\frac{\cos^2(c+dx)(7b-5a\tan(c+dx))(a+b\tan(c+dx))}{8d}-\frac{2ab\log(\cos(c+dx))}{d}+\frac{\sin(c+dx)\cos^3(c+dx)(a+b\tan(c+dx))^2}{4d}+\frac{b^2\tan(c+dx)}{d}$$

```
[In] Int[Sin[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (3*(a^2 - 5*b^2)*x)/8 - (2*a*b*Log[Cos[c + d*x]])/d + (b^2*Tan[c + d*x])/d
+ (Cos[c + d*x]^2*(7*b - 5*a*Tan[c + d*x])*(a + b*Tan[c + d*x]))/(8*d) + (C
os[c + d*x]^3*Sin[c + d*x]*(a + b*Tan[c + d*x])^2)/(4*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{b\text{Subst}\left(\int \frac{x^4(a+x)^2}{(b^2+x^2)^3}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{\cos^3(c+dx)\sin(c+dx)(a+b\tan(c+dx))^2}{4d} - \frac{\text{Subst}\left(\int \frac{(a+x)(ab^4+3b^4x-4ab^2x^2-4b^2x^3)}{(b^2+x^2)^2}\,dx, x, b\tan(c+dx)\right)}{4bd}$$

$$= \frac{\cos^2(c+dx)(7b-5a\tan(c+dx))(a+b\tan(c+dx))}{8d} + \frac{\cos^3(c+dx)\sin(c+dx)(a+b\tan(c+dx))^2}{4d} + \frac{\text{Subst}\left(\int \frac{b^4(3a^2-7b^2)+16ab^4x+8b^4x^2}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{8b^3d}$$

$$= \frac{\cos^2(c+dx)(7b-5a\tan(c+dx))(a+b\tan(c+dx))}{8d} + \frac{\cos^3(c+dx)\sin(c+dx)(a+b\tan(c+dx))^2}{4d} + \frac{\text{Subst}\left(\int \left(8b^4+\frac{3b^4(a^2-5b^2)+16ab^4x}{b^2+x^2}\right)\,dx, x, b\tan(c+dx)\right)}{8b^3d}$$

$$= \frac{b^2\tan(c+dx)}{d} + \frac{\cos^2(c+dx)(7b-5a\tan(c+dx))(a+b\tan(c+dx))}{8d} + \frac{\cos^3(c+dx)\sin(c+dx)(a+b\tan(c+dx))^2}{4d} + \frac{\text{Subst}\left(\int \frac{3b^4(a^2-5b^2)+16ab^4x}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{8b^3d}$$

$$= \frac{b^2\tan(c+dx)}{d} + \frac{\cos^2(c+dx)(7b-5a\tan(c+dx))(a+b\tan(c+dx))}{8d} + \frac{\cos^3(c+dx)\sin(c+dx)(a+b\tan(c+dx))^2}{4d} + \frac{(2ab)\text{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{d} + \frac{(3b(a^2-5b^2))\,\text{Subst}\left(\int \frac{1}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{8d}$$

$$= \frac{3}{8}\left(a^2-5b^2\right)x - \frac{2ab\log(\cos(c+dx))}{d} + \frac{b^2\tan(c+dx)}{d} + \frac{\cos^2(c+dx)(7b-5a\tan(c+dx))(a+b\tan(c+dx))}{8d} + \frac{\cos^3(c+dx)\sin(c+dx)(a+b\tan(c+dx))^2}{4d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 250 vs. 2(113) = 226.

Time = 3.12 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.21

$$\int \sin^4(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{b\left(\frac{3(a^2-b^2)\arctan(\tan(c+dx))}{b}+\frac{4(-2a^2+3b^2)\arctan(\tan(c+dx))}{b}+16a\cos^2(c+dx)-4a\cos^4(c+dx)+4\left(2a+\frac{a^2-3b}{\sqrt{-b^2}}\right.\right.}{}$$

[In] Integrate[Sin[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]

[Out] (b*((3*(a^2 - b^2)*ArcTan[Tan[c + d*x]])/b + (4*(-2*a^2 + 3*b^2)*ArcTan[Tan[c + d*x]])/b + 16*a*Cos[c + d*x]^2 - 4*a*Cos[c + d*x]^4 + 4*(2*a + (a^2 - 3*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]] + 4*(2*a + (-a^2 + 3*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + (2*(a^2 - b^2)*Cos[c + d*x]^3*Sin[c + d*x])/b + (3*(a - b)*(a + b)*Sin[2*(c + d*x)])/(2*b) + (2*(-2*a^2 + 3*b^2)*Sin[2*(c + d*x)])/b + 8*b*Tan[c + d*x]))/(8*d)

Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.24

method	result
derivativedivides	$\frac{a^2\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(-\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+b^2\left(\frac{\sin^7(}{\cos(}\right.}{d}$
default	$\frac{a^2\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(-\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+b^2\left(\frac{\sin^7(}{\cos(}\right.}{d}$
risch	$2iabx+\frac{3a^2x}{8}-\frac{15b^2x}{8}+\frac{3\,e^{2i(dx+c)}ab}{8d}-\frac{ie^{-2i(dx+c)}a^2}{8d}-\frac{ie^{2i(dx+c)}b^2}{4d}+\frac{3\,e^{-2i(dx+c)}ab}{8d}+\frac{ie^{2i(dx+c)}a^2}{8d}$

[In] int(sin(d*x+c)^4*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^2*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)+2*a*b*(-1/4*sin(d*x+c)^4-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))+b^2*(sin(d*x+c)^7/cos(d*x+c)+(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)-15/8*d*x-15/8*c))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.21

$$\int \sin^4(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{8\,ab\cos\left(dx+c\right)^5 - 32\,ab\cos\left(dx+c\right)^3 + 32\,ab\cos\left(dx+c\right)\log\left(-\cos\left(dx+c\right)\right) - (6\,(a^2-5\,b^2)dx - 13\,a}{16\,d\cos\left(dx+\right.}$$

```
[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/16*(8*a*b*cos(d*x + c)^5 - 32*a*b*cos(d*x + c)^3 + 32*a*b*cos(d*x + c)*l
og(-cos(d*x + c)) - (6*(a^2 - 5*b^2)*d*x - 13*a*b)*cos(d*x + c) - 2*(2*(a^2
 - b^2)*cos(d*x + c)^4 - (5*a^2 - 9*b^2)*cos(d*x + c)^2 + 8*b^2)*sin(d*x +
c))/(d*cos(d*x + c))
```

Sympy [F]

$$\int \sin^4(c+dx)(a+b\tan(c+dx))^2\,dx = \int \left(a+b\tan\left(c+dx\right)\right)^2\sin^4\left(c+dx\right)dx$$

```
[In] integrate(sin(d*x+c)**4*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*sin(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.13

$$\int \sin^4(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{8\,ab\log\left(\tan\left(dx+c\right)^2+1\right) + 8\,b^2\tan\left(dx+c\right) + 3\,(a^2-5\,b^2)(dx+c) + \frac{16\,ab\tan(dx+c)^2-(5\,a^2-9\,b^2)\tan(dx+c)^3+12}{\tan(dx+c)^4+2\tan(dx+c}}{8\,d}$$

```
[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*(8*a*b*log(tan(d*x + c)^2 + 1) + 8*b^2*tan(d*x + c) + 3*(a^2 - 5*b^2)*(
d*x + c) + (16*a*b*tan(d*x + c)^2 - (5*a^2 - 9*b^2)*tan(d*x + c)^3 + 12*a*b
 - (3*a^2 - 7*b^2)*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5604 vs. 2(107) = 214.

Time = 2.74 (sec) , antiderivative size = 5604, normalized size of antiderivative = 49.59

$$\int \sin^4(c+dx)(a+b\tan(c+dx))^2\,dx = \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/64*(3*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*
tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^5*tan(c)^5 + 24*a^2*d*x
*tan(d*x)^5*tan(c)^5 - 120*b^2*d*x*tan(d*x)^5*tan(c)^5 + 3*pi*b^2*sgn(-2*ta
n(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^5*t
an(c)^5 + 6*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c)
+ 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^5*tan(c)^3 - 3*pi*b
^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan
(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^4 + 6*pi*b^2*sgn(2*tan(d*x
)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*
x) - 2*tan(c))*tan(d*x)^3*tan(c)^5 + 6*b^2*arctan((tan(d*x) + tan(c))/(tan(
d*x)*tan(c) - 1))*tan(d*x)^5*tan(c)^5 - 6*b^2*arctan(-(tan(d*x) - tan(c))/(
tan(d*x)*tan(c) + 1))*tan(d*x)^5*tan(c)^5 - 64*a*b*log(4*(tan(d*x)^2*tan(c)
^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 +
1))*tan(d*x)^5*tan(c)^5 + 48*a^2*d*x*tan(d*x)^5*tan(c)^3 - 240*b^2*d*x*tan(
d*x)^5*tan(c)^3 + 6*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 +
 2*tan(d*x) - 2*tan(c))*tan(d*x)^5*tan(c)^3 - 24*a^2*d*x*tan(d*x)^4*tan(c)^
4 + 120*b^2*d*x*tan(d*x)^4*tan(c)^4 - 3*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2
*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^4 + 48*a^2*d*
x*tan(d*x)^3*tan(c)^5 - 240*b^2*d*x*tan(d*x)^3*tan(c)^5 + 6*pi*b^2*sgn(-2*t
an(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^3*
tan(c)^5 + 44*a*b*tan(d*x)^5*tan(c)^5 + 3*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2
- 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c)
)*tan(d*x)^5*tan(c) - 6*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*
x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c
)^2 + 12*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2
*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^3*tan(c)^3 + 12*b^2*ar
ctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^5*tan(c)^3 - 12*b^
2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^5*tan(c)^3 -
128*a*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan
(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^5*tan(c)^3 - 6*pi*b^2*sgn(2*ta
n(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*t
an(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^4 - 6*b^2*arctan((tan(d*x) + tan(c))/
(tan(d*x)*tan(c) - 1))*tan(d*x)^4*tan(c)^4 + 6*b^2*arctan(-(tan(d*x) - tan(
c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)^4 + 64*a*b*log(4*(tan(d*x)^2*t
an(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)
```

```
^2 + 1))*tan(d*x)^4*tan(c)^4 + 24*a^2*tan(d*x)^5*tan(c)^4 - 120*b^2*tan(d*x
)^5*tan(c)^4 + 3*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*ta
n(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)*tan(c)^5 + 12*
b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^3*tan(c)^5 -
 12*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^3*tan(c
)^5 - 128*a*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)
^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^3*tan(c)^5 + 24*a^2*tan(
d*x)^4*tan(c)^5 - 120*b^2*tan(d*x)^4*tan(c)^5 + 24*a^2*d*x*tan(d*x)^5*tan(c
) - 120*b^2*d*x*tan(d*x)^5*tan(c) + 3*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*t
an(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^5*tan(c) - 48*a^2*d*x*ta
n(d*x)^4*tan(c)^2 + 240*b^2*d*x*tan(d*x)^4*tan(c)^2 - 6*pi*b^2*sgn(-2*tan(d
*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(
c)^2 + 96*a^2*d*x*tan(d*x)^3*tan(c)^3 - 480*b^2*d*x*tan(d*x)^3*tan(c)^3 + 1
2*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*ta
n(c))*tan(d*x)^3*tan(c)^3 + 24*a*b*tan(d*x)^5*tan(c)^3 - 48*a^2*d*x*tan(d*x
)^2*tan(c)^4 + 240*b^2*d*x*tan(d*x)^2*tan(c)^4 - 6*pi*b^2*sgn(-2*tan(d*x)^2
*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^4
- 172*a*b*tan(d*x)^4*tan(c)^4 + 24*a^2*d*x*tan(d*x)*tan(c)^5 - 120*b^2*d*x*
tan(d*x)*tan(c)^5 + 3*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2
 + 2*tan(d*x) - 2*tan(c))*tan(d*x)*tan(c)^5 + 24*a*b*tan(d*x)^3*tan(c)^5 -
3*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*
x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4 + 6*pi*b^2*sgn(2*tan(d*x)^2
*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x)
- 2*tan(c))*tan(d*x)^3*tan(c) + 6*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*
tan(c) - 1))*tan(d*x)^5*tan(c) - 6*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x
)*tan(c) + 1))*tan(d*x)^5*tan(c) - 64*a*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*ta
n(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d
*x)^5*tan(c) - 12*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*t
an(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^2 -
12*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^4*tan(c)^
2 + 12*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^4*ta
n(c)^2 + 128*a*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d
*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^4*tan(c)^2 + 40*a^2*t
an(d*x)^5*tan(c)^2 - 200*b^2*tan(d*x)^5*tan(c)^2 + 6*pi*b^2*sgn(2*tan(d*x)^
2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x)
 - 2*tan(c))*tan(d*x)*tan(c)^3 + 24*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x
)*tan(c) - 1))*tan(d*x)^3*tan(c)^3 - 24*b^2*arctan(-(tan(d*x) - tan(c))/(ta
n(d*x)*tan(c) + 1))*tan(d*x)^3*tan(c)^3 - 256*a*b*log(4*(tan(d*x)^2*tan(c)^
2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1
))*tan(d*x)^3*tan(c)^3 + 24*a^2*tan(d*x)^4*tan(c)^3 - 120*b^2*tan(d*x)^4*ta
n(c)^3 - 3*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) +
 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^4 - 12*b^2*arctan((tan
(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^2*tan(c)^4 + 12*b^2*arctan(
-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^4 + 128*a*b*l
og(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + t
```

```
an(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^2*tan(c)^4 + 24*a^2*tan(d*x)^3*tan(c)^4
 - 120*b^2*tan(d*x)^3*tan(c)^4 + 6*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)
*tan(c) - 1))*tan(d*x)*tan(c)^5 - 6*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*
x)*tan(c) + 1))*tan(d*x)*tan(c)^5 - 64*a*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*t
an(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(
d*x)*tan(c)^5 + 40*a^2*tan(d*x)^2*tan(c)^5 - 200*b^2*tan(d*x)^2*tan(c)^5 -
24*a^2*d*x*tan(d*x)^4 + 120*b^2*d*x*tan(d*x)^4 - 3*pi*b^2*sgn(-2*tan(d*x)^2
*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4 + 48*a^2*
d*x*tan(d*x)^3*tan(c) - 240*b^2*d*x*tan(d*x)^3*tan(c) + 6*pi*b^2*sgn(-2*tan
(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^3*ta
n(c) - 52*a*b*tan(d*x)^5*tan(c) - 96*a^2*d*x*tan(d*x)^2*tan(c)^2 + 480*b^2*
d*x*tan(d*x)^2*tan(c)^2 - 12*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*t
an(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^2 - 280*a*b*tan(d*x)^4*t
an(c)^2 + 48*a^2*d*x*tan(d*x)*tan(c)^3 - 240*b^2*d*x*tan(d*x)*tan(c)^3 + 6*
pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(
c))*tan(d*x)*tan(c)^3 - 16*a*b*tan(d*x)^3*tan(c)^3 - 24*a^2*d*x*tan(c)^4 +
120*b^2*d*x*tan(c)^4 - 3*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c
)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^4 - 280*a*b*tan(d*x)^2*tan(c)^4 - 52*a*
b*tan(d*x)*tan(c)^5 - 6*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*
x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2 - 6*b
^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^4 + 6*b^2*arc
tan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^4 + 64*a*b*log(4*(
tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x
)^2 + tan(c)^2 + 1))*tan(d*x)^4 - 64*b^2*tan(d*x)^5 + 3*pi*b^2*sgn(2*tan(d*
x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d
*x) - 2*tan(c))*tan(d*x)*tan(c) + 12*b^2*arctan((tan(d*x) + tan(c))/(tan(d*
x)*tan(c) - 1))*tan(d*x)^3*tan(c) - 12*b^2*arctan(-(tan(d*x) - tan(c))/(tan
(d*x)*tan(c) + 1))*tan(d*x)^3*tan(c) - 128*a*b*log(4*(tan(d*x)^2*tan(c)^2 -
 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*
tan(d*x)^3*tan(c) - 80*a^2*tan(d*x)^4*tan(c) + 80*b^2*tan(d*x)^4*tan(c) - 6
*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x
)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^2 - 24*b^2*arctan((tan(d*x) + ta
n(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^2*tan(c)^2 + 24*b^2*arctan(-(tan(d*x)
 - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^2 + 256*a*b*log(4*(tan(
d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2
+ tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 - 96*a^2*tan(d*x)^3*tan(c)^2 - 160*b^2
*tan(d*x)^3*tan(c)^2 + 12*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) -
 1))*tan(d*x)*tan(c)^3 - 12*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c
) + 1))*tan(d*x)*tan(c)^3 - 128*a*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)
*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)*ta
n(c)^3 - 96*a^2*tan(d*x)^2*tan(c)^3 - 160*b^2*tan(d*x)^2*tan(c)^3 - 6*b^2*a
rctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(c)^4 + 6*b^2*arctan(-(
tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(c)^4 + 64*a*b*log(4*(tan(d*x)
^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + ta
n(c)^2 + 1))*tan(c)^4 - 80*a^2*tan(d*x)*tan(c)^4 + 80*b^2*tan(d*x)*tan(c)^4
```

```
 - 64*b^2*tan(c)^5 - 48*a^2*d*x*tan(d*x)^2 + 240*b^2*d*x*tan(d*x)^2 - 6*pi*
b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))
*tan(d*x)^2 + 52*a*b*tan(d*x)^4 + 24*a^2*d*x*tan(d*x)*tan(c) - 120*b^2*d*x*
tan(d*x)*tan(c) + 3*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 +
 2*tan(d*x) - 2*tan(c))*tan(d*x)*tan(c) + 280*a*b*tan(d*x)^3*tan(c) - 48*a^
2*d*x*tan(c)^2 + 240*b^2*d*x*tan(c)^2 - 6*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) +
 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^2 + 16*a*b*tan(d*x)^2*
tan(c)^2 + 280*a*b*tan(d*x)*tan(c)^3 + 52*a*b*tan(c)^4 - 3*pi*b^2*sgn(2*tan
(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*ta
n(d*x) - 2*tan(c)) - 12*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1
))*tan(d*x)^2 + 12*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*t
an(d*x)^2 + 128*a*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(ta
n(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^2 + 40*a^2*tan(d*x
)^3 - 200*b^2*tan(d*x)^3 + 6*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c
) - 1))*tan(d*x)*tan(c) - 6*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c
) + 1))*tan(d*x)*tan(c) - 64*a*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*ta
n(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)*tan(c
) + 24*a^2*tan(d*x)^2*tan(c) - 120*b^2*tan(d*x)^2*tan(c) - 12*b^2*arctan((t
an(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(c)^2 + 12*b^2*arctan(-(tan(d*x
) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(c)^2 + 128*a*b*log(4*(tan(d*x)^2*tan
(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2
 + 1))*tan(c)^2 + 24*a^2*tan(d*x)*tan(c)^2 - 120*b^2*tan(d*x)*tan(c)^2 + 40
*a^2*tan(c)^3 - 200*b^2*tan(c)^3 - 24*a^2*d*x + 120*b^2*d*x - 3*pi*b^2*sgn(
-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c)) - 24*a*
b*tan(d*x)^2 + 172*a*b*tan(d*x)*tan(c) - 24*a*b*tan(c)^2 - 6*b^2*arctan((ta
n(d*x) + tan(c))/(tan(d*x)*tan(c) - 1)) + 6*b^2*arctan(-(tan(d*x) - tan(c))
/(tan(d*x)*tan(c) + 1)) + 64*a*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*ta
n(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1)) + 24*a^2*tan(d
*x) - 120*b^2*tan(d*x) + 24*a^2*tan(c) - 120*b^2*tan(c) - 44*a*b)/(d*tan(d*
x)^5*tan(c)^5 + 2*d*tan(d*x)^5*tan(c)^3 - d*tan(d*x)^4*tan(c)^4 + 2*d*tan(d
*x)^3*tan(c)^5 + d*tan(d*x)^5*tan(c) - 2*d*tan(d*x)^4*tan(c)^2 + 4*d*tan(d*
x)^3*tan(c)^3 - 2*d*tan(d*x)^2*tan(c)^4 + d*tan(d*x)*tan(c)^5 - d*tan(d*x)^
4 + 2*d*tan(d*x)^3*tan(c) - 4*d*tan(d*x)^2*tan(c)^2 + 2*d*tan(d*x)*tan(c)^3
 - d*tan(c)^4 - 2*d*tan(d*x)^2 + d*tan(d*x)*tan(c) - 2*d*tan(c)^2 - d)
```

Mupad [B] (verification not implemented)

Time = 4.15 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.12

$$\int \sin^4(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= x\left(\frac{3\,a^2}{8}-\frac{15\,b^2}{8}\right)+\frac{b^2\tan(c+d\,x)}{d}$$
$$+\frac{\left(\frac{9\,b^2}{8}-\frac{5\,a^2}{8}\right)\,\tan(c+d\,x)^3+2\,a\,b\tan(c+d\,x)^2+\left(\frac{7\,b^2}{8}-\frac{3\,a^2}{8}\right)\,\tan(c+d\,x)+\frac{3\,a\,b}{2}}{d\,\left(\tan\left(c+d\,x\right)^4+2\tan\left(c+d\,x\right)^2+1\right)}$$
$$+\frac{a\,b\,\ln\left(\tan(c+d\,x)^2+1\right)}{d}$$

```
[In] int(sin(c + d*x)^4*(a + b*tan(c + d*x))^2,x)
```

```
[Out] x*((3*a^2)/8 - (15*b^2)/8) + (b^2*tan(c + d*x))/d + ((3*a*b)/2 - tan(c + d*
x)*((3*a^2)/8 - (7*b^2)/8) - tan(c + d*x)^3*((5*a^2)/8 - (9*b^2)/8) + 2*a*b
*tan(c + d*x)^2)/(d*(2*tan(c + d*x)^2 + tan(c + d*x)^4 + 1)) + (a*b*log(tan
(c + d*x)^2 + 1))/d
```

3.23 $\int \sin^3(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 122

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{2ab\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{a^2\cos(c+dx)}{d} + \frac{2b^2\cos(c+dx)}{d} + \frac{a^2\cos^3(c+dx)}{3d} - \frac{b^2\cos^3(c+dx)}{3d} + \frac{b^2\sec(c+dx)}{d} - \frac{2ab\sin(c+dx)}{d} - \frac{2ab\sin^3(c+dx)}{3d}$$

```
[Out] 2*a*b*arctanh(sin(d*x+c))/d-a^2*cos(d*x+c)/d+2*b^2*cos(d*x+c)/d+1/3*a^2*cos
(d*x+c)^3/d-1/3*b^2*cos(d*x+c)^3/d+b^2*sec(d*x+c)/d-2*a*b*sin(d*x+c)/d-2/3*
a*b*sin(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3598, 2713, 2672, 308, 212, 2670, 276}

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{a^2\cos^3(c+dx)}{3d} - \frac{a^2\cos(c+dx)}{d} + \frac{2ab\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{2ab\sin^3(c+dx)}{3d} - \frac{2ab\sin(c+dx)}{d} - \frac{b^2\cos^3(c+dx)}{3d} + \frac{2b^2\cos(c+dx)}{d} + \frac{b^2\sec(c+dx)}{d}$$

```
[In] Int[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (2*a*b*ArcTanh[Sin[c + d*x]])/d - (a^2*Cos[c + d*x])/d + (2*b^2*Cos[c + d*x
])/d + (a^2*Cos[c + d*x]^3)/(3*d) - (b^2*Cos[c + d*x]^3)/(3*d) + (b^2*Sec[c
 + d*x])/d - (2*a*b*Sin[c + d*x])/d - (2*a*b*Sin[c + d*x]^3)/(3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
```

```
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^2 \sin^3(c+dx) + 2ab\sin^3(c+dx)\tan(c+dx) + b^2\sin^3(c+dx)\tan^2(c+dx)\right) dx \\
&= a^2 \int \sin^3(c+dx)\,dx + (2ab)\int \sin^3(c+dx)\tan(c+dx)\,dx + b^2\int \sin^3(c+dx)\tan^2(c+dx)\,dx \\
&= -\frac{a^2\text{Subst}\left(\int (1-x^2)\,dx, x, \cos(c+dx)\right)}{d} + \frac{(2ab)\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d} - \frac{b^2\text{Subst}\left(\int \frac{(1-x^2)^2}{x^2}\,dx, x, \cos(c+dx)\right)}{d} \\
&= -\frac{a^2\cos(c+dx)}{d} + \frac{a^2\cos^3(c+dx)}{3d} + \frac{(2ab)\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)dx, x, \sin(c+dx)\right)}{d} - \frac{b^2\text{Subst}\left(\int \left(-2+\frac{1}{x^2}+x^2\right)dx, x, \cos(c+dx)\right)}{d} \\
&= -\frac{a^2\cos(c+dx)}{d} + \frac{2b^2\cos(c+dx)}{d} + \frac{a^2\cos^3(c+dx)}{3d} - \frac{b^2\cos^3(c+dx)}{3d} + \frac{b^2\sec(c+dx)}{d} - \frac{2ab\sin(c+dx)}{d} - \frac{2ab\sin^3(c+dx)}{3d} + \frac{(2ab)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d} \\
&= \frac{2ab\,\text{arctanh}(\sin(c+dx))}{d} - \frac{a^2\cos(c+dx)}{d} + \frac{2b^2\cos(c+dx)}{d} + \frac{a^2\cos^3(c+dx)}{3d} - \frac{b^2\cos^3(c+dx)}{3d} + \frac{b^2\sec(c+dx)}{d} - \frac{2ab\sin(c+dx)}{d} - \frac{2ab\sin^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.25

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{\sec(c+dx)\left(-9a^2+45b^2+(-8a^2+20b^2)\cos(2(c+dx))+(a^2-b^2)\cos(4(c+dx))-48ab\cos(c+dx)\log\right.}{}$$

```
[In] Integrate[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]*(-9*a^2 + 45*b^2 + (-8*a^2 + 20*b^2)*Cos[2*(c + d*x)] + (a^2
- b^2)*Cos[4*(c + d*x)] - 48*a*b*Cos[c + d*x]*Log[Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]] + 48*a*b*Cos[c + d*x]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
- 28*a*b*Sin[2*(c + d*x)] + 2*a*b*Sin[4*(c + d*x)]))/(24*d)
```

Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{-\frac{a^2\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+2ab\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+b^2\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\sin\right.\right.}{d}$
default	$\frac{-\frac{a^2\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+2ab\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+b^2\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\sin\right.\right.}{d}$
risch	$\frac{5ie^{i(dx+c)}ab}{4d}-\frac{3\,\mathrm{e}^{i(dx+c)}a^2}{8d}+\frac{7\,\mathrm{e}^{i(dx+c)}b^2}{8d}-\frac{5ie^{-i(dx+c)}ab}{4d}-\frac{3\,\mathrm{e}^{-i(dx+c)}a^2}{8d}+\frac{7\,\mathrm{e}^{-i(dx+c)}b^2}{8d}+\frac{2b^2\mathrm{e}^{i(dx+c}}{d\left(\mathrm{e}^{2i(dx+c)}+\right.}$

```
[In] int(sin(d*x+c)^3*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a^2*(2+sin(d*x+c)^2)*cos(d*x+c)+2*a*b*(-1/3*sin(d*x+c)^3-sin(d*x+
c)+ln(sec(d*x+c)+tan(d*x+c)))+b^2*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^
4+4/3*sin(d*x+c)^2)*cos(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.03

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{(a^2-b^2)\cos{(dx+c)}^4+3\,ab\cos{(dx+c)}\log{(\sin{(dx+c)}+1)}-3\,ab\cos{(dx+c)}\log{(-\sin{(dx+c)}+1)}}{3\,d\cos{(dx+c)}}$$

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*((a^2 - b^2)*cos(d*x + c)^4 + 3*a*b*cos(d*x + c)*log(sin(d*x + c) + 1)
- 3*a*b*cos(d*x + c)*log(-sin(d*x + c) + 1) - 3*(a^2 - 2*b^2)*cos(d*x + c)^
2 + 3*b^2 + 2*(a*b*cos(d*x + c)^3 - 4*a*b*cos(d*x + c))*sin(d*x + c))/(d*co
s(d*x + c))
```

Sympy [F]

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^2\,dx = \int (a+b\tan(c+dx))^2\sin^3(c+dx)\,dx$$

```
[In] integrate(sin(d*x+c)**3*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*sin(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.85

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{(\cos(dx+c)^3 - 3\cos(dx+c))a^2 - (2\sin(dx+c)^3 - 3\log(\sin(dx+c)+1) + 3\log(\sin(dx+c)-1) +}{3\,d}$$

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*((cos(d*x + c)^3 - 3*cos(d*x + c))*a^2 - (2*sin(d*x + c)^3 - 3*log(sin(
d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin(d*x + c))*a*b - (cos(d*x +
c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*b^2)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32694 vs. 2(116) = 232.

Time = 16.95 (sec) , antiderivative size = 32694, normalized size of antiderivative = 267.98

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^2\,dx = \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/192*(15*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2
*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*
tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2
+ 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^8 + 15*pi*b^2*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2
*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*
tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*
d*x)^8*tan(1/2*c)^8 - 15*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2
```

```
*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*ta
n(1/2*d*x)^8*tan(1/2*c)^8 + 15*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*t
an(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) -
 1)*tan(1/2*d*x)^8*tan(1/2*c)^8 - 30*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d
*x)^8*tan(1/2*c)^8 + 30*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
 tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^6 + 30*pi*b^2
*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*
x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2
*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x)
 - 1)*tan(1/2*d*x)^8*tan(1/2*c)^6 - 60*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)
^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/
2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - t
an(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^7*tan(1/2*c
)^7 - 60*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*
c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*t
an(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 -
 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^7*tan(1/2*c)^7 + 30*pi*b^2*sgn(tan(1/2*d*
x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*
c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*t
an(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d
*x)^6*tan(1/2*c)^8 + 30*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*
d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
 tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^8 + 30*pi*b^2
*tan(1/2*d*x)^8*tan(1/2*c)^8 - 30*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan
(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1
/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 - 30*b^2*arctan((tan(1/2*d*x)*tan(1
/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d
*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 + 30*b^2*arctan((tan(1/2
*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c)
- tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 + 30*b^2*arct
an((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*
tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 -
192*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) +
2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x
) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1
/2*c)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 + 192*a*b*log(2*(tan(1/2*d*x)^2*t
an(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + t
an(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*
d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8*ta
n(1/2*c)^8 - 30*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan
(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x
```

)^8*tan(1/2*c)^6 + 30*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*
x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1
/2*d*x)^8*tan(1/2*c)^6 - 60*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/
2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^8*tan
(1/2*c)^6 + 60*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(
1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)
^7*tan(1/2*c)^7 - 60*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x
)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/
2*d*x)^7*tan(1/2*c)^7 + 120*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/
2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^7*tan
(1/2*c)^7 - 30*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(
1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)
^6*tan(1/2*c)^8 + 30*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x
)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/
2*d*x)^6*tan(1/2*c)^8 - 60*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2
*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(
1/2*c)^8 - 128*a^2*tan(1/2*d*x)^8*tan(1/2*c)^8 + 512*b^2*tan(1/2*d*x)^8*tan
(1/2*c)^8 - 180*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*t
an(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2
*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^7*tan(1/2*c)^5 - 180*pi*b^2*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/
2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*t
an(1/2*d*x)^7*tan(1/2*c)^5 + 60*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*
tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) -
1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*
d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 6
0*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + ta
n(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*
c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(
1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 60*pi*b^2*tan(1/2*d*x)^8*tan(1/
2*c)^6 - 60*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c)
 + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*
x)^8*tan(1/2*c)^6 - 60*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) +
 tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))
*tan(1/2*d*x)^8*tan(1/2*c)^6 + 60*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan
(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1
/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^6 + 60*b^2*arctan((tan(1/2*d*x)*tan(1
/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d
*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^6 - 384*a*b*log(2*(tan(1/2
*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2
*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/
(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*
d*x)^8*tan(1/2*c)^6 + 384*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/

```
2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/
2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 +
tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^6 - 180*pi*b^
2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d
*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 +
2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x
) - 1)*tan(1/2*d*x)^5*tan(1/2*c)^7 - 180*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*
c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(
1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 -
 tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^5*tan(1/2
*c)^7 - 120*pi*b^2*tan(1/2*d*x)^7*tan(1/2*c)^7 + 120*b^2*arctan((tan(1/2*d*
x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - t
an(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^7*tan(1/2*c)^7 + 120*b^2*arctan
((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*ta
n(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^7*tan(1/2*c)^7 - 12
0*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan
(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^7*tan(1
/2*c)^7 - 120*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*
c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*
d*x)^7*tan(1/2*c)^7 + 768*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/
2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/
2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 +
tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^7*tan(1/2*c)^7 - 768*a*b*l
og(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2
*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan
(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 +
 1))*tan(1/2*d*x)^7*tan(1/2*c)^7 + 768*a*b*tan(1/2*d*x)^8*tan(1/2*c)^7 + 60
*pi*b^2*tan(1/2*d*x)^6*tan(1/2*c)^8 - 60*b^2*arctan((tan(1/2*d*x)*tan(1/2*c
) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x)
+ tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^8 - 60*b^2*arctan((tan(1/2*d*x
)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + ta
n(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^8 + 60*b^2*arctan((
tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(
1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^8 + 60*b
^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/
2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*
c)^8 - 384*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/
2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(
1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2
+ tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^8 + 384*a*b*log(2*(tan(1/2*d
*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c
)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(t
an(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*
x)^6*tan(1/2*c)^8 + 768*a*b*tan(1/2*d*x)^7*tan(1/2*c)^8 + 180*pi*b^2*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
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tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^7*tan(1/2*c)^5 - 180*pi*b^2
*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*
x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^7*tan(1/2*c)^5 + 360
*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*t
an(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^7*tan(1/2*c)^5 - 60*pi*b^2*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
 tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 60*pi*b^2
*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*
x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 - 120
*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*t
an(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 - 256*a^2*tan(1/2
*d*x)^8*tan(1/2*c)^6 + 1024*b^2*tan(1/2*d*x)^8*tan(1/2*c)^6 + 180*pi*b^2*sg
n(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^
2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^5*tan(1/2*c)^7 - 180*pi
*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/
2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^5*tan(1/2*c)^7 +
 360*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*
x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^5*tan(1/2*c)^7 + 512*a^2*tan
(1/2*d*x)^7*tan(1/2*c)^7 - 2048*b^2*tan(1/2*d*x)^7*tan(1/2*c)^7 - 256*a^2*t
an(1/2*d*x)^6*tan(1/2*c)^8 + 1024*b^2*tan(1/2*d*x)^6*tan(1/2*c)^8 - 30*pi*b
^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*
d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 +
 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*
x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^2 - 30*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*
c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(
1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 -
 tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2
*c)^2 - 180*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1
/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^
2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^
2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^7*tan(1/2*c)^3 - 180*pi*b^2*sgn(tan(1/
2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(
1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*
x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1
/2*d*x)^7*tan(1/2*c)^3 - 540*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan
(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*
sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x
)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^5*tan(1/2*c)^5 - 540*
pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(
1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)
^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/
2*d*x) - 1)*tan(1/2*d*x)^5*tan(1/2*c)^5 - 360*pi*b^2*tan(1/2*d*x)^7*tan(1/2
*c)^5 + 360*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c)
 + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*
x)^7*tan(1/2*c)^5 + 360*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x)
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+ tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)
)*tan(1/2*d*x)^7*tan(1/2*c)^5 - 360*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + t
an(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan
(1/2*c) - 1))*tan(1/2*d*x)^7*tan(1/2*c)^5 - 360*b^2*arctan((tan(1/2*d*x)*ta
n(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/
2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^7*tan(1/2*c)^5 + 2304*a*b*log(2*(tan
(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan
(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) +
 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(
1/2*d*x)^7*tan(1/2*c)^5 - 2304*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*t
an(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + t
an(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)
^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^7*tan(1/2*c)^5 + 1792
*a*b*tan(1/2*d*x)^8*tan(1/2*c)^5 + 120*pi*b^2*tan(1/2*d*x)^6*tan(1/2*c)^6 -
 120*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(
tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6*ta
n(1/2*c)^6 - 120*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1
/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1
/2*d*x)^6*tan(1/2*c)^6 + 120*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*
d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c)
 - 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 + 120*b^2*arctan((tan(1/2*d*x)*tan(1/2*c
) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x)
+ tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 - 768*a*b*log(2*(tan(1/2*d*x
)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^
2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan
(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)
^6*tan(1/2*c)^6 + 768*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*
x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)
^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(
1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 - 180*pi*b^2*sg
n(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^
2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) -
1)*tan(1/2*d*x)^3*tan(1/2*c)^7 - 180*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*
c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan
(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^3*tan(1/2*c)^
7 - 360*pi*b^2*tan(1/2*d*x)^5*tan(1/2*c)^7 + 360*b^2*arctan((tan(1/2*d*x)*t
an(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1
/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^5*tan(1/2*c)^7 + 360*b^2*arctan((ta
n(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/
2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^5*tan(1/2*c)^7 - 360*b^
2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2
*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^5*tan(1/2*c
)^7 - 360*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) -
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 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)
^5*tan(1/2*c)^7 + 2304*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d
*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c
)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan
(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^5*tan(1/2*c)^7 - 2304*a*b*log
(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d
*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1
/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1
))*tan(1/2*d*x)^5*tan(1/2*c)^7 - 30*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
+ 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c
) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(
1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^8
 - 30*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c)
+ tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(
1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*
tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^8 + 1792*a*b*tan(1/2*d*x)^5*tan
(1/2*c)^8 + 30*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(
1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)
^8*tan(1/2*c)^2 - 30*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x
)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/
2*d*x)^8*tan(1/2*c)^2 + 60*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2
*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^8*tan(
1/2*c)^2 + 180*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(
1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)
^7*tan(1/2*c)^3 - 180*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*
x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1
/2*d*x)^7*tan(1/2*c)^3 + 360*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1
/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^7*ta
n(1/2*c)^3 + 768*a^2*tan(1/2*d*x)^8*tan(1/2*c)^4 + 540*pi*b^2*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2
*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^5*tan(1/2*c)^5 - 540*pi*b^2*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^5*tan(1/2*c)^5 + 1080*pi*b^
2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2
*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^5*tan(1/2*c)^5 + 4608*a^2*tan(1/2*d*x)
^7*tan(1/2*c)^5 - 6144*b^2*tan(1/2*d*x)^7*tan(1/2*c)^5 + 4096*a^2*tan(1/2*d
*x)^6*tan(1/2*c)^6 + 2048*b^2*tan(1/2*d*x)^6*tan(1/2*c)^6 + 180*pi*b^2*sgn(
tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2
+ tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^3*tan(1/2*c)^7 - 180*pi*b
^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*
d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^3*tan(1/2*c)^7 + 3
60*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)
*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^3*tan(1/2*c)^7 + 4608*a^2*tan(
1/2*d*x)^5*tan(1/2*c)^7 - 6144*b^2*tan(1/2*d*x)^5*tan(1/2*c)^7 + 30*pi*b^2*
sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x
```

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)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^8 - 30*p
i*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1
/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^8
+ 60*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*
x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2*tan(1/2*c)^8 + 768*a^2*tan
(1/2*d*x)^4*tan(1/2*c)^8 - 15*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)
*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*
x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8 - 15*pi*b^2*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/
2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*t
an(1/2*d*x)^8 - 60*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^
2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/
2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(
1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^7*tan(1/2*c) - 60*pi*b^2*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/
2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*t
an(1/2*d*x)^7*tan(1/2*c) - 60*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)
*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*
x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 - 60*
pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(
1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)
^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/
2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 - 60*pi*b^2*tan(1/2*d*x)^8*tan(1/2*
c)^2 + 60*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) +
 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)
^8*tan(1/2*c)^2 + 60*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + t
an(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*t
an(1/2*d*x)^8*tan(1/2*c)^2 - 60*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1
/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2
*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^2 - 60*b^2*arctan((tan(1/2*d*x)*tan(1/2
*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x
) + tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^2 + 384*a*b*log(2*(tan(1/2*d
*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c
)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(t
an(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*
x)^8*tan(1/2*c)^2 - 384*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*
d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*
c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + ta
n(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^2 - 540*pi*b^2*
sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x
)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*
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tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x)
- 1)*tan(1/2*d*x)^5*tan(1/2*c)^3 - 540*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)
^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/
2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - t
an(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^5*tan(1/2*c
)^3 - 360*pi*b^2*tan(1/2*d*x)^7*tan(1/2*c)^3 + 360*b^2*arctan((tan(1/2*d*x)
*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan
(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^7*tan(1/2*c)^3 + 360*b^2*arctan((
tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(
1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^7*tan(1/2*c)^3 - 360*
b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1
/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^7*tan(1/2
*c)^3 - 360*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c)
 - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*
x)^7*tan(1/2*c)^3 + 2304*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2
*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2
*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + t
an(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^7*tan(1/2*c)^3 - 2304*a*b*l
og(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2
*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan
(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 +
 1))*tan(1/2*d*x)^7*tan(1/2*c)^3 - 1792*a*b*tan(1/2*d*x)^8*tan(1/2*c)^3 - 1
7920*a*b*tan(1/2*d*x)^7*tan(1/2*c)^4 - 540*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/
2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*ta
n(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2
 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^3*tan(1
/2*c)^5 - 540*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan
(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x
)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c
)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^3*tan(1/2*c)^5 - 1080*pi*b^2*tan(1/2
*d*x)^5*tan(1/2*c)^5 + 1080*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d
*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c)
+ 1))*tan(1/2*d*x)^5*tan(1/2*c)^5 + 1080*b^2*arctan((tan(1/2*d*x)*tan(1/2*c
) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x)
- tan(1/2*c) + 1))*tan(1/2*d*x)^5*tan(1/2*c)^5 - 1080*b^2*arctan((tan(1/2*d
*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) -
tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^5*tan(1/2*c)^5 - 1080*b^2*arct
an((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*
tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^5*tan(1/2*c)^5 +
6912*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) +
 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*
x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(
1/2*c)^2 + 1))*tan(1/2*d*x)^5*tan(1/2*c)^5 - 6912*a*b*log(2*(tan(1/2*d*x)^2
*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 +
 tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/
```

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2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^5*
tan(1/2*c)^5 - 28672*a*b*tan(1/2*d*x)^6*tan(1/2*c)^5 - 60*pi*b^2*sgn(tan(1/
2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(
1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*
x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1
/2*d*x)^2*tan(1/2*c)^6 - 60*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(
1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*s
gn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)
^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^6 - 28672
*a*b*tan(1/2*d*x)^5*tan(1/2*c)^6 - 60*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2
*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - ta
n(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)*tan(1/2*c)^7
 - 60*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c)
+ tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(
1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*
tan(1/2*d*x) - 1)*tan(1/2*d*x)*tan(1/2*c)^7 - 360*pi*b^2*tan(1/2*d*x)^3*tan
(1/2*c)^7 + 360*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/
2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/
2*d*x)^3*tan(1/2*c)^7 + 360*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d
*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c)
+ 1))*tan(1/2*d*x)^3*tan(1/2*c)^7 - 360*b^2*arctan((tan(1/2*d*x)*tan(1/2*c)
 + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) -
 tan(1/2*c) - 1))*tan(1/2*d*x)^3*tan(1/2*c)^7 - 360*b^2*arctan((tan(1/2*d*x
)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + ta
n(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^3*tan(1/2*c)^7 + 2304*a*b*log(2*
(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)
*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*
c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*
tan(1/2*d*x)^3*tan(1/2*c)^7 - 2304*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 -
 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2
 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/
2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c)^7 -
17920*a*b*tan(1/2*d*x)^4*tan(1/2*c)^7 - 15*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/
2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*ta
n(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2
 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^8 - 15*pi
*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/
2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*
d*x) - 1)*tan(1/2*c)^8 - 60*pi*b^2*tan(1/2*d*x)^2*tan(1/2*c)^8 + 60*b^2*arc
tan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)
*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^8 +
 60*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(t
an(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan
```

(1/2*c)^8 - 60*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^8 - 60*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^8 + 384*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^8 - 384*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^8 - 1792*a*b*tan(1/2*d*x)^3*tan(1/2*c)^8 + 15*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8 - 15*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8 + 30*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^8 + 60*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^7*tan(1/2*c) - 60*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^7*tan(1/2*c) + 120*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^7*tan(1/2*c) + 60*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 - 60*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 + 120*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 - 256*a^2*tan(1/2*d*x)^8*tan(1/2*c)^2 + 1024*b^2*tan(1/2*d*x)^8*tan(1/2*c)^2 + 540*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^5*tan(1/2*c)^3 - 540*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^5*tan(1/2*c)^3 + 1080*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^5*tan(1/2*c)^3 - 4608*a^2*tan(1/2*d*x)^7*tan(1/2*c)^3 + 6144*b^2*tan(1/2*d*x)^7*tan(1/2*c)^3 - 19968*a^2*tan(1/2*d*x)^6*tan(1/2*c)^4 + 30720*b^2*tan(1/2*d*x)^6*tan(1/2*c)^4 + 540*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^3*tan(1/2*c)^5 - 540*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^3*tan(1/2*c)^5 + 1080*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^3*tan(1/2*c)^5 - 24064*a^2*tan(1/2*d

```
*x)^5*tan(1/2*c)^5 + 22528*b^2*tan(1/2*d*x)^5*tan(1/2*c)^5 + 60*pi*b^2*sgn(
tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2
+ tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^6 - 60*pi*b^
2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d
*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^6 + 12
0*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*
tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2*tan(1/2*c)^6 - 19968*a^2*tan(
1/2*d*x)^4*tan(1/2*c)^6 + 30720*b^2*tan(1/2*d*x)^4*tan(1/2*c)^6 + 60*pi*b^2
*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*
x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)*tan(1/2*c)^7 - 60*pi
*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/
2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)*tan(1/2*c)^7 + 1
20*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)
*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)*tan(1/2*c)^7 - 4608*a^2*tan(1/
2*d*x)^3*tan(1/2*c)^7 + 6144*b^2*tan(1/2*d*x)^3*tan(1/2*c)^7 + 15*pi*b^2*sg
n(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^
2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^8 - 15*pi*b^2*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1
/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^8 + 30*pi*b^2*sgn(tan(1/2*d*x)^2*t
an(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1
)*tan(1/2*c)^8 - 256*a^2*tan(1/2*d*x)^2*tan(1/2*c)^8 + 1024*b^2*tan(1/2*d*x
)^2*tan(1/2*c)^8 - 30*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*
x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan
(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + t
an(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6 - 30*pi*b^2*sgn(tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*
c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*t
an(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d
*x)^6 - 30*pi*b^2*tan(1/2*d*x)^8 + 30*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) +
 tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + t
an(1/2*c) + 1))*tan(1/2*d*x)^8 + 30*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - t
an(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan
(1/2*c) + 1))*tan(1/2*d*x)^8 - 30*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan
(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1
/2*c) - 1))*tan(1/2*d*x)^8 - 30*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1
/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2
*c) - 1))*tan(1/2*d*x)^8 + 192*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*t
an(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + t
an(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)
^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8 - 192*a*b*log(2*(ta
n(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*ta
n(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c)
+ 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan
(1/2*d*x)^8 - 180*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2
*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2
```

*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1
/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^5*tan(1/2*c) - 180*pi*b^2*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/
2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*t
an(1/2*d*x)^5*tan(1/2*c) - 120*pi*b^2*tan(1/2*d*x)^7*tan(1/2*c) + 120*b^2*a
rctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*
x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^7*tan(1/2*c) +
120*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(
tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^7*ta
n(1/2*c) - 120*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2
*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2
*d*x)^7*tan(1/2*c) - 120*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x)
- tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1
))*tan(1/2*d*x)^7*tan(1/2*c) + 768*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 +
2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2
+ tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/
2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^7*tan(1/2*c) - 76
8*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*
tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x)
+ 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2
*c)^2 + 1))*tan(1/2*d*x)^7*tan(1/2*c) - 768*a*b*tan(1/2*d*x)^8*tan(1/2*c) -
120*pi*b^2*tan(1/2*d*x)^6*tan(1/2*c)^2 + 120*b^2*arctan((tan(1/2*d*x)*tan(
1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*
d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 + 120*b^2*arctan((tan(1
/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c
) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 - 120*b^2*a
rctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*
x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^2
- 120*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)
/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*
tan(1/2*c)^2 + 768*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^
2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2
- 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2
*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 - 768*a*b*log(2*(t
an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*t
an(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c)
+ 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*ta
n(1/2*d*x)^6*tan(1/2*c)^2 - 540*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*
tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) -
1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*
d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^3*tan(1/2*c)^3 - 5
40*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + t
an(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2
*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan

```
(1/2*d*x) - 1)*tan(1/2*d*x)^3*tan(1/2*c)^3 - 1080*pi*b^2*tan(1/2*d*x)^5*tan
(1/2*c)^3 + 1080*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1
/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1
/2*d*x)^5*tan(1/2*c)^3 + 1080*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2
*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c
) + 1))*tan(1/2*d*x)^5*tan(1/2*c)^3 - 1080*b^2*arctan((tan(1/2*d*x)*tan(1/2
*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x
) - tan(1/2*c) - 1))*tan(1/2*d*x)^5*tan(1/2*c)^3 - 1080*b^2*arctan((tan(1/2
*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c)
+ tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^5*tan(1/2*c)^3 + 6912*a*b*lo
g(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*
d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(
1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 +
1))*tan(1/2*d*x)^5*tan(1/2*c)^3 - 6912*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)
^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*
x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*ta
n(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^5*tan(1/2*c)^
3 + 28672*a*b*tan(1/2*d*x)^6*tan(1/2*c)^3 + 53760*a*b*tan(1/2*d*x)^5*tan(1/
2*c)^4 - 180*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(
1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)
^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)
^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)*tan(1/2*c)^5 - 180*pi*b^2*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1
/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x
)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/
2*d*x)*tan(1/2*c)^5 - 1080*pi*b^2*tan(1/2*d*x)^3*tan(1/2*c)^5 + 1080*b^2*ar
ctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x
)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^3*tan(1/2*c)^5
+ 1080*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)
/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^3*
tan(1/2*c)^5 - 1080*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + ta
n(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*ta
n(1/2*d*x)^3*tan(1/2*c)^5 - 1080*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(
1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/
2*c) - 1))*tan(1/2*d*x)^3*tan(1/2*c)^5 + 6912*a*b*log(2*(tan(1/2*d*x)^2*tan
(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan
(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*
x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(
1/2*c)^5 - 6912*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*t
an(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2
*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*
x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c)^5 + 53760*a*b*tan(1/2*d
*x)^4*tan(1/2*c)^5 - 30*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
```

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 tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^6 - 30*pi*b^2*sgn(tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*
c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*t
an(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c
)^6 - 120*pi*b^2*tan(1/2*d*x)^2*tan(1/2*c)^6 + 120*b^2*arctan((tan(1/2*d*x)
*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan
(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 + 120*b^2*arctan((
tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(
1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 - 120*
b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1
/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2
*c)^6 - 120*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c)
 - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*
x)^2*tan(1/2*c)^6 + 768*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*
c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + ta
n(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 - 768*a*b*log
(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d
*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1
/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1
))*tan(1/2*d*x)^2*tan(1/2*c)^6 + 28672*a*b*tan(1/2*d*x)^3*tan(1/2*c)^6 - 12
0*pi*b^2*tan(1/2*d*x)*tan(1/2*c)^7 + 120*b^2*arctan((tan(1/2*d*x)*tan(1/2*c
) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x)
+ tan(1/2*c) + 1))*tan(1/2*d*x)*tan(1/2*c)^7 + 120*b^2*arctan((tan(1/2*d*x)
*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan
(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)*tan(1/2*c)^7 - 120*b^2*arctan((ta
n(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/
2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)*tan(1/2*c)^7 - 120*b^2*
arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d
*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)*tan(1/2*c)^7
+ 768*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c)
+ 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d
*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan
(1/2*c)^2 + 1))*tan(1/2*d*x)*tan(1/2*c)^7 - 768*a*b*log(2*(tan(1/2*d*x)^2*t
an(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + t
an(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*
d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)*tan(
1/2*c)^7 - 30*pi*b^2*tan(1/2*c)^8 + 30*b^2*arctan((tan(1/2*d*x)*tan(1/2*c)
+ tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) +
tan(1/2*c) + 1))*tan(1/2*c)^8 + 30*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - ta
n(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(
1/2*c) + 1))*tan(1/2*c)^8 - 30*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/
2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*
c) - 1))*tan(1/2*c)^8 - 30*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*
x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) -
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 1))*tan(1/2*c)^8 + 192*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*
c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + ta
n(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^8 - 192*a*b*log(2*(tan(1/2*d*x
)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^
2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan
(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^8
 - 768*a*b*tan(1/2*d*x)*tan(1/2*c)^8 + 30*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2
*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan
(1/2*d*x) - 1)*tan(1/2*d*x)^6 - 30*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 -
 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*
x) - 1)*tan(1/2*d*x)^6 + 60*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/
2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6 - 1
28*a^2*tan(1/2*d*x)^8 + 512*b^2*tan(1/2*d*x)^8 + 180*pi*b^2*sgn(tan(1/2*d*x
)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c
)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^5*tan(1/2*c) - 180*pi*b^2*sgn(tan(1/
2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(
1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^5*tan(1/2*c) + 360*pi*b^2*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - t
an(1/2*c)^2 + 1)*tan(1/2*d*x)^5*tan(1/2*c) - 512*a^2*tan(1/2*d*x)^7*tan(1/2
*c) + 2048*b^2*tan(1/2*d*x)^7*tan(1/2*c) + 4096*a^2*tan(1/2*d*x)^6*tan(1/2*
c)^2 + 2048*b^2*tan(1/2*d*x)^6*tan(1/2*c)^2 + 540*pi*b^2*sgn(tan(1/2*d*x)^2
*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2
 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^3*tan(1/2*c)^3 - 540*pi*b^2*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1
/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^3*tan(1/2*c)^3 + 1080*pi*b^2*sgn
(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) -
 tan(1/2*c)^2 + 1)*tan(1/2*d*x)^3*tan(1/2*c)^3 + 24064*a^2*tan(1/2*d*x)^5*t
an(1/2*c)^3 - 22528*b^2*tan(1/2*d*x)^5*tan(1/2*c)^3 + 30208*a^2*tan(1/2*d*x
)^4*tan(1/2*c)^4 - 10240*b^2*tan(1/2*d*x)^4*tan(1/2*c)^4 + 180*pi*b^2*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
 tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)*tan(1/2*c)^5 - 180*pi*b^2*
sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x
)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)*tan(1/2*c)^5 + 360*pi
*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(
1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)*tan(1/2*c)^5 + 24064*a^2*tan(1/2*d*
x)^3*tan(1/2*c)^5 - 22528*b^2*tan(1/2*d*x)^3*tan(1/2*c)^5 + 30*pi*b^2*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
 tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^6 - 30*pi*b^2*sgn(tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*
c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^6 + 60*pi*b^2*sgn(tan(1/2*d*x)^2*tan(
1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*t
an(1/2*c)^6 + 4096*a^2*tan(1/2*d*x)^2*tan(1/2*c)^6 + 2048*b^2*tan(1/2*d*x)^
2*tan(1/2*c)^6 - 512*a^2*tan(1/2*d*x)*tan(1/2*c)^7 + 2048*b^2*tan(1/2*d*x)*
tan(1/2*c)^7 - 128*a^2*tan(1/2*c)^8 + 512*b^2*tan(1/2*c)^8 - 60*pi*b^2*tan(
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1/2*d*x)^6 + 60*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/
2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/
2*d*x)^6 + 60*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*
c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*
d*x)^6 - 60*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c)
 - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*
x)^6 - 60*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) -
 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)
^6 + 384*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*
c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/
2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6 - 384*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c
)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d
*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*t
an(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6 + 768*a*b*
tan(1/2*d*x)^7 - 180*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x
)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(
1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + ta
n(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^3*tan(1/2*c) - 180*pi*b^2*sgn
(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2
 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan
(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1
)*tan(1/2*d*x)^3*tan(1/2*c) - 360*pi*b^2*tan(1/2*d*x)^5*tan(1/2*c) + 360*b^
2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2
*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^5*tan(1/2*c
) + 360*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1
)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^5
*tan(1/2*c) - 360*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(
1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(
1/2*d*x)^5*tan(1/2*c) - 360*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d
*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c)
- 1))*tan(1/2*d*x)^5*tan(1/2*c) + 2304*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)
^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*
x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*ta
n(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^5*tan(1/2*c)
- 2304*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c)
 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*
d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + ta
n(1/2*c)^2 + 1))*tan(1/2*d*x)^5*tan(1/2*c) + 60*pi*b^2*sgn(tan(1/2*d*x)^2*t
an(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 +
 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2
*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*
tan(1/2*c)^2 + 60*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2
*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1
```

/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 28672*a*b*tan(1
/2*d*x)^5*tan(1/2*c)^2 - 180*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan
(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*
sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x
)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)*tan(1/2*c)^3 - 180*pi
*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/
2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*
d*x) - 1)*tan(1/2*d*x)*tan(1/2*c)^3 - 1080*pi*b^2*tan(1/2*d*x)^3*tan(1/2*c)
^3 + 1080*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) +
 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)
^3*tan(1/2*c)^3 + 1080*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) +
 tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))
*tan(1/2*d*x)^3*tan(1/2*c)^3 - 1080*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + t
an(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan
(1/2*c) - 1))*tan(1/2*d*x)^3*tan(1/2*c)^3 - 1080*b^2*arctan((tan(1/2*d*x)*t
an(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1
/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^3*tan(1/2*c)^3 + 6912*a*b*log(2*(ta
n(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*ta
n(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c)
+ 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan
(1/2*d*x)^3*tan(1/2*c)^3 - 6912*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*
tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c
)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c)^3 - 537
60*a*b*tan(1/2*d*x)^4*tan(1/2*c)^3 - 53760*a*b*tan(1/2*d*x)^3*tan(1/2*c)^4
- 360*pi*b^2*tan(1/2*d*x)*tan(1/2*c)^5 + 360*b^2*arctan((tan(1/2*d*x)*tan(1
/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d
*x) + tan(1/2*c) + 1))*tan(1/2*d*x)*tan(1/2*c)^5 + 360*b^2*arctan((tan(1/2*
d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) +
 tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)*tan(1/2*c)^5 - 360*b^2*arctan
((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*ta
n(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)*tan(1/2*c)^5 - 360*
b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1
/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)*tan(1/2*c
)^5 + 2304*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/
2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(
1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2
+ tan(1/2*c)^2 + 1))*tan(1/2*d*x)*tan(1/2*c)^5 - 2304*a*b*log(2*(tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)
^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(ta
n(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x
)*tan(1/2*c)^5 - 28672*a*b*tan(1/2*d*x)^2*tan(1/2*c)^5 - 60*pi*b^2*tan(1/2*
c)^6 + 60*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) +
 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*c)^6

+ 60*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*c)^6 - 60*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*c)^6 - 60*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*c)^6 + 384*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^6 - 384*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^6 + 768*a*b*tan(1/2*c)^7 - 256*a^2*tan(1/2*d*x)^6 + 1024*b^2*tan(1/2*d*x)^6 + 180*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^3*tan(1/2*c) - 180*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^3*tan(1/2*c) + 360*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^3*tan(1/2*c) - 4608*a^2*tan(1/2*d*x)^5*tan(1/2*c) + 6144*b^2*tan(1/2*d*x)^5*tan(1/2*c) - 60*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 60*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 120*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 19968*a^2*tan(1/2*d*x)^4*tan(1/2*c)^2 + 30720*b^2*tan(1/2*d*x)^4*tan(1/2*c)^2 + 180*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)*tan(1/2*c)^3 - 180*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)*tan(1/2*c)^3 + 360*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)*tan(1/2*c)^3 - 24064*a^2*tan(1/2*d*x)^3*tan(1/2*c)^3 + 22528*b^2*tan(1/2*d*x)^3*tan(1/2*c)^3 - 19968*a^2*tan(1/2*d*x)^2*tan(1/2*c)^4 + 30720*b^2*tan(1/2*d*x)^2*tan(1/2*c)^4 - 4608*a^2*tan(1/2*d*x)*tan(1/2*c)^5 + 6144*b^2*tan(1/2*d*x)*tan(1/2*c)^5 - 256*a^2*tan(1/2*c)^6 + 1024*b^2*tan(1/2*c)^6 + 30*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2 + 30*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2 + 1792*a*b*tan(1/2*d*x)^5 - 60*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/

2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*ta
n(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2
 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)*tan(1/2
*c) - 60*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*
c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*t
an(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 -
 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)*tan(1/2*c) - 360*pi*b^2*tan(1/2*d*x)^3*ta
n(1/2*c) + 360*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2
*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2
*d*x)^3*tan(1/2*c) + 360*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x)
 + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1
))*tan(1/2*d*x)^3*tan(1/2*c) - 360*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + ta
n(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(
1/2*c) - 1))*tan(1/2*d*x)^3*tan(1/2*c) - 360*b^2*arctan((tan(1/2*d*x)*tan(1
/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d
*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^3*tan(1/2*c) + 2304*a*b*log(2*(tan(1/2*
d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*
c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(
tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d
*x)^3*tan(1/2*c) - 2304*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*
d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*
c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + ta
n(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c) + 17920*a*b*tan
(1/2*d*x)^4*tan(1/2*c) + 30*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(
1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*s
gn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)
^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^2 + 30*pi*b^2*sgn(tan(1/
2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(
1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*
x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1
/2*c)^2 + 120*pi*b^2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 120*b^2*arctan((tan(1/2*
d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) -
 tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 120*b^2*arct
an((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*
tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 +
120*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(t
an(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan
(1/2*c)^2 + 120*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/
2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/
2*d*x)^2*tan(1/2*c)^2 - 768*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(
1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(
1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2
+ tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 768*a*b
log(2(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1
/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*t

an(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 28672*a*b*tan(1/2*d*x)^3*tan(1/2*c)^2 - 360*pi*b^2*tan(1/2*d*x)*tan(1/2*c)^3 + 360*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)*tan(1/2*c)^3 + 360*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)*tan(1/2*c)^3 - 360*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)*tan(1/2*c)^3 - 360*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)*tan(1/2*c)^3 + 2304*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)*tan(1/2*c)^3 - 2304*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)*tan(1/2*c)^3 + 28672*a*b*tan(1/2*d*x)^2*tan(1/2*c)^3 + 17920*a*b*tan(1/2*d*x)*tan(1/2*c)^4 + 1792*a*b*tan(1/2*c)^5 - 30*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2 + 30*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2 - 60*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2 + 768*a^2*tan(1/2*d*x)^4 + 60*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)*tan(1/2*c) - 60*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)*tan(1/2*c) + 120*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)*tan(1/2*c) + 4608*a^2*tan(1/2*d*x)^3*tan(1/2*c) - 6144*b^2*tan(1/2*d*x)^3*tan(1/2*c) - 30*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^2 + 30*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^2 - 60*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^2 + 4096*a^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2048*b^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4608*a^2*tan(1/2*d*x)*tan(1/2*c)^3 - 6144*b^2*tan(1/2*d*x)*tan(1/2*c)^3 + 768*a^2*tan(1/2*c)^4 + 15*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1) + 15*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*

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x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*
c)^2 - 2*tan(1/2*d*x) - 1) + 60*pi*b^2*tan(1/2*d*x)^2 - 60*b^2*arctan((tan(
1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*
c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2 - 60*b^2*arctan((tan(1/
2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c)
 + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2 + 60*b^2*arctan((tan(1/2*
d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) -
 tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2 + 60*b^2*arctan((tan(1/2*d*
x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + t
an(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^2 - 384*a*b*log(2*(tan(1/2*d*x)
^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2
 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(
1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^
2 + 384*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c
) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2
*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + t
an(1/2*c)^2 + 1))*tan(1/2*d*x)^2 - 1792*a*b*tan(1/2*d*x)^3 - 120*pi*b^2*tan
(1/2*d*x)*tan(1/2*c) + 120*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*
x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) +
 1))*tan(1/2*d*x)*tan(1/2*c) + 120*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - ta
n(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(
1/2*c) + 1))*tan(1/2*d*x)*tan(1/2*c) - 120*b^2*arctan((tan(1/2*d*x)*tan(1/2
*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x
) - tan(1/2*c) - 1))*tan(1/2*d*x)*tan(1/2*c) - 120*b^2*arctan((tan(1/2*d*x)
*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan
(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)*tan(1/2*c) + 768*a*b*log(2*(tan(1
/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1
/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1
)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/
2*d*x)*tan(1/2*c) - 768*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*
d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*
c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + ta
n(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)*tan(1/2*c) + 60*pi*b^2*tan(1
/2*c)^2 - 60*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c
) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*c
)^2 - 60*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) +
1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*c)^2
+ 60*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(
tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*c)^2 + 60
*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(
1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*c)^2 - 384*a*
b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(
1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*
tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^
2 + 1))*tan(1/2*c)^2 + 384*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1
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/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1
/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 +
 tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^2 - 1792*a*b*tan(1/2*c)^3 -
 15*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 -
tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1) + 15*pi*b^2*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1
/2*c)^2 - 2*tan(1/2*d*x) - 1) - 30*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 -
 tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1) - 256*a^2*t
an(1/2*d*x)^2 + 1024*b^2*tan(1/2*d*x)^2 + 512*a^2*tan(1/2*d*x)*tan(1/2*c) -
 2048*b^2*tan(1/2*d*x)*tan(1/2*c) - 256*a^2*tan(1/2*c)^2 + 1024*b^2*tan(1/2
*c)^2 + 30*pi*b^2 - 30*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) -
 tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))
 - 30*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/
(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)) + 30*b^2*arctan(
(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan
(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)) + 30*b^2*arctan((tan(1/2*d*x)*tan
(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2
*d*x) + tan(1/2*c) - 1)) - 192*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*t
an(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + t
an(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)
^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1)) + 192*a*b*log(2*(tan(1/2*d*x)^2*ta
n(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + ta
n(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d
*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1)) - 768*a*b*tan(1/2*
d*x) - 768*a*b*tan(1/2*c) - 128*a^2 + 512*b^2)/(d*tan(1/2*d*x)^8*tan(1/2*c)
^8 + 2*d*tan(1/2*d*x)^8*tan(1/2*c)^6 - 4*d*tan(1/2*d*x)^7*tan(1/2*c)^7 + 2*
d*tan(1/2*d*x)^6*tan(1/2*c)^8 - 12*d*tan(1/2*d*x)^7*tan(1/2*c)^5 + 4*d*tan(
1/2*d*x)^6*tan(1/2*c)^6 - 12*d*tan(1/2*d*x)^5*tan(1/2*c)^7 - 2*d*tan(1/2*d*
x)^8*tan(1/2*c)^2 - 12*d*tan(1/2*d*x)^7*tan(1/2*c)^3 - 36*d*tan(1/2*d*x)^5*
tan(1/2*c)^5 - 12*d*tan(1/2*d*x)^3*tan(1/2*c)^7 - 2*d*tan(1/2*d*x)^2*tan(1/
2*c)^8 - d*tan(1/2*d*x)^8 - 4*d*tan(1/2*d*x)^7*tan(1/2*c) - 4*d*tan(1/2*d*x
)^6*tan(1/2*c)^2 - 36*d*tan(1/2*d*x)^5*tan(1/2*c)^3 - 36*d*tan(1/2*d*x)^3*t
an(1/2*c)^5 - 4*d*tan(1/2*d*x)^2*tan(1/2*c)^6 - 4*d*tan(1/2*d*x)*tan(1/2*c)
^7 - d*tan(1/2*c)^8 - 2*d*tan(1/2*d*x)^6 - 12*d*tan(1/2*d*x)^5*tan(1/2*c) -
 36*d*tan(1/2*d*x)^3*tan(1/2*c)^3 - 12*d*tan(1/2*d*x)*tan(1/2*c)^5 - 2*d*ta
n(1/2*c)^6 - 12*d*tan(1/2*d*x)^3*tan(1/2*c) + 4*d*tan(1/2*d*x)^2*tan(1/2*c)
^2 - 12*d*tan(1/2*d*x)*tan(1/2*c)^3 + 2*d*tan(1/2*d*x)^2 - 4*d*tan(1/2*d*x)
*tan(1/2*c) + 2*d*tan(1/2*c)^2 + d)
```

Mupad [B] (verification not implemented)

Time = 7.45 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.43

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{4\,a\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(\frac{8\,a^2}{3}-\frac{32\,b^2}{3}\right)-4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{4\,a^2}{3}-\frac{16\,b^2}{3}+\frac{28\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}-\frac{28\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{3}-4\,a}{d\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

```
[In] int(sin(c + d*x)^3*(a + b*tan(c + d*x))^2,x)
```

```
[Out] (4*a*b*atanh(tan(c/2 + (d*x)/2)))/d - (tan(c/2 + (d*x)/2)^2*((8*a^2)/3 - (3
2*b^2)/3) - 4*a^2*tan(c/2 + (d*x)/2)^4 + (4*a^2)/3 - (16*b^2)/3 + (28*a*b*t
an(c/2 + (d*x)/2)^3)/3 - (28*a*b*tan(c/2 + (d*x)/2)^5)/3 - 4*a*b*tan(c/2 +
(d*x)/2)^7 + 4*a*b*tan(c/2 + (d*x)/2))/(d*(2*tan(c/2 + (d*x)/2)^2 - 2*tan(c
/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^8 + 1))
```

3.24 $\int \sin^2(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 76

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{1}{2}\left(a^2-3b^2\right)x-\frac{2ab\log(\cos(c+dx))}{d}+\frac{3b^2\tan(c+dx)}{2d}-\frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^2}{2d}$$

```
[Out] 1/2*(a^2-3*b^2)*x-2*a*b*ln(cos(d*x+c))/d+3/2*b^2*tan(d*x+c)/d-1/2*cos(d*x+c
)*sin(d*x+c)*(a+b*tan(d*x+c))^2/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3597, 1659, 788, 649, 209, 266}

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{1}{2}x\left(a^2-3b^2\right)-\frac{2ab\log(\cos(c+dx))}{d}-\frac{\sin(c+dx)\cos(c+dx)(a+b\tan(c+dx))^2}{2d}+\frac{3b^2\tan(c+dx)}{2d}$$

```
[In] Int[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((a^2 - 3*b^2)*x)/2 - (2*a*b*Log[Cos[c + d*x]])/d + (3*b^2*Tan[c + d*x])/(2
*d) - (Cos[c + d*x]*Sin[c + d*x]*(a + b*Tan[c + d*x])^2)/(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 788

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{b\text{Subst}\left(\int \frac{x^2(a+x)^2}{(b^2+x^2)^2}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^2}{2d} - \frac{\operatorname{Subst}\left(\int \frac{(a+x)(-ab^2-3b^2x)}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{2bd}$$

$$= \frac{3b^2\tan(c+dx)}{2d} - \frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^2}{2d} - \frac{\operatorname{Subst}\left(\int \frac{-a^2b^2+3b^4-4ab^2x}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{2bd}$$

$$= \frac{3b^2\tan(c+dx)}{2d} - \frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^2}{2d} + \frac{(2ab)\operatorname{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{d} + \frac{(b(a^2-3b^2))\operatorname{Subst}\left(\int \frac{1}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{2d}$$

$$= \frac{1}{2}\left(a^2-3b^2\right)x - \frac{2ab\log(\cos(c+dx))}{d} + \frac{3b^2\tan(c+dx)}{2d} - \frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^2}{2d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 162 vs. 2(76) = 152.

Time = 2.70 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.13

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{b\left(\frac{(-a^2+b^2)\arctan(\tan(c+dx))}{b} + 2a\cos^2(c+dx) + \left(2a+\frac{a^2-2b^2}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right) + \left(2a+\frac{-a^2+2b^2}{\sqrt{-b^2}}\right)}{2d}$$

```
[In] Integrate[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (b*(((-a^2 + b^2)*ArcTan[Tan[c + d*x]])/b + 2*a*Cos[c + d*x]^2 + (2*a + (a^
2 - 2*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]] + (2*a + (-a^2 + 2*
b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + ((-a^2 + b^2)*Sin[2*(c
+ d*x)])/(2*b) + 2*b*Tan[c + d*x]))/(2*d)
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.43

method	result
derivativedivides	$\frac{a^2\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2ab\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+b^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\right.}{d}$
default	$\frac{a^2\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2ab\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+b^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\right.}{d}$
risch	$2iabx+\frac{a^2x}{2}-\frac{3b^2x}{2}+\frac{e^{2i(dx+c)}ab}{4d}+\frac{ie^{2i(dx+c)}a^2}{8d}-\frac{ie^{2i(dx+c)}b^2}{8d}+\frac{e^{-2i(dx+c)}ab}{4d}-\frac{ie^{-2i(dx+c)}a^2}{8d}+ie^{-}$

```
[In] int(sin(d*x+c)^2*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+2*a*b*(-1/2*sin(d*x+c)^
2-ln(cos(d*x+c)))+b^2*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c)
)*cos(d*x+c)-3/2*d*x-3/2*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.33

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{2\,ab\cos\left(dx+c\right)^3-4\,ab\cos\left(dx+c\right)\log\left(-\cos\left(dx+c\right)\right)+\left(\left(a^2-3\,b^2\right)dx-ab\right)\cos\left(dx+c\right)-\left(\left(a^2-b^2\right)\right.}{2\,d\cos\left(dx+c\right)}$$

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*cos(d*x + c)^3 - 4*a*b*cos(d*x + c)*log(-cos(d*x + c)) + ((a^2 -
 3*b^2)*d*x - a*b)*cos(d*x + c) - ((a^2 - b^2)*cos(d*x + c)^2 - 2*b^2)*sin(
d*x + c))/(d*cos(d*x + c))
```

Sympy [F]

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^2\,dx=\int\left(a+b\tan\left(c+dx\right)\right)^2\sin^2\left(c+dx\right)dx$$

```
[In] integrate(sin(d*x+c)**2*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*sin(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{2\,ab\log\left(\tan\left(dx+c\right)^2+1\right)+2\,b^2\tan\left(dx+c\right)+(a^2-3\,b^2)(dx+c)+\frac{2\,ab-(a^2-b^2)\tan(dx+c)}{\tan(dx+c)^2+1}}{2\,d}$$

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*a*b*log(tan(d*x + c)^2 + 1) + 2*b^2*tan(d*x + c) + (a^2 - 3*b^2)*(d*
x + c) + (2*a*b - (a^2 - b^2)*tan(d*x + c))/(tan(d*x + c)^2 + 1))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 981 vs. 2(70) = 140.

Time = 0.62 (sec) , antiderivative size = 981, normalized size of antiderivative = 12.91

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^2\,dx = \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(a^2*d*x*tan(d*x)^3*tan(c)^3 - 3*b^2*d*x*tan(d*x)^3*tan(c)^3 - 2*a*b*lo
g(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + ta
n(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 + a^2*d*x*tan(d*x)^3*tan(c) -
 3*b^2*d*x*tan(d*x)^3*tan(c) - a^2*d*x*tan(d*x)^2*tan(c)^2 + 3*b^2*d*x*tan(
d*x)^2*tan(c)^2 + a^2*d*x*tan(d*x)*tan(c)^3 - 3*b^2*d*x*tan(d*x)*tan(c)^3 +
 a*b*tan(d*x)^3*tan(c)^3 - 2*a*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*ta
n(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^3*tan
(c) + 2*a*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2
*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 + a^2*tan(d*x)^
3*tan(c)^2 - 3*b^2*tan(d*x)^3*tan(c)^2 - 2*a*b*log(4*(tan(d*x)^2*tan(c)^2 -
 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*
tan(d*x)*tan(c)^3 + a^2*tan(d*x)^2*tan(c)^3 - 3*b^2*tan(d*x)^2*tan(c)^3 - a
^2*d*x*tan(d*x)^2 + 3*b^2*d*x*tan(d*x)^2 + a^2*d*x*tan(d*x)*tan(c) - 3*b^2*
d*x*tan(d*x)*tan(c) - a*b*tan(d*x)^3*tan(c) - a^2*d*x*tan(c)^2 + 3*b^2*d*x*
tan(c)^2 - 5*a*b*tan(d*x)^2*tan(c)^2 - a*b*tan(d*x)*tan(c)^3 + 2*a*b*log(4*
(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*
x)^2 + tan(c)^2 + 1))*tan(d*x)^2 - 2*b^2*tan(d*x)^3 - 2*a*b*log(4*(tan(d*x)
^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + ta
n(c)^2 + 1))*tan(d*x)*tan(c) - 2*a^2*tan(d*x)^2*tan(c) + 2*a*b*log(4*(tan(d
```

```
*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 +
 tan(c)^2 + 1))*tan(c)^2 - 2*a^2*tan(d*x)*tan(c)^2 - 2*b^2*tan(c)^3 - a^2*d
*x + 3*b^2*d*x + a*b*tan(d*x)^2 + 5*a*b*tan(d*x)*tan(c) + a*b*tan(c)^2 + 2*
a*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^
2 + tan(d*x)^2 + tan(c)^2 + 1)) + a^2*tan(d*x) - 3*b^2*tan(d*x) + a^2*tan(c
) - 3*b^2*tan(c) - a*b)/(d*tan(d*x)^3*tan(c)^3 + d*tan(d*x)^3*tan(c) - d*ta
n(d*x)^2*tan(c)^2 + d*tan(d*x)*tan(c)^3 - d*tan(d*x)^2 + d*tan(d*x)*tan(c)
- d*tan(c)^2 - d)
```

Mupad [B] (verification not implemented)

Time = 4.62 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{\cos\left(c+d\,x\right)^2\,\left(a\,b-\tan(c+d\,x)\,\left(\frac{a^2}{2}-\frac{b^2}{2}\right)\right)+b^2\tan(c+d\,x)+a\,b\,\ln\left(\tan(c+d\,x)^2+1\right)+d\,x\,\left(\frac{a^2}{2}-\frac{3}{\ldots}\right.}{d}$$

```
[In] int(sin(c + d*x)^2*(a + b*tan(c + d*x))^2,x)
```

```
[Out] (cos(c + d*x)^2*(a*b - tan(c + d*x)*(a^2/2 - b^2/2)) + b^2*tan(c + d*x) + a
*b*log(tan(c + d*x)^2 + 1) + d*x*(a^2/2 - (3*b^2)/2))/d
```

3.25 $\int \sin(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 19, antiderivative size = 68

$$\int \sin(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{2ab\text{arctanh}(\sin(c+dx))}{d} - \frac{a^2\cos(c+dx)}{d} + \frac{b^2\cos(c+dx)}{d} + \frac{b^2\sec(c+dx)}{d} - \frac{2ab\sin(c+dx)}{d}$$

```
[Out] 2*a*b*arctanh(sin(d*x+c))/d-a^2*cos(d*x+c)/d+b^2*cos(d*x+c)/d+b^2*sec(d*x+c
)/d-2*a*b*sin(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3598, 2718, 2672, 327, 212, 2670, 14}

$$\int \sin(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{a^2\cos(c+dx)}{d} + \frac{2ab\text{arctanh}(\sin(c+dx))}{d} - \frac{2ab\sin(c+dx)}{d} + \frac{b^2\cos(c+dx)}{d} + \frac{b^2\sec(c+dx)}{d}$$

```
[In] Int[Sin[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (2*a*b*ArcTanh[Sin[c + d*x]])/d - (a^2*Cos[c + d*x])/d + (b^2*Cos[c + d*x])
/d + (b^2*Sec[c + d*x])/d - (2*a*b*Sin[c + d*x])/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(a^2 \sin(c+dx) + 2ab\sin(c+dx)\tan(c+dx) + b^2\sin(c+dx)\tan^2(c+dx)\right)\,dx$$

$$= a^2 \int \sin(c+dx)\,dx + (2ab)\int \sin(c+dx)\tan(c+dx)\,dx + b^2\int \sin(c+dx)\tan^2(c+dx)\,dx$$

$$= -\frac{a^2\cos(c+dx)}{d} + \frac{(2ab)\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d} - \frac{b^2\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{a^2\cos(c+dx)}{d} - \frac{2ab\sin(c+dx)}{d} + \frac{(2ab)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d} - \frac{b^2\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)\,dx, x, \cos(c+dx)\right)}{d}$$

$$= \frac{2ab\text{arctanh}(\sin(c+dx))}{d} - \frac{a^2\cos(c+dx)}{d} + \frac{b^2\cos(c+dx)}{d} + \frac{b^2\sec(c+dx)}{d} - \frac{2ab\sin(c+dx)}{d}$$

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.63

$$\int \sin(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{\sec(c+dx)\left(a^2-3b^2+\left(a^2-b^2\right)\cos(2(c+dx))+4ab\cos(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\sin\left(\frac{1}{2}(c+dx\right.\right.\right.}{2d}$$

```
[In] Integrate[Sin[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -1/2*(Sec[c + d*x]*(a^2 - 3*b^2 + (a^2 - b^2)*Cos[2*(c + d*x)] + 4*a*b*Cos[
c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]]) + 2*a*b*Sin[2*(c + d*x)]))/d
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{-a^{2} \cos(dx+c)+2ab(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+b^{2}\left(\frac{\sin^{4}(dx+c)}{\cos(dx+c)}+(2+\sin^{2}(dx+c))\cos(dx+c)\right)}{d}$
default	$\frac{-a^{2} \cos(dx+c)+2ab(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+b^{2}\left(\frac{\sin^{4}(dx+c)}{\cos(dx+c)}+(2+\sin^{2}(dx+c))\cos(dx+c)\right)}{d}$
risch	$\frac{i e^{i(dx+c)} ab}{d}-\frac{e^{i(dx+c)} a^{2}}{2d}+\frac{e^{i(dx+c)} b^{2}}{2d}-\frac{i e^{-i(dx+c)} ab}{d}-\frac{e^{-i(dx+c)} a^{2}}{2d}+\frac{e^{-i(dx+c)} b^{2}}{2d}+\frac{2b^{2} e^{i(dx+c)}}{d(e^{2i(dx+c)}+1)}-$ 2

```
[In] int(sin(d*x+c)*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(-a^2*cos(d*x+c)+2*a*b*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+b^2*(sin
(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.32

$$\int \sin(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{ab\cos(dx+c)\log(\sin(dx+c)+1)-ab\cos(dx+c)\log(-\sin(dx+c)+1)-2\,ab\cos(dx+c)\sin(dx+}{d\cos(dx+c)}$$

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
[Out] (a*b*cos(d*x + c)*log(sin(d*x + c) + 1) - a*b*cos(d*x + c)*log(-sin(d*x + c
) + 1) - 2*a*b*cos(d*x + c)*sin(d*x + c) - (a^2 - b^2)*cos(d*x + c)^2 + b^2
)/(d*cos(d*x + c))
```

Sympy [F]

$$\int \sin(c+dx)(a+b\tan(c+dx))^2\,dx = \int (a+b\tan(c+dx))^2 \sin(c+dx)\,dx$$

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))**2,x)
[Out] Integral((a + b*tan(c + d*x))**2*sin(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.99

$$\int \sin(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{b^2\left(\frac{1}{\cos(dx+c)}+\cos\left(dx+c\right)\right)+ab(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)-2\,\sin\left(dx+c\right))-a^2\cos\left(\right.}{d}$$

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (b^2*(1/cos(d*x + c) + cos(d*x + c)) + a*b*(log(sin(d*x + c) + 1) - log(sin
(d*x + c) - 1) - 2*sin(d*x + c)) - a^2*cos(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2405 vs. 2(68) = 136.

Time = 1.08 (sec) , antiderivative size = 2405, normalized size of antiderivative = 35.37

$$\int \sin(c+dx)(a+b\tan(c+dx))^2\,dx = \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*
tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x)
- 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2
*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 - a*b*log(2*(tan(1/2*d*x)^2*tan(1/2
*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2
*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2
*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*
c)^4 + a^2*tan(1/2*d*x)^4*tan(1/2*c)^4 - 2*b^2*tan(1/2*d*x)^4*tan(1/2*c)^4
- 4*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) +
2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x
) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1
/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c)^3 + 4*a*b*log(2*(tan(1/2*d*x)^2*tan
(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan
(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*
x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(
1/2*c)^3 - 4*a*b*tan(1/2*d*x)^4*tan(1/2*c)^3 - 4*a*b*tan(1/2*d*x)^3*tan(1/2
*c)^4 - 2*a^2*tan(1/2*d*x)^4*tan(1/2*c)^2 - 8*a^2*tan(1/2*d*x)^3*tan(1/2*c)
^3 + 8*b^2*tan(1/2*d*x)^3*tan(1/2*c)^3 - 2*a^2*tan(1/2*d*x)^2*tan(1/2*c)^4
- a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*
```

```
tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x)
- 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2
*c)^2 + 1))*tan(1/2*d*x)^4 + a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan
(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan
(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2
 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4 - 4*a*b*log(2*(tan(1/
2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/
2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)
/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2
*d*x)^3*tan(1/2*c) + 4*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d
*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c
)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan
(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c) + 4*a*b*tan(1/2*
d*x)^4*tan(1/2*c) + 24*a*b*tan(1/2*d*x)^3*tan(1/2*c)^2 - 4*a*b*log(2*(tan(1
/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1
/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1
)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/
2*d*x)*tan(1/2*c)^3 + 4*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*
d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*
c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + ta
n(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)*tan(1/2*c)^3 + 24*a*b*tan(1/
2*d*x)^2*tan(1/2*c)^3 - a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*
c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + ta
n(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^4 + a*b*log(2*(tan(1/2*d*x)^2*
tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 +
tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2
*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^4 + 4
*a*b*tan(1/2*d*x)*tan(1/2*c)^4 + a^2*tan(1/2*d*x)^4 - 2*b^2*tan(1/2*d*x)^4
+ 8*a^2*tan(1/2*d*x)^3*tan(1/2*c) - 8*b^2*tan(1/2*d*x)^3*tan(1/2*c) + 20*a^
2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 24*b^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 8*a^2*
tan(1/2*d*x)*tan(1/2*c)^3 - 8*b^2*tan(1/2*d*x)*tan(1/2*c)^3 + a^2*tan(1/2*c
)^4 - 2*b^2*tan(1/2*c)^4 - 4*a*b*tan(1/2*d*x)^3 - 4*a*b*log(2*(tan(1/2*d*x)
^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2
 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(
1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)*
tan(1/2*c) + 4*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*ta
n(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*
tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x
)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)*tan(1/2*c) - 24*a*b*tan(1/2*d*x)^2*ta
n(1/2*c) - 24*a*b*tan(1/2*d*x)*tan(1/2*c)^2 - 4*a*b*tan(1/2*c)^3 - 2*a^2*ta
n(1/2*d*x)^2 - 8*a^2*tan(1/2*d*x)*tan(1/2*c) + 8*b^2*tan(1/2*d*x)*tan(1/2*c
) - 2*a^2*tan(1/2*c)^2 + a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2
*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2
*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + t
```

```
an(1/2*d*x)^2 + tan(1/2*c)^2 + 1)) - a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2
 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)
^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(
1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1)) + 4*a*b*tan(1/2*d*x) + 4*a*b
*tan(1/2*c) + a^2 - 2*b^2)/(d*tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*d*tan(1/2*d*x
)^3*tan(1/2*c)^3 - d*tan(1/2*d*x)^4 - 4*d*tan(1/2*d*x)^3*tan(1/2*c) - 4*d*t
an(1/2*d*x)*tan(1/2*c)^3 - d*tan(1/2*c)^4 - 4*d*tan(1/2*d*x)*tan(1/2*c) + d
)
```

Mupad [B] (verification not implemented)

Time = 4.79 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.37

$$\int \sin(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{4\,a\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-2\,a^2+4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+4\,b^2}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-1\right)}$$

```
[In] int(sin(c + d*x)*(a + b*tan(c + d*x))^2,x)
```

```
[Out] (4*a*b*atanh(tan(c/2 + (d*x)/2)))/d - (2*a^2*tan(c/2 + (d*x)/2)^2 - 2*a^2 +
 4*b^2 + 4*a*b*tan(c/2 + (d*x)/2)^3 - 4*a*b*tan(c/2 + (d*x)/2))/(d*(tan(c/2
 + (d*x)/2)^4 - 1))
```

3.26 $\int \csc(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 19, antiderivative size = 43

$$\int \csc(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{a^2\text{arctanh}(\cos(c+dx))}{d} + \frac{2ab\text{arctanh}(\sin(c+dx))}{d} + \frac{b^2\sec(c+dx)}{d}$$

```
[Out] -a^2*arctanh(cos(d*x+c))/d+2*a*b*arctanh(sin(d*x+c))/d+b^2*sec(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3598, 3855, 2686, 8}

$$\int \csc(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{a^2\text{arctanh}(\cos(c+dx))}{d} + \frac{2ab\text{arctanh}(\sin(c+dx))}{d} + \frac{b^2\sec(c+dx)}{d}$$

```
[In] Int[Csc[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -((a^2*ArcTanh[Cos[c + d*x]])/d) + (2*a*b*ArcTanh[Sin[c + d*x]])/d + (b^2*S
ec[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^2\csc(c+dx)+2ab\sec(c+dx)+b^2\sec(c+dx)\tan(c+dx)\right)\,dx\\ &= a^2\int \csc(c+dx)\,dx+(2ab)\int \sec(c+dx)\,dx+b^2\int \sec(c+dx)\tan(c+dx)\,dx\\ &= -\frac{a^2\text{arctanh}(\cos(c+dx))}{d}+\frac{2ab\text{arctanh}(\sin(c+dx))}{d}+\frac{b^2\text{Subst}(\int 1\,dx,x,\sec(c+dx))}{d}\\ &= -\frac{a^2\text{arctanh}(\cos(c+dx))}{d}+\frac{2ab\text{arctanh}(\sin(c+dx))}{d}+\frac{b^2\sec(c+dx)}{d}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 97 vs. $2(43) = 86$.

Time = 1.15 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.26

$$\int \csc(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{a\left(-a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+2b\log\right.}{d}$$

```
[In] Integrate[Csc[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (a*(-(a*Log[Cos[(c + d*x)/2]]) - 2*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2
]] + a*Log[Sin[(c + d*x)/2]] + 2*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
) + b^2*Sec[c + d*x])/d
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.30

method	result	size
derivativedivides	$\frac{\frac{b^2}{\cos(dx+c)}+2ab\ln(\sec(dx+c)+\tan(dx+c))+a^2\ln(\csc(dx+c)-\cot(dx+c))}{d}$	56
default	$\frac{\frac{b^2}{\cos(dx+c)}+2ab\ln(\sec(dx+c)+\tan(dx+c))+a^2\ln(\csc(dx+c)-\cot(dx+c))}{d}$	56
risch	$\frac{2b^2 e^{i(dx+c)}}{d(e^{2i(dx+c)}+1)}+\frac{2ab\ln(e^{i(dx+c)}+i)}{d}-\frac{2ab\ln(e^{i(dx+c)}-i)}{d}+\frac{a^2\ln(e^{i(dx+c)}-1)}{d}-\frac{a^2\ln(e^{i(dx+c)}+1)}{d}$	111

```
[In] int(csc(d*x+c)*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(b^2/cos(d*x+c)+2*a*b*ln(sec(d*x+c)+tan(d*x+c))+a^2*ln(csc(d*x+c)-cot(d
*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(43) = 86.

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.37

$$\int \csc(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{a^2\cos(dx+c)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-a^2\cos(dx+c)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-2\,ab\cos(dx+c)\,l}{2\,d\cos(dx+c)}$$

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
[Out] -1/2*(a^2*cos(d*x + c)*log(1/2*cos(d*x + c) + 1/2) - a^2*cos(d*x + c)*log(-
1/2*cos(d*x + c) + 1/2) - 2*a*b*cos(d*x + c)*log(sin(d*x + c) + 1) + 2*a*b*
cos(d*x + c)*log(-sin(d*x + c) + 1) - 2*b^2)/(d*cos(d*x + c))
```

Sympy [F]

$$\int \csc(c+dx)(a+b\tan(c+dx))^2\,dx = \int (a+b\tan(c+dx))^2\csc(c+dx)\,dx$$

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))**2,x)
[Out] Integral((a + b*tan(c + d*x))**2*csc(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.40

$$\int \csc(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{ab(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))-a^2\log(\cot(dx+c)+\csc(dx+c))+\frac{b^2}{\cos(dx+c)}}{d}$$

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (a*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) - a^2*log(cot(d*x + c)
 + csc(d*x + c)) + b^2/cos(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.72

$$\int \csc(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{2\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-2\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\,b^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}$$

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*a*b*log(abs(tan(1/2*d*x + 1/2
*c) - 1)) + a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 2*b^2/(tan(1/2*d*x + 1/2*c
)^2 - 1))/d
```

Mupad [B] (verification not implemented)

Time = 4.39 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.91

$$\int \csc(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{2\,b^2}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$
$$-\frac{4\,a\,b\,\mathrm{atanh}\left(\frac{16\,a^2\,b^2}{8\,a^3\,b-16\,a^2\,b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{8\,a^3\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a^3\,b-16\,a^2\,b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

```
[In] int((a + b*tan(c + d*x))^2/sin(c + d*x),x)
```

```
[Out] (a^2*log(tan(c/2 + (d*x)/2)))/d - (2*b^2)/(d*(tan(c/2 + (d*x)/2)^2 - 1)) -
(4*a*b*atanh((16*a^2*b^2)/(8*a^3*b - 16*a^2*b^2*tan(c/2 + (d*x)/2)) - (8*a^
3*b*tan(c/2 + (d*x)/2))/(8*a^3*b - 16*a^2*b^2*tan(c/2 + (d*x)/2))))/d
```

3.27 $\int \csc^2(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 42

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^2\,dx=-\frac{a^2\cot(c+dx)}{d}+\frac{2ab\log(\tan(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

```
[Out] -a^2*cot(d*x+c)/d+2*a*b*ln(tan(d*x+c))/d+b^2*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3597, 45}

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^2\,dx=-\frac{a^2\cot(c+dx)}{d}+\frac{2ab\log(\tan(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

```
[In] Int[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -((a^2*Cot[c + d*x])/d) + (2*a*b*Log[Tan[c + d*x]])/d + (b^2*Tan[c + d*x])/
d
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{b\text{Subst}\left(\int \frac{(a+x)^2}{x^2}\,dx, x, b\tan(c+dx)\right)}{d}\\ &= \frac{b\text{Subst}\left(\int \left(1+\frac{a^2}{x^2}+\frac{2a}{x}\right)\,dx, x, b\tan(c+dx)\right)}{d}\\ &= -\frac{a^2\cot(c+dx)}{d}+\frac{2ab\log(\tan(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 91 vs. $2(42) = 84$.

Time = 1.40 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.17

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{\cos(c+dx)\left(a\cos(c+dx)(a\cot(c+dx)+2b(\log(\cos(c+dx))-\log(\sin(c+dx))))-b^2\sin(c+dx)\right)(}{d(a\cos(c+dx)+b\sin(c+dx))^2}$$

```
[In] Integrate[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
[Out] -((Cos[c + d*x]*(a*Cos[c + d*x]*(a*Cot[c + d*x] + 2*b*(Log[Cos[c + d*x]] -
Log[Sin[c + d*x]])) - b^2*Sin[c + d*x])*(a + b*Tan[c + d*x])^2)/(d*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^2))
```

Maple [A] (verified)

Time = 1.91 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\frac{b^2\tan(dx+c)+2ab\ln(\tan(dx+c))-a^2\cot(dx+c)}{d}$	38
default	$\frac{b^2\tan(dx+c)+2ab\ln(\tan(dx+c))-a^2\cot(dx+c)}{d}$	38
risch	$-\frac{2i(a^2e^{2i(dx+c)}-b^2e^{2i(dx+c)}+a^2+b^2)}{d(e^{2i(dx+c)}-1)(e^{2i(dx+c)}+1)}+\frac{2ab\ln(e^{2i(dx+c)}-1)}{d}-\frac{2ab\ln(e^{2i(dx+c)}+1)}{d}$	106

```
[In] int(csc(d*x+c)^2*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(b^2*tan(d*x+c)+2*a*b*ln(tan(d*x+c))-a^2*cot(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(42) = 84$.

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.29

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{ab\cos(dx+c)\log\left(\cos(dx+c)^2\right)\sin(dx+c)-ab\cos(dx+c)\log\left(-\frac{1}{4}\cos(dx+c)^2+\frac{1}{4}\right)\sin(dx+c)+}{d\cos(dx+c)\sin(dx+c)}$$

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
[Out] -(a*b*cos(d*x + c)*log(cos(d*x + c)^2)*sin(d*x + c) - a*b*cos(d*x + c)*log(
-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) + (a^2 + b^2)*cos(d*x + c)^2 - b^2)
/(d*cos(d*x + c)*sin(d*x + c))
```

Sympy [F]

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^2\,dx = \int (a+b\tan(c+dx))^2\csc^2(c+dx)\,dx$$

```
[In] integrate(csc(d*x+c)**2*(a+b*tan(d*x+c))**2,x)
[Out] Integral((a + b*tan(c + d*x))**2*csc(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{2\,ab\log\left(\tan\left(dx+c\right)\right)+b^2\tan\left(dx+c\right)-\frac{a^2}{\tan(dx+c)}}{d}$$

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
[Out] (2*a*b*log(tan(d*x + c)) + b^2*tan(d*x + c) - a^2/tan(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{2\,ab\log\left(\left|\tan\left(dx+c\right)\right|\right)+b^2\tan\left(dx+c\right)-\frac{2\,ab\tan(dx+c)+a^2}{\tan(dx+c)}}{d}$$

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="giac")
[Out] (2*a*b*log(abs(tan(d*x + c))) + b^2*tan(d*x + c) - (2*a*b*tan(d*x + c) + a^
2)/tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 4.56 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{b^2\tan(c+d\,x)}{d}-\frac{a^2}{d\tan\left(c+d\,x\right)}+\frac{2\,a\,b\,\ln\left(\tan(c+d\,x)\right)}{d}$$

```
[In] int((a + b*tan(c + d*x))^2/sin(c + d*x)^2,x)
[Out] (b^2*tan(c + d*x))/d - a^2/(d*tan(c + d*x)) + (2*a*b*log(tan(c + d*x)))/d
```

3.28 $\int \csc^3(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 95

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{a^2\text{arctanh}(\cos(c+dx))}{2d} - \frac{b^2\text{arctanh}(\cos(c+dx))}{d} + \frac{2ab\text{arctanh}(\sin(c+dx))}{d} - \frac{2ab\csc(c+dx)}{d} - \frac{a^2\cot(c+dx)\csc(c+dx)}{2d} + \frac{b^2\sec(c+dx)}{d}$$

```
[Out] -1/2*a^2*arctanh(cos(d*x+c))/d-b^2*arctanh(cos(d*x+c))/d+2*a*b*arctanh(sin(
d*x+c))/d-2*a*b*csc(d*x+c)/d-1/2*a^2*cot(d*x+c)*csc(d*x+c)/d+b^2*sec(d*x+c)
/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3598, 3853, 3855, 2701, 327, 213, 2702}

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{a^2\text{arctanh}(\cos(c+dx))}{2d} - \frac{a^2\cot(c+dx)\csc(c+dx)}{2d} + \frac{2ab\text{arctanh}(\sin(c+dx))}{d} - \frac{2ab\csc(c+dx)}{d} - \frac{b^2\text{arctanh}(\cos(c+dx))}{d} + \frac{b^2\sec(c+dx)}{d}$$

```
[In] Int[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -1/2*(a^2*ArcTanh[Cos[c + d*x]])/d - (b^2*ArcTanh[Cos[c + d*x]])/d + (2*a*b
*ArcTanh[Sin[c + d*x]])/d - (2*a*b*Csc[c + d*x])/d - (a^2*Cot[c + d*x]*Csc[
c + d*x])/(2*d) + (b^2*Sec[c + d*x])/d
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^2\csc^3(c+dx)+2ab\csc^2(c+dx)\sec(c+dx)+b^2\csc(c+dx)\sec^2(c+dx)\right)\,dx\\
&= a^2\int \csc^3(c+dx)\,dx+(2ab)\int \csc^2(c+dx)\sec(c+dx)\,dx+b^2\int \csc(c+dx)\sec^2(c+dx)\,dx\\
&= -\frac{a^2\cot(c+dx)\csc(c+dx)}{2d}+\frac{1}{2}a^2\int \csc(c+dx)\,dx\\
&\quad -\frac{(2ab)\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx,x,\csc(c+dx)\right)}{d}+\frac{b^2\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx,x,\sec(c+dx)\right)}{d}\\
&= -\frac{a^2\text{arctanh}(\cos(c+dx))}{2d}-\frac{2ab\csc(c+dx)}{d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{2d}+\frac{b^2\sec(c+dx)}{d}\\
&\quad -\frac{(2ab)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\csc(c+dx)\right)}{d}+\frac{b^2\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\sec(c+dx)\right)}{d}\\
&= -\frac{a^2\text{arctanh}(\cos(c+dx))}{2d}-\frac{b^2\text{arctanh}(\cos(c+dx))}{d}+\frac{2ab\text{arctanh}(\sin(c+dx))}{d}\\
&\quad -\frac{2ab\csc(c+dx)}{d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{2d}+\frac{b^2\sec(c+dx)}{d}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 250 vs. $2(95) = 190$.

Time = 2.78 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.63

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{8b^2-8ab\cot\left(\frac{1}{2}(c+dx)\right)-a^2\csc^2\left(\frac{1}{2}(c+dx)\right)-4(a^2+2b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-16ab\log\left(\cos\left(\frac{1}{2}(c+\right.\right.}{}$$

```
[In] Integrate[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (8*b^2 - 8*a*b*Cot[(c + d*x)/2] - a^2*Csc[(c + d*x)/2]^2 - 4*(a^2 + 2*b^2)*
Log[Cos[(c + d*x)/2]] - 16*a*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4
*(a^2 + 2*b^2)*Log[Sin[(c + d*x)/2]] + 16*a*b*Log[Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]] + a^2*Sec[(c + d*x)/2]^2 + (8*b^2*Sin[(c + d*x)/2])/(Cos[(c + d
*x)/2] - Sin[(c + d*x)/2]) - (8*b^2*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + S
in[(c + d*x)/2]) - 8*a*b*Tan[(c + d*x)/2])/(8*d)
```

Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{b^2\left(\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)+2ab\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+a^2\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\right.}{d}$
default	$\frac{b^2\left(\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)+2ab\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+a^2\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\right.}{d}$
risch	$\frac{e^{i(dx+c)}\left(a^2e^{4i(dx+c)}+2b^2e^{4i(dx+c)}-4iab\,e^{4i(dx+c)}+2a^2e^{2i(dx+c)}-4b^2e^{2i(dx+c)}+a^2+2b^2+4iab\right)}{d\left(e^{2i(dx+c)}-1\right)^2\left(e^{2i(dx+c)}+1\right)}+\frac{a^2\ln\left(e^{i(dx+c)}-1\right)}{2d}$

```
[In] int(csc(d*x+c)^3*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(1/cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+2*a*b*(-1/sin(d*x+c)+ln(s
ec(d*x+c)+tan(d*x+c)))+a^2*(-1/2*csc(d*x+c)*cot(d*x+c)+1/2*ln(csc(d*x+c)-co
t(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(91) = 182$.

Time = 0.31 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.42

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{8\,ab\cos{(dx+c)}\sin{(dx+c)}+2\,(a^2+2\,b^2)\cos{(dx+c)}^2-4\,b^2-\left((a^2+2\,b^2)\cos{(dx+c)}^3-(a^2+2\,b^2)\,\mathrm{c}\right.}{}$$

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(8*a*b*cos(d*x + c)*sin(d*x + c) + 2*(a^2 + 2*b^2)*cos(d*x + c)^2 - 4*b
^2 - ((a^2 + 2*b^2)*cos(d*x + c)^3 - (a^2 + 2*b^2)*cos(d*x + c))*log(1/2*co
s(d*x + c) + 1/2) + ((a^2 + 2*b^2)*cos(d*x + c)^3 - (a^2 + 2*b^2)*cos(d*x +
 c))*log(-1/2*cos(d*x + c) + 1/2) + 4*(a*b*cos(d*x + c)^3 - a*b*cos(d*x + c
))*log(sin(d*x + c) + 1) - 4*(a*b*cos(d*x + c)^3 - a*b*cos(d*x + c))*log(-s
in(d*x + c) + 1))/(d*cos(d*x + c)^3 - d*cos(d*x + c))
```

Sympy [F]

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^2\,dx = \int (a+b\tan(c+dx))^2\csc^3(c+dx)\,dx$$

```
[In] integrate(csc(d*x+c)**3*(a+b*tan(d*x+c))**2,x)
[Out] Integral((a + b*tan(c + d*x))**2*csc(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.28

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{a^2\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1} - \log(\cos(dx+c)+1) + \log(\cos(dx+c)-1)\right) + 2b^2\left(\frac{2}{\cos(dx+c)} - \log(\cos(dx+c)+1) + \right.}{4d}$$

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
[Out] 1/4*(a^2*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - log(cos(d*x + c) + 1) + log
(cos(d*x + c) - 1)) + 2*b^2*(2/cos(d*x + c) - log(cos(d*x + c) + 1) + log(c
os(d*x + c) - 1)) - 4*a*b*(2/sin(d*x + c) - log(sin(d*x + c) + 1) + log(sin
(d*x + c) - 1)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.51 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.81

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + 16ab\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right) - 16ab\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right) - 8ab\tan\left(\frac{1}{2}d\right.}{}$$

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="giac")
[Out] 1/8*(a^2*tan(1/2*d*x + 1/2*c)^2 + 16*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1))
 - 16*a*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 8*a*b*tan(1/2*d*x + 1/2*c) +
 4*(a^2 + 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) - 16*b^2/(tan(1/2*d*x + 1/2
*c)^2 - 1) - (6*a^2*tan(1/2*d*x + 1/2*c)^2 + 12*b^2*tan(1/2*d*x + 1/2*c)^2
+ 8*a*b*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*d*x + 1/2*c)^2)/d
```

Mupad [B] (verification not implemented)

Time = 4.16 (sec) , antiderivative size = 292, normalized size of antiderivative = 3.07

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,\left(\frac{a^2}{2}+b^2\right)}{d}$$
$$+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\,\left(\frac{a^2}{2}+8\,b^2\right)-\frac{a^2}{2}+4\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-4\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\,\left(4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\right)}$$
$$+\frac{4\,a\,b\,\mathrm{atanh}\left(\frac{8\,a\,b^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4\,a^3\,b-16\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,a^2\,b^2+8\,a\,b^3}-\frac{16\,a^2\,b^2}{4\,a^3\,b-16\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,a^2\,b^2+8\,a\,b^3}+\frac{4\,a^3\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4\,a^3\,b-16\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,a^2\,b^2+8\,a\,b^3}\right)}{d}$$
$$-\frac{a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}$$

```
[In] int((a + b*tan(c + d*x))^2/sin(c + d*x)^3,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^2)/(8*d) + (log(tan(c/2 + (d*x)/2))*(a^2/2 + b^2))/
d + (tan(c/2 + (d*x)/2)^2*(a^2/2 + 8*b^2) - a^2/2 + 4*a*b*tan(c/2 + (d*x)/2
)^3 - 4*a*b*tan(c/2 + (d*x)/2))/(d*(4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d
*x)/2)^4)) + (4*a*b*atanh((8*a*b^3*tan(c/2 + (d*x)/2))/(8*a*b^3 + 4*a^3*b -
 16*a^2*b^2*tan(c/2 + (d*x)/2)) - (16*a^2*b^2)/(8*a*b^3 + 4*a^3*b - 16*a^2*
b^2*tan(c/2 + (d*x)/2)) + (4*a^3*b*tan(c/2 + (d*x)/2))/(8*a*b^3 + 4*a^3*b -
 16*a^2*b^2*tan(c/2 + (d*x)/2))))/d - (a*b*tan(c/2 + (d*x)/2))/d
```

3.29 $\int \csc^4(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 79

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{(a^2+b^2)\cot(c+dx)}{d} - \frac{ab\cot^2(c+dx)}{d} - \frac{a^2\cot^3(c+dx)}{3d} + \frac{2ab\log(\tan(c+dx))}{d} + \frac{b^2\tan(c+dx)}{d}$$

```
[Out] -(a^2+b^2)*cot(d*x+c)/d-a*b*cot(d*x+c)^2/d-1/3*a^2*cot(d*x+c)^3/d+2*a*b*ln(
tan(d*x+c))/d+b^2*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3597, 908}

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{(a^2+b^2)\cot(c+dx)}{d} - \frac{a^2\cot^3(c+dx)}{3d} - \frac{ab\cot^2(c+dx)}{d} + \frac{2ab\log(\tan(c+dx))}{d} + \frac{b^2\tan(c+dx)}{d}$$

```
[In] Int[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(((a^2 + b^2)*Cot[c + d*x])/d) - (a*b*Cot[c + d*x]^2)/d - (a^2*Cot[c + d*x
]^3)/(3*d) + (2*a*b*Log[Tan[c + d*x]])/d + (b^2*Tan[c + d*x])/d
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b\text{Subst}\left(\int \frac{(a+x)^2(b^2+x^2)}{x^4}\,dx, x, b\tan(c+dx)\right)}{d}\\
&= \frac{b\text{Subst}\left(\int \left(1+\frac{a^2b^2}{x^4}+\frac{2ab^2}{x^3}+\frac{a^2+b^2}{x^2}+\frac{2a}{x}\right)dx, x, b\tan(c+dx)\right)}{d}\\
&= -\frac{(a^2+b^2)\cot(c+dx)}{d}-\frac{ab\cot^2(c+dx)}{d}\\
&\quad -\frac{a^2\cot^3(c+dx)}{3d}+\frac{2ab\log(\tan(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.75 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.61

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{(3ab\cot^2(c+dx)+a^2\cot^3(c+dx)+\cos^2(c+dx)\left((2a^2+3b^2)\cot(c+dx)+6ab(\log(\cos(c+dx))-\text{l}\right.}{3d(a\cos(c+dx)+b\sin(c+dx))^2}$$

```
[In] Integrate[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -1/3*((3*a*b*Cot[c + d*x]^2 + a^2*Cot[c + d*x]^3 + Cos[c + d*x]^2*((2*a^2 +
 3*b^2)*Cot[c + d*x] + 6*a*b*(Log[Cos[c + d*x]] - Log[Sin[c + d*x]])) - (3*
b^2*Sin[2*(c + d*x)])/2)*(a + b*Tan[c + d*x])^2)/(d*(a*Cos[c + d*x] + b*Sin
[c + d*x])^2)
```

Maple [A] (verified)

Time = 3.49 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{b^2\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+2ab\left(-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a^2\left(-\frac{2}{3}-\frac{\left(\csc^2(dx+c)\right)}{3}\right)\cot(dx+c)}{d}$
default	$\frac{b^2\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+2ab\left(-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a^2\left(-\frac{2}{3}-\frac{\left(\csc^2(dx+c)\right)}{3}\right)\cot(dx+c)}{d}$
risch	$\frac{4ab\,e^{6i(dx+c)}+4ia^2e^{4i(dx+c)}-4ib^2e^{4i(dx+c)}+\frac{8ia^2e^{2i(dx+c)}}{3}+8ib^2e^{2i(dx+c)}-4ab\,e^{2i(dx+c)}-\frac{4ia^2}{3}-4ib^2}{d\left(e^{2i(dx+c)}-1\right)^3\left(e^{2i(dx+c)}+1\right)}+\frac{2ab\ln\left(e^{2i(dx+}\right.}{d}$

```
[In] int(csc(d*x+c)^4*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(1/sin(d*x+c)/cos(d*x+c)-2*cot(d*x+c))+2*a*b*(-1/2/sin(d*x+c)^2+ln
(tan(d*x+c)))+a^2*(-2/3-1/3*csc(d*x+c)^2)*cot(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(77) = 154$.

Time = 0.27 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.20

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^2\,dx =$$
$$-\frac{2\left(a^2+3\,b^2\right)\cos\left(dx+c\right)^4-3\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)-3\left(a^2+3\,b^2\right)\cos\left(dx+c\right)^2+3\left(ab\cos\left(dx+c\right.\right.}{3\left(d\cos\right.}$$

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(2*(a^2 + 3*b^2)*cos(d*x + c)^4 - 3*a*b*cos(d*x + c)*sin(d*x + c) - 3*
(a^2 + 3*b^2)*cos(d*x + c)^2 + 3*(a*b*cos(d*x + c)^3 - a*b*cos(d*x + c))*lo
g(cos(d*x + c)^2)*sin(d*x + c) - 3*(a*b*cos(d*x + c)^3 - a*b*cos(d*x + c))*
log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) + 3*b^2)/((d*cos(d*x + c)^3 - d
*cos(d*x + c))*sin(d*x + c))
```

Sympy [F]

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^2\,dx = \int (a+b\tan{(c+dx)})^2\csc^4{(c+dx)}\,dx$$

```
[In] integrate(csc(d*x+c)**4*(a+b*tan(d*x+c))**2,x)
[Out] Integral((a + b*tan(c + d*x))**2*csc(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.87

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{6\,ab\log{(\tan{(dx+c)})} + 3\,b^2\tan{(dx+c)} - \frac{3\,ab\tan(dx+c)+3\,(a^2+b^2)\tan(dx+c)^2+a^2}{\tan(dx+c)^3}}{3\,d}$$

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
[Out] 1/3*(6*a*b*log(tan(d*x + c)) + 3*b^2*tan(d*x + c) - (3*a*b*tan(d*x + c) + 3
*(a^2 + b^2)*tan(d*x + c)^2 + a^2)/tan(d*x + c)^3)/d
```

Giac [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.15

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{6\,ab\log{(|\tan{(dx+c)}|)} + 3\,b^2\tan{(dx+c)} - \frac{11\,ab\tan(dx+c)^3+3\,a^2\tan(dx+c)^2+3\,b^2\tan(dx+c)^2+3\,ab\tan(dx+c)+a^2}{\tan(dx+c)^3}}{3\,d}$$

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="giac")
[Out] 1/3*(6*a*b*log(abs(tan(d*x + c))) + 3*b^2*tan(d*x + c) - (11*a*b*tan(d*x +
c)^3 + 3*a^2*tan(d*x + c)^2 + 3*b^2*tan(d*x + c)^2 + 3*a*b*tan(d*x + c) + a
^2)/tan(d*x + c)^3)/d
```

Mupad [B] (verification not implemented)

Time = 4.67 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.91

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{b^2\tan(c+d\,x)}{d} - \frac{\tan(c+d\,x)^2\,(a^2+b^2)+\frac{a^2}{3}+a\,b\tan(c+d\,x)}{d\tan\left(c+d\,x\right)^3} + \frac{2\,a\,b\,\ln\left(\tan(c+d\,x)\right)}{d}$$

```
[In] int((a + b*tan(c + d*x))^2/sin(c + d*x)^4,x)
```

```
[Out] (b^2*tan(c + d*x))/d - (tan(c + d*x)^2*(a^2 + b^2) + a^2/3 + a*b*tan(c + d*
x))/(d*tan(c + d*x)^3) + (2*a*b*log(tan(c + d*x)))/d
```

3.30 $\int \csc^5(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 165

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{3a^2\text{arctanh}(\cos(c+dx))}{8d} - \frac{3b^2\text{arctanh}(\cos(c+dx))}{2d} + \frac{2ab\text{arctanh}(\sin(c+dx))}{d} - \frac{2ab\csc(c+dx)}{d} - \frac{3a^2\cot(c+dx)\csc(c+dx)}{8d} - \frac{2ab\csc^3(c+dx)}{3d} - \frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d} + \frac{3b^2\sec(c+dx)}{2d} - \frac{b^2\csc^2(c+dx)\sec(c+dx)}{2d}$$

```
[Out] -3/8*a^2*arctanh(cos(d*x+c))/d-3/2*b^2*arctanh(cos(d*x+c))/d+2*a*b*arctanh(
sin(d*x+c))/d-2*a*b*csc(d*x+c)/d-3/8*a^2*cot(d*x+c)*csc(d*x+c)/d-2/3*a*b*cs
c(d*x+c)^3/d-1/4*a^2*cot(d*x+c)*csc(d*x+c)^3/d+3/2*b^2*sec(d*x+c)/d-1/2*b^2
*csc(d*x+c)^2*sec(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used

$= \{3598, 3853, 3855, 2701, 308, 213, 2702, 294, 327\}$

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{3a^2\text{arctanh}(\cos(c+dx))}{8d} - \frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d} - \frac{3a^2\cot(c+dx)\csc(c+dx)}{8d} + \frac{2ab\text{arctanh}(\sin(c+dx))}{d} - \frac{2ab\csc^3(c+dx)}{3d} - \frac{2ab\csc(c+dx)}{d} - \frac{3b^2\text{arctanh}(\cos(c+dx))}{2d} + \frac{3b^2\sec(c+dx)}{2d} - \frac{b^2\csc^2(c+dx)\sec(c+dx)}{2d}$$

```
[In] Int[Csc[c + d*x]^5*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-3*a^2*ArcTanh[Cos[c + d*x]])/(8*d) - (3*b^2*ArcTanh[Cos[c + d*x]])/(2*d)
+ (2*a*b*ArcTanh[Sin[c + d*x]])/d - (2*a*b*Csc[c + d*x])/d - (3*a^2*Cot[c +
 d*x]*Csc[c + d*x])/(8*d) - (2*a*b*Csc[c + d*x]^3)/(3*d) - (a^2*Cot[c + d*x
]*Csc[c + d*x]^3)/(4*d) + (3*b^2*Sec[c + d*x])/(2*d) - (b^2*Csc[c + d*x]^2*
Sec[c + d*x])/(2*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^2 \csc^5(c+dx) + 2ab\csc^4(c+dx)\sec(c+dx) + b^2\csc^3(c+dx)\sec^2(c+dx)\right)\,dx \\ &= a^2\int \csc^5(c+dx)\,dx + (2ab)\int \csc^4(c+dx)\sec(c+dx)\,dx + b^2\int \csc^3(c+dx)\sec^2(c+dx)\,dx\end{aligned}$$

$$= -\frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d} + \frac{1}{4}(3a^2)\int \csc^3(c+dx)\,dx - \frac{(2ab)\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d} + \frac{b^2\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx, x, \sec(c+dx)\right)}{d}$$

$$= -\frac{3a^2\cot(c+dx)\csc(c+dx)}{8d} - \frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d} - \frac{b^2\csc^2(c+dx)\sec(c+dx)}{2d} + \frac{1}{8}(3a^2)\int \csc(c+dx)\,dx - \frac{(2ab)\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)\,dx, x, \csc(c+dx)\right)}{d} + \frac{(3b^2)\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{2d}$$

$$= -\frac{3a^2\text{arctanh}(\cos(c+dx))}{8d} - \frac{2ab\csc(c+dx)}{d} - \frac{3a^2\cot(c+dx)\csc(c+dx)}{8d} - \frac{2ab\csc^3(c+dx)}{3d} - \frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d} + \frac{3b^2\sec(c+dx)}{2d} - \frac{b^2\csc^2(c+dx)\sec(c+dx)}{2d} - \frac{(2ab)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d} + \frac{(3b^2)\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{2d}$$

$$= -\frac{3a^2\text{arctanh}(\cos(c+dx))}{8d} - \frac{3b^2\text{arctanh}(\cos(c+dx))}{2d} + \frac{2ab\text{arctanh}(\sin(c+dx))}{d} - \frac{2ab\csc(c+dx)}{d} - \frac{3a^2\cot(c+dx)\csc(c+dx)}{8d} - \frac{2ab\csc^3(c+dx)}{3d} - \frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d} + \frac{3b^2\sec(c+dx)}{2d} - \frac{b^2\csc^2(c+dx)\sec(c+dx)}{2d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 994 vs. $2(165) = 330$.

Time = 7.25 (sec) , antiderivative size = 994, normalized size of antiderivative = 6.02

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{b^2\cos^2(c+dx)(a+b\tan(c+dx))^2}{d(a\cos(c+dx)+b\sin(c+dx))^2} - \frac{7ab\cos^2(c+dx)\cot\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^2}{6d(a\cos(c+dx)+b\sin(c+dx))^2} + \frac{(-3a^2-4b^2)\cos^2(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^2}{32d(a\cos(c+dx)+b\sin(c+dx))^2} - \frac{ab\cos^2(c+dx)\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^2}{12d(a\cos(c+dx)+b\sin(c+dx))^2} - \frac{a^2\cos^2(c+dx)\csc^4\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^2}{64d(a\cos(c+dx)+b\sin(c+dx))^2} - \frac{3(a^2+4b^2)\cos^2(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^2}{8d(a\cos(c+dx)+b\sin(c+dx))^2} - \frac{2ab\cos^2(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^2}{d(a\cos(c+dx)+b\sin(c+dx))^2} + \frac{3(a^2+4b^2)\cos^2(c+dx)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^2}{8d(a\cos(c+dx)+b\sin(c+dx))^2} + \frac{2ab\cos^2(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^2}{d(a\cos(c+dx)+b\sin(c+dx))^2} + \frac{(3a^2+4b^2)\cos^2(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^2}{32d(a\cos(c+dx)+b\sin(c+dx))^2} + \frac{a^2\cos^2(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^2}{64d(a\cos(c+dx)+b\sin(c+dx))^2} + \frac{b^2\cos^2(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^2}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^2} - \frac{b^2\cos^2(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^2}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^2} - \frac{7ab\cos^2(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^2}{6d(a\cos(c+dx)+b\sin(c+dx))^2} - \frac{ab\cos^2(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^2}{12d(a\cos(c+dx)+b\sin(c+dx))^2}$$

```
[In] Integrate[Csc[c + d*x]^5*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (b^2*Cos[c + d*x]^2*(a + b*Tan[c + d*x])^2)/(d*(a*Cos[c + d*x] + b*Sin[c +
d*x])^2) - (7*a*b*Cos[c + d*x]^2*Cot[(c + d*x)/2]*(a + b*Tan[c + d*x])^2)/(
6*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) + ((-3*a^2 - 4*b^2)*Cos[c + d*x]^2
```

```
*Csc[(c + d*x)/2]^2*(a + b*Tan[c + d*x])^2)/(32*d*(a*Cos[c + d*x] + b*Sin[c
 + d*x])^2) - (a*b*Cos[c + d*x]^2*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2*(a +
b*Tan[c + d*x])^2)/(12*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) - (a^2*Cos[c
+ d*x]^2*Csc[(c + d*x)/2]^4*(a + b*Tan[c + d*x])^2)/(64*d*(a*Cos[c + d*x] +
 b*Sin[c + d*x])^2) - (3*(a^2 + 4*b^2)*Cos[c + d*x]^2*Log[Cos[(c + d*x)/2]]
*(a + b*Tan[c + d*x])^2)/(8*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) - (2*a*b
*Cos[c + d*x]^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x
])^2)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) + (3*(a^2 + 4*b^2)*Cos[c + d*
x]^2*Log[Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^2)/(8*d*(a*Cos[c + d*x] + b
*Sin[c + d*x])^2) + (2*a*b*Cos[c + d*x]^2*Log[Cos[(c + d*x)/2] + Sin[(c + d
*x)/2]]*(a + b*Tan[c + d*x])^2)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) + (
(3*a^2 + 4*b^2)*Cos[c + d*x]^2*Sec[(c + d*x)/2]^2*(a + b*Tan[c + d*x])^2)/(
32*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) + (a^2*Cos[c + d*x]^2*Sec[(c + d*
x)/2]^4*(a + b*Tan[c + d*x])^2)/(64*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)
+ (b^2*Cos[c + d*x]^2*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^2)/(d*(Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) - (b^2*Co
s[c + d*x]^2*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^2)/(d*(Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) - (7*a*b*Cos[c + d
*x]^2*Tan[(c + d*x)/2]*(a + b*Tan[c + d*x])^2)/(6*d*(a*Cos[c + d*x] + b*Sin
[c + d*x])^2) - (a*b*Cos[c + d*x]^2*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]*(a
+ b*Tan[c + d*x])^2)/(12*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)
```

Maple [A] (verified)

Time = 6.33 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{b^2\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c)}+\frac{3}{2\cos(dx+c)}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+2ab\left(-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(\right.}{d}$
default	$\frac{b^2\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c)}+\frac{3}{2\cos(dx+c)}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+2ab\left(-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(\right.}{d}$
risch	$\frac{e^{i(dx+c)}\left(9a^2e^{8i(dx+c)}+36b^2e^{8i(dx+c)}-48iab\,e^{8i(dx+c)}-24a^2e^{6i(dx+c)}-96b^2e^{6i(dx+c)}+160iab\,e^{6i(dx+c)}-66a^2e^{4i(dx+c)}+\right.}{12d\left(e^{2i(dx+c)}-1\right)^4\left(e^{2i(dx+c)}+1\right)}$

```
[In] int(csc(d*x+c)^5*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(-1/2/sin(d*x+c)^2/cos(d*x+c)+3/2/cos(d*x+c)+3/2*ln(csc(d*x+c)-cot
(d*x+c)))+2*a*b*(-1/3/sin(d*x+c)^3-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+
a^2*((-1/4*csc(d*x+c)^3-3/8*csc(d*x+c))*cot(d*x+c)+3/8*ln(csc(d*x+c)-cot(d*
x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(151) = 302$.

Time = 0.30 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.02

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{18\,(a^2+4\,b^2)\cos{(dx+c)}^4 - 30\,(a^2+4\,b^2)\cos{(dx+c)}^2 + 48\,b^2 - 9\,\left((a^2+4\,b^2)\cos{(dx+c)}^5 - 2\,(a^2+4\right.}{}$$

[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="fricas")

[Out] 1/48*(18*(a^2 + 4*b^2)*cos(d*x + c)^4 - 30*(a^2 + 4*b^2)*cos(d*x + c)^2 + 48*b^2 - 9*((a^2 + 4*b^2)*cos(d*x + c)^5 - 2*(a^2 + 4*b^2)*cos(d*x + c)^3 + (a^2 + 4*b^2)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 9*((a^2 + 4*b^2)*cos(d*x + c)^5 - 2*(a^2 + 4*b^2)*cos(d*x + c)^3 + (a^2 + 4*b^2)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 48*(a*b*cos(d*x + c)^5 - 2*a*b*cos(d*x + c)^3 + a*b*cos(d*x + c))*log(sin(d*x + c) + 1) - 48*(a*b*cos(d*x + c)^5 - 2*a*b*cos(d*x + c)^3 + a*b*cos(d*x + c))*log(-sin(d*x + c) + 1) + 32*(3*a*b*cos(d*x + c)^3 - 4*a*b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5 - 2*d*cos(d*x + c)^3 + d*cos(d*x + c))

Sympy [F]

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^2\,dx = \int (a+b\tan{(c+dx)})^2\csc^5{(c+dx)}\,dx$$

[In] integrate(csc(d*x+c)**5*(a+b*tan(d*x+c))**2,x)

[Out] Integral((a + b*tan(c + d*x))**2*csc(c + d*x)**5, x)

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.13

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{3\,a^2\left(\frac{2\left(3\,\cos(dx+c)^3-5\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1} - 3\,\log\left(\cos{(dx+c)}+1\right) + 3\,\log\left(\cos{(dx+c)}-1\right)\right) + 12\,b^2\left(\frac{2\left(3\,\cos(dx+c)\right.}{\cos(dx+c)^3-\cos}\right.}{}$$

[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="maxima")

```
[Out] 1/48*(3*a^2*(2*(3*cos(d*x + c)^3 - 5*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(
d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 12*b
^2*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*log(cos(d*
x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 16*a*b*(2*(3*sin(d*x + c)^2 + 1)/s
in(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.51 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.63

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-16\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+24\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+384\,ab\,\mathrm{l}}{}$$

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^2*tan(1/2*d*x + 1/2*c)^4 - 16*a*b*tan(1/2*d*x + 1/2*c)^3 + 24*a^
2*tan(1/2*d*x + 1/2*c)^2 + 24*b^2*tan(1/2*d*x + 1/2*c)^2 + 384*a*b*log(abs(
tan(1/2*d*x + 1/2*c) + 1)) - 384*a*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2
40*a*b*tan(1/2*d*x + 1/2*c) + 72*(a^2 + 4*b^2)*log(abs(tan(1/2*d*x + 1/2*c)
)) - 384*b^2/(tan(1/2*d*x + 1/2*c)^2 - 1) - (150*a^2*tan(1/2*d*x + 1/2*c)^4
 + 600*b^2*tan(1/2*d*x + 1/2*c)^4 + 240*a*b*tan(1/2*d*x + 1/2*c)^3 + 24*a^2
*tan(1/2*d*x + 1/2*c)^2 + 24*b^2*tan(1/2*d*x + 1/2*c)^2 + 16*a*b*tan(1/2*d*
x + 1/2*c) + 3*a^2)/tan(1/2*d*x + 1/2*c)^4)/d
```

Mupad [B] (verification not implemented)

Time = 4.22 (sec) , antiderivative size = 378, normalized size of antiderivative = 2.29

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^2\,dx=\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{3\,a^2}{8}+\frac{3\,b^2}{2}\right)}{d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}$$
$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{7\,a^2}{4}+2\,b^2\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(2\,a^2+34\,b^2\right)+\frac{a^2}{4}+\frac{56\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}-20\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+}{d\,\left(16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\right)}$$
$$+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{a^2}{8}+\frac{b^2}{8}\right)}{d}-\frac{a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12\,d}$$
$$+\frac{4\,a\,b\,\mathrm{atanh}\left(\frac{12\,a\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3\,a^3\,b-16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b^2+12\,a\,b^3}-\frac{16\,a^2\,b^2}{3\,a^3\,b-16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b^2+12\,a\,b^3}+\frac{3\,a^3\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3\,a^3\,b-16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b^2+12\,a\,b^3}\right)}{d}$$
$$-\frac{5\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,d}$$

```
[In] int((a + b*tan(c + d*x))^2/sin(c + d*x)^5,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2))*((3*a^2)/8 + (3*b^2)/2))/d + (a^2*tan(c/2 + (d*x)/
2)^4)/(64*d) - (tan(c/2 + (d*x)/2)^2*((7*a^2)/4 + 2*b^2) - tan(c/2 + (d*x)/
2)^4*(2*a^2 + 34*b^2) + a^2/4 + (56*a*b*tan(c/2 + (d*x)/2)^3)/3 - 20*a*b*ta
n(c/2 + (d*x)/2)^5 + (4*a*b*tan(c/2 + (d*x)/2))/3)/(d*(16*tan(c/2 + (d*x)/2
)^4 - 16*tan(c/2 + (d*x)/2)^6)) + (tan(c/2 + (d*x)/2)^2*(a^2/8 + b^2/8))/d
- (a*b*tan(c/2 + (d*x)/2)^3)/(12*d) + (4*a*b*atanh((12*a*b^3*tan(c/2 + (d*x
)/2))/(12*a*b^3 + 3*a^3*b - 16*a^2*b^2*tan(c/2 + (d*x)/2)) - (16*a^2*b^2)/(
12*a*b^3 + 3*a^3*b - 16*a^2*b^2*tan(c/2 + (d*x)/2)) + (3*a^3*b*tan(c/2 + (d
*x)/2))/(12*a*b^3 + 3*a^3*b - 16*a^2*b^2*tan(c/2 + (d*x)/2))))/d - (5*a*b*t
an(c/2 + (d*x)/2))/(4*d)
```

3.31 $\int \csc^6(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 122

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^2\,dx=-\frac{(a^2+2b^2)\cot(c+dx)}{d}-\frac{2ab\cot^2(c+dx)}{d}-\frac{(2a^2+b^2)\cot^3(c+dx)}{3d}-\frac{ab\cot^4(c+dx)}{2d}-\frac{a^2\cot^5(c+dx)}{5d}+\frac{2ab\log(\tan(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

```
[Out] -(a^2+2*b^2)*cot(d*x+c)/d-2*a*b*cot(d*x+c)^2/d-1/3*(2*a^2+b^2)*cot(d*x+c)^3
/d-1/2*a*b*cot(d*x+c)^4/d-1/5*a^2*cot(d*x+c)^5/d+2*a*b*ln(tan(d*x+c))/d+b^2
*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3597, 962}

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^2\,dx=-\frac{(2a^2+b^2)\cot^3(c+dx)}{3d}-\frac{(a^2+2b^2)\cot(c+dx)}{d}-\frac{a^2\cot^5(c+dx)}{5d}-\frac{ab\cot^4(c+dx)}{2d}-\frac{2ab\cot^2(c+dx)}{d}+\frac{2ab\log(\tan(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

```
[In] Int[Csc[c + d*x]^6*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(((a^2 + 2*b^2)*Cot[c + d*x])/d) - (2*a*b*Cot[c + d*x]^2)/d - ((2*a^2 + b^
2)*Cot[c + d*x]^3)/(3*d) - (a*b*Cot[c + d*x]^4)/(2*d) - (a^2*Cot[c + d*x]^5
)/(5*d) + (2*a*b*Log[Tan[c + d*x]])/d + (b^2*Tan[c + d*x])/d
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b\text{Subst}\left(\int \frac{(a+x)^2(b^2+x^2)^2}{x^6}\,dx, x, b\tan(c+dx)\right)}{d}\\
&= \frac{b\text{Subst}\left(\int \left(1+\frac{a^2b^4}{x^6}+\frac{2ab^4}{x^5}+\frac{2a^2b^2+b^4}{x^4}+\frac{4ab^2}{x^3}+\frac{a^2+2b^2}{x^2}+\frac{2a}{x}\right)dx, x, b\tan(c+dx)\right)}{d}\\
&= -\frac{(a^2+2b^2)\cot(c+dx)}{d}-\frac{2ab\cot^2(c+dx)}{d}-\frac{(2a^2+b^2)\cot^3(c+dx)}{3d}\\
&\quad -\frac{ab\cot^4(c+dx)}{2d}-\frac{a^2\cot^5(c+dx)}{5d}+\frac{2ab\log(\tan(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.82 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.93

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{2\cot(c+dx)\left(8a^2+25b^2+(4a^2+5b^2)\csc^2(c+dx)+3a^2\csc^4(c+dx)\right)+15b(2a\csc^2(c+dx)+a\csc^4}{30d}$$

```
[In] Integrate[Csc[c + d*x]^6*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -1/30*(2*Cot[c + d*x]*(8*a^2 + 25*b^2 + (4*a^2 + 5*b^2)*Csc[c + d*x]^2 + 3*
a^2*Csc[c + d*x]^4) + 15*b*(2*a*Csc[c + d*x]^2 + a*Csc[c + d*x]^4 + 4*a*Log
[Cos[c + d*x]] - 4*a*Log[Sin[c + d*x]] - 2*b*Tan[c + d*x]))/d
```

Maple [A] (verified)

Time = 12.76 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{b^2\left(-\frac{1}{3\sin(dx+c)^3\cos(dx+c)}+\frac{4}{3\sin(dx+c)\cos(dx+c)}-\frac{8\cot(dx+c)}{3}\right)+2ab\left(-\frac{1}{4\sin(dx+c)^4}-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a^2\left(\right.}{d}$
default	$\frac{b^2\left(-\frac{1}{3\sin(dx+c)^3\cos(dx+c)}+\frac{4}{3\sin(dx+c)\cos(dx+c)}-\frac{8\cot(dx+c)}{3}\right)+2ab\left(-\frac{1}{4\sin(dx+c)^4}-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a^2\left(\right.}{d}$
risch	$\frac{4ab\,e^{10i(dx+c)}-16ab\,e^{8i(dx+c)}-\frac{32ia^2e^{6i(dx+c)}}{3}+\frac{32ib^2e^{6i(dx+c)}}{3}-\frac{16ia^2e^{4i(dx+c)}}{3}-\frac{80ib^2e^{4i(dx+c)}}{3}+16ab\,e^{4i(dx+c)}+\frac{64ia^2e}{}}{d\left(e^{2i(dx+c)}-1\right)^5\left(e^{2i(dx+c)}+1\right)}$

```
[In] int(csc(d*x+c)^6*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(-1/3/sin(d*x+c)^3/cos(d*x+c)+4/3/sin(d*x+c)/cos(d*x+c)-8/3*cot(d*
x+c))+2*a*b*(-1/4/sin(d*x+c)^4-1/2/sin(d*x+c)^2+ln(tan(d*x+c)))+a^2*(-8/15-
1/5*csc(d*x+c)^4-4/15*csc(d*x+c)^2)*cot(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(116) = 232$.

Time = 0.27 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.97

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^2\,dx =$$

$$-\frac{16\left(a^2+5\,b^2\right)\cos\left(dx+c\right)^6-40\left(a^2+5\,b^2\right)\cos\left(dx+c\right)^4+30\left(a^2+5\,b^2\right)\cos\left(dx+c\right)^2+30\left(ab\cos\left(dx+\right.\right.}{}$$

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/30*(16*(a^2 + 5*b^2)*cos(d*x + c)^6 - 40*(a^2 + 5*b^2)*cos(d*x + c)^4 +
30*(a^2 + 5*b^2)*cos(d*x + c)^2 + 30*(a*b*cos(d*x + c)^5 - 2*a*b*cos(d*x +
c)^3 + a*b*cos(d*x + c))*log(cos(d*x + c)^2)*sin(d*x + c) - 30*(a*b*cos(d*x
 + c)^5 - 2*a*b*cos(d*x + c)^3 + a*b*cos(d*x + c))*log(-1/4*cos(d*x + c)^2
+ 1/4)*sin(d*x + c) - 30*b^2 - 15*(2*a*b*cos(d*x + c)^3 - 3*a*b*cos(d*x + c
))*sin(d*x + c))/((d*cos(d*x + c)^5 - 2*d*cos(d*x + c)^3 + d*cos(d*x + c))*
sin(d*x + c))
```

Sympy [F]

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^2\,dx = \int (a+b\tan(c+dx))^2\csc^6(c+dx)\,dx$$

```
[In] integrate(csc(d*x+c)**6*(a+b*tan(d*x+c))**2,x)
[Out] Integral((a + b*tan(c + d*x))**2*csc(c + d*x)**6, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.85

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{60\,ab\log(\tan(dx+c)) + 30\,b^2\tan(dx+c) - \frac{60\,ab\tan(dx+c)^3+30\,(a^2+2\,b^2)\tan(dx+c)^4+15\,ab\tan(dx+c)+10\,(2\,a^2+b^2)\tan(}{\tan(dx+c)^5}}{30\,d}$$

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
[Out] 1/30*(60*a*b*log(tan(d*x + c)) + 30*b^2*tan(d*x + c) - (60*a*b*tan(d*x + c)
^3 + 30*(a^2 + 2*b^2)*tan(d*x + c)^4 + 15*a*b*tan(d*x + c) + 10*(2*a^2 + b^
2)*tan(d*x + c)^2 + 6*a^2)/tan(d*x + c)^5)/d
```

Giac [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.07

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{60\,ab\log(|\tan(dx+c)|) + 30\,b^2\tan(dx+c) - \frac{137\,ab\tan(dx+c)^5+30\,a^2\tan(dx+c)^4+60\,b^2\tan(dx+c)^4+60\,ab\tan(dx+c)^3+20}{\tan(dx+c)^5}}{30\,d}$$

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^2,x, algorithm="giac")
[Out] 1/30*(60*a*b*log(abs(tan(d*x + c))) + 30*b^2*tan(d*x + c) - (137*a*b*tan(d*
x + c)^5 + 30*a^2*tan(d*x + c)^4 + 60*b^2*tan(d*x + c)^4 + 60*a*b*tan(d*x +
 c)^3 + 20*a^2*tan(d*x + c)^2 + 10*b^2*tan(d*x + c)^2 + 15*a*b*tan(d*x + c)
 + 6*a^2)/tan(d*x + c)^5)/d
```

Mupad [B] (verification not implemented)

Time = 4.95 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.88

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{b^2\tan(c+d\,x)}{d} - \frac{\tan(c+d\,x)^4\,(a^2+2\,b^2)+\frac{a^2}{5}+\tan(c+d\,x)^2\,\left(\frac{2\,a^2}{3}+\frac{b^2}{3}\right)+\frac{a\,b\tan(c+d\,x)}{2}+2\,a\,b\tan(c+d\,x)^3}{d\tan\left(c+d\,x\right)^5} + \frac{2\,a\,b\,\ln\left(\tan(c+d\,x)\right)}{d}$$

```
[In] int((a + b*tan(c + d*x))^2/sin(c + d*x)^6,x)
```

```
[Out] (b^2*tan(c + d*x))/d - (tan(c + d*x)^4*(a^2 + 2*b^2) + a^2/5 + tan(c + d*x)
^2*((2*a^2)/3 + b^2/3) + (a*b*tan(c + d*x))/2 + 2*a*b*tan(c + d*x)^3)/(d*ta
n(c + d*x)^5) + (2*a*b*log(tan(c + d*x)))/d
```

3.32 $\int \sin^3(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 205

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{3a^2b\text{arctanh}(\sin(c+dx))}{d} - \frac{5b^3\text{arctanh}(\sin(c+dx))}{2d} - \frac{a^3\cos(c+dx)}{d} + \frac{6ab^2\cos(c+dx)}{d} + \frac{a^3\cos^3(c+dx)}{3d} - \frac{ab^2\cos^3(c+dx)}{d} + \frac{3ab^2\sec(c+dx)}{d} - \frac{3a^2b\sin(c+dx)}{d} + \frac{5b^3\sin(c+dx)}{2d} - \frac{a^2b\sin^3(c+dx)}{d} + \frac{5b^3\sin^3(c+dx)}{6d} + \frac{b^3\sin^3(c+dx)\tan^2(c+dx)}{2d}$$

```
[Out] 3*a^2*b*arctanh(sin(d*x+c))/d-5/2*b^3*arctanh(sin(d*x+c))/d-a^3*cos(d*x+c)/
d+6*a*b^2*cos(d*x+c)/d+1/3*a^3*cos(d*x+c)^3/d-a*b^2*cos(d*x+c)^3/d+3*a*b^2*
sec(d*x+c)/d-3*a^2*b*sin(d*x+c)/d+5/2*b^3*sin(d*x+c)/d-a^2*b*sin(d*x+c)^3/d
+5/6*b^3*sin(d*x+c)^3/d+1/2*b^3*sin(d*x+c)^3*tan(d*x+c)^2/d
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3598, 2713, 2672, 308, 212, 2670, 276, 294}

$$\begin{aligned}\int \sin^3(c+dx)(a+b\tan(c+dx))^3\,dx = {} & \frac{a^3\cos^3(c+dx)}{3d}-\frac{a^3\cos(c+dx)}{d} \\ & +\frac{3a^2b\text{arctanh}(\sin(c+dx))}{d}-\frac{a^2b\sin^3(c+dx)}{d} \\ & -\frac{3a^2b\sin(c+dx)}{d}-\frac{ab^2\cos^3(c+dx)}{d} \\ & +\frac{6ab^2\cos(c+dx)}{d}+\frac{3ab^2\sec(c+dx)}{d} \\ & -\frac{5b^3\text{arctanh}(\sin(c+dx))}{2d}+\frac{5b^3\sin^3(c+dx)}{6d} \\ & +\frac{5b^3\sin(c+dx)}{2d}+\frac{b^3\sin^3(c+dx)\tan^2(c+dx)}{2d}\end{aligned}$$

```
[In] Int[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a^2*b*ArcTanh[Sin[c + d*x]])/d - (5*b^3*ArcTanh[Sin[c + d*x]])/(2*d) - (
a^3*Cos[c + d*x])/d + (6*a*b^2*Cos[c + d*x])/d + (a^3*Cos[c + d*x]^3)/(3*d)
 - (a*b^2*Cos[c + d*x]^3)/d + (3*a*b^2*Sec[c + d*x])/d - (3*a^2*b*Sin[c + d
*x])/d + (5*b^3*Sin[c + d*x])/(2*d) - (a^2*b*Sin[c + d*x]^3)/d + (5*b^3*Sin
[c + d*x]^3)/(6*d) + (b^3*Sin[c + d*x]^3*Tan[c + d*x]^2)/(2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^3 \sin^3(c+dx) + 3a^2 b \sin^3(c+dx)\tan(c+dx) + 3ab^2 \sin^3(c+dx)\tan^2(c+dx) \right. \\
&\qquad \left. + b^3 \sin^3(c+dx)\tan^3(c+dx)\right)\,dx \\
&= a^3 \int \sin^3(c+dx)\,dx + \left(3a^2 b\right) \int \sin^3(c+dx)\tan(c+dx)\,dx \\
&\quad + \left(3ab^2\right) \int \sin^3(c+dx)\tan^2(c+dx)\,dx + b^3 \int \sin^3(c+dx)\tan^3(c+dx)\,dx \\
&= -\frac{a^3 \text{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cos(c+dx)\right)}{d} + \frac{\left(3a^2 b\right)\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d} \\
&\quad - \frac{\left(3ab^2\right)\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^2}\,dx, x, \cos(c+dx)\right)}{d} + \frac{b^3 \text{Subst}\left(\int \frac{x^6}{\left(1-x^2\right)^2}\,dx, x, \sin(c+dx)\right)}{d}
\end{aligned}$$

$$= -\frac{a^3\cos(c+dx)}{d} + \frac{a^3\cos^3(c+dx)}{3d} + \frac{b^3\sin^3(c+dx)\tan^2(c+dx)}{2d} + \frac{(3a^2b)\,\text{Subst}\left(\int\left(-1-x^2+\frac{1}{1-x^2}\right)\,dx,x,\sin(c+dx)\right)}{d} - \frac{(3ab^2)\,\text{Subst}\left(\int\left(-2+\frac{1}{x^2}+x^2\right)\,dx,x,\cos(c+dx)\right)}{d} - \frac{(5b^3)\,\text{Subst}\left(\int\frac{x^4}{1-x^2}\,dx,x,\sin(c+dx)\right)}{2d}$$

$$= -\frac{a^3\cos(c+dx)}{d} + \frac{6ab^2\cos(c+dx)}{d} + \frac{a^3\cos^3(c+dx)}{3d} - \frac{ab^2\cos^3(c+dx)}{d} + \frac{3ab^2\sec(c+dx)}{d} - \frac{3a^2b\sin(c+dx)}{d} - \frac{a^2b\sin^3(c+dx)}{d} + \frac{b^3\sin^3(c+dx)\tan^2(c+dx)}{2d} + \frac{(3a^2b)\,\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\sin(c+dx)\right)}{d} - \frac{(5b^3)\,\text{Subst}\left(\int\left(-1-x^2+\frac{1}{1-x^2}\right)\,dx,x,\sin(c+dx)\right)}{2d}$$

$$= \frac{3a^2b\text{arctanh}(\sin(c+dx))}{d} - \frac{a^3\cos(c+dx)}{d} + \frac{6ab^2\cos(c+dx)}{d} + \frac{a^3\cos^3(c+dx)}{3d} - \frac{ab^2\cos^3(c+dx)}{d} + \frac{3ab^2\sec(c+dx)}{d} - \frac{3a^2b\sin(c+dx)}{d} + \frac{5b^3\sin(c+dx)}{2d} - \frac{a^2b\sin^3(c+dx)}{d} + \frac{5b^3\sin^3(c+dx)}{6d} + \frac{b^3\sin^3(c+dx)\tan^2(c+dx)}{2d} - \frac{(5b^3)\,\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\sin(c+dx)\right)}{2d}$$

$$= \frac{3a^2b\text{arctanh}(\sin(c+dx))}{d} - \frac{5b^3\text{arctanh}(\sin(c+dx))}{2d} - \frac{a^3\cos(c+dx)}{d} + \frac{6ab^2\cos(c+dx)}{d} + \frac{a^3\cos^3(c+dx)}{3d} - \frac{ab^2\cos^3(c+dx)}{d} + \frac{3ab^2\sec(c+dx)}{d} - \frac{3a^2b\sin(c+dx)}{d} + \frac{5b^3\sin(c+dx)}{2d} - \frac{a^2b\sin^3(c+dx)}{d} + \frac{5b^3\sin^3(c+dx)}{6d} + \frac{b^3\sin^3(c+dx)\tan^2(c+dx)}{2d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 771 vs. 2(205) = 410.

Time = 7.29 (sec) , antiderivative size = 771, normalized size of antiderivative = 3.76

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{3ab^2\cos^3(c+dx)(a+b\tan(c+dx))^3}{d(a\cos(c+dx)+b\sin(c+dx))^3} - \frac{3a(a^2-7b^2)\cos^4(c+dx)(a+b\tan(c+dx))^3}{4d(a\cos(c+dx)+b\sin(c+dx))^3}$$
$$+ \frac{a(a^2-3b^2)\cos^3(c+dx)\cos(3(c+dx))(a+b\tan(c+dx))^3}{12d(a\cos(c+dx)+b\sin(c+dx))^3}$$
$$+ \frac{(-6a^2b+5b^3)\cos^3(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^3}{2d(a\cos(c+dx)+b\sin(c+dx))^3}$$
$$+ \frac{(6a^2b-5b^3)\cos^3(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^3}{2d(a\cos(c+dx)+b\sin(c+dx))^3}$$
$$+ \frac{b^3\cos^3(c+dx)(a+b\tan(c+dx))^3}{4d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a\cos(c+dx)+b\sin(c+dx))^3}$$
$$+ \frac{3ab^2\cos^3(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^3}$$
$$- \frac{b^3\cos^3(c+dx)(a+b\tan(c+dx))^3}{4d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a\cos(c+dx)+b\sin(c+dx))^3}$$
$$- \frac{3ab^2\cos^3(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^3}$$
$$- \frac{3b(5a^2-3b^2)\cos^3(c+dx)\sin(c+dx)(a+b\tan(c+dx))^3}{4d(a\cos(c+dx)+b\sin(c+dx))^3}$$
$$+ \frac{b(3a^2-b^2)\cos^3(c+dx)\sin(3(c+dx))(a+b\tan(c+dx))^3}{12d(a\cos(c+dx)+b\sin(c+dx))^3}$$

```
[In] Integrate[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a*b^2*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(d*(a*Cos[c + d*x] + b*Sin[
c + d*x])^3) - (3*a*(a^2 - 7*b^2)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^3)/(4
*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (a*(a^2 - 3*b^2)*Cos[c + d*x]^3*C
os[3*(c + d*x)]*(a + b*Tan[c + d*x])^3)/(12*d*(a*Cos[c + d*x] + b*Sin[c + d
*x])^3) + ((-6*a^2*b + 5*b^3)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]]*(a + b*Tan[c + d*x])^3)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3
) + ((6*a^2*b - 5*b^3)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/
2]]*(a + b*Tan[c + d*x])^3)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (b^
3*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(4*d*(Cos[(c + d*x)/2] - Sin[(c +
d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (3*a*b^2*Cos[c + d*x]^3*S
in[(c + d*x)/2]*(a + b*Tan[c + d*x])^3)/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x
)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (b^3*Cos[c + d*x]^3*(a + b*Tan
```

```
[c + d*x])^3)/(4*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(a*Cos[c + d*x]
+ b*Sin[c + d*x])^3) - (3*a*b^2*Cos[c + d*x]^3*Sin[(c + d*x)/2]*(a + b*Tan[
c + d*x])^3)/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*S
in[c + d*x])^3) - (3*b*(5*a^2 - 3*b^2)*Cos[c + d*x]^3*Sin[c + d*x]*(a + b*T
an[c + d*x])^3)/(4*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (b*(3*a^2 - b^2
)*Cos[c + d*x]^3*Sin[3*(c + d*x)]*(a + b*Tan[c + d*x])^3)/(12*d*(a*Cos[c +
d*x] + b*Sin[c + d*x])^3)
```

Maple [A] (verified)

Time = 2.72 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.90

method	result
derivativedivides	$-\frac{a^3(2+\sin^2(dx+c))\cos(dx+c)}{3}+3a^2b\left(-\frac{(\sin^3(dx+c))}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+3ab^2\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\right.\right.$
default	$-\frac{a^3(2+\sin^2(dx+c))\cos(dx+c)}{3}+3a^2b\left(-\frac{(\sin^3(dx+c))}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+3ab^2\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\right.\right.$
risch	$\frac{i e^{-3i(dx+c)}b a^2}{8d}+\frac{15i e^{i(dx+c)}b a^2}{8d}+\frac{e^{3i(dx+c)}a^3}{24d}-\frac{e^{3i(dx+c)}a b^2}{8d}-\frac{i b^2 e^{i(dx+c)}(6ia e^{2i(dx+c)}+b e^{2i(dx+c)}+6ia-b)}{d(e^{2i(dx+c)}+1)^2}$

```
[In] int(sin(d*x+c)^3*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a^3*(2+sin(d*x+c)^2)*cos(d*x+c)+3*a^2*b*(-1/3*sin(d*x+c)^3-sin(d*
x+c)+ln(sec(d*x+c)+tan(d*x+c)))+3*a*b^2*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d
*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+b^3*(1/2*sin(d*x+c)^7/cos(d*x+c)^2+1/
2*sin(d*x+c)^5+5/6*sin(d*x+c)^3+5/2*sin(d*x+c)-5/2*ln(sec(d*x+c)+tan(d*x+c)
)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.92

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$=\frac{4\,(a^3-3\,ab^2)\cos{(dx+c)}^5+36\,ab^2\cos{(dx+c)}-12\,(a^3-6\,ab^2)\cos{(dx+c)}^3+3\,(6\,a^2b-5\,b^3)\cos{(dx+}}{}$$

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*(a^3 - 3*a*b^2)*cos(d*x + c)^5 + 36*a*b^2*cos(d*x + c) - 12*(a^3 -
6*a*b^2)*cos(d*x + c)^3 + 3*(6*a^2*b - 5*b^3)*cos(d*x + c)^2*log(sin(d*x +
c) + 1) - 3*(6*a^2*b - 5*b^3)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*
(3*a^2*b - b^3)*cos(d*x + c)^4 + 3*b^3 - 2*(12*a^2*b - 7*b^3)*cos(d*x + c)^
2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^3\,dx = \int (a+b\tan{(c+dx)})^3\sin^3{(c+dx)}\,dx$$

```
[In] integrate(sin(d*x+c)**3*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*sin(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.84

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{4\left(\cos{(dx+c)}^3-3\cos{(dx+c)}\right)a^3-6\left(2\sin{(dx+c)}^3-3\log{(\sin{(dx+c)}+1)}+3\log{(\sin{(dx+c)}-\right.}{}$$

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/12*(4*(cos(d*x + c)^3 - 3*cos(d*x + c))*a^3 - 6*(2*sin(d*x + c)^3 - 3*log
(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin(d*x + c))*a^2*b - 12*(
cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a*b^2 + (4*sin(d*x + c)^3
 - 6*sin(d*x + c)/(sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(
sin(d*x + c) - 1) + 24*sin(d*x + c))*b^3)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66584 vs. 2(195) = 390.

Time = 147.41 (sec) , antiderivative size = 66584, normalized size of antiderivative = 324.80

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^3\,dx = \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/192*(45*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(
1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)
^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)
^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 + 45*pi*a*b^2*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/
```

2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*t
an(1/2*d*x)^10*tan(1/2*c)^10 + 45*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
+ 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c
) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 + 45*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1
/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*t
an(1/2*c) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 - 360*pi*a*b^2*sgn(tan(1/2*d*x
)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^
2 + 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 + 45*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1
/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*t
an(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^
2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan
(1/2*c)^8 + 45*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*
tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*
d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/
2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^8 - 360*pi*a*b^2*sg
n(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^
2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) -
1)*tan(1/2*d*x)^9*tan(1/2*c)^9 - 360*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)
^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/
2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - t
an(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^9*tan(1/2*c
)^9 + 45*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/
2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2
*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2
 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^10 + 45*pi*a*b^2*sgn(tan(1
/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan
(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d
*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(
1/2*d*x)^8*tan(1/2*c)^10 - 90*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1
/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2
*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^10 - 90*a*b^2*arctan((tan(1/2*d*x)*tan
(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2
*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^10 + 90*a*b^2*arctan((t
an(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1
/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^10 + 90*
a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan
(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(
1/2*c)^10 + 288*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2
*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 -
 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*
d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^10*tan(1/2*c)^10 - 240*b^3*log(2*(
tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*
tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c
) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*t

```
an(1/2*d*x)^10*tan(1/2*c)^10 - 288*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2
 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)
^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(
1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^10*tan(1/2*c)^1
0 + 240*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c
) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2
*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + t
an(1/2*c)^2 + 1))*tan(1/2*d*x)^10*tan(1/2*c)^10 + 45*pi*a*b^2*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2
*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^8 + 45*pi*a*b^2*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^8 - 360*pi*a*b^
2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2
*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^10*tan(1/2*c)^8 - 360*pi*a*b^2*sgn(tan
(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - t
an(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^9*tan(1/2*c)^9 - 360*pi*a*b^2*
sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x
)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^9*tan(1/2*c)^9 + 2880*p
i*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*t
an(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^9*tan(1/2*c)^9 + 45*pi*a*b^2*sgn
(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2
 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^10 + 45*pi*a*
b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2
*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^10 - 3
60*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*
x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^8*tan(1/2*c)^10 + 128*a^3*ta
n(1/2*d*x)^10*tan(1/2*c)^10 - 1536*a*b^2*tan(1/2*d*x)^10*tan(1/2*c)^10 - 90
*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + t
an(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2
*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan
(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^6 - 90*pi*a*b^2*sgn(tan(1/2*d*x)^
2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^
2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(
1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)
^10*tan(1/2*c)^6 - 720*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2
*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(
tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2
+ tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^9*tan(1/2*c)^7 - 720*pi*a
*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/
2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*
d*x) - 1)*tan(1/2*d*x)^9*tan(1/2*c)^7 + 765*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan
(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2
*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c
)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*ta
```

n(1/2*c)^8 + 765*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^8 - 90*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^8 - 90*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^8 + 90*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^8 + 90*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^8 + 288*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^10*tan(1/2*c)^8 - 240*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^10*tan(1/2*c)^8 - 288*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^10*tan(1/2*c)^8 + 240*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^10*tan(1/2*c)^8 - 720*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^7*tan(1/2*c)^9 - 720*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^7*tan(1/2*c)^9 + 720*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^9*tan(1/2*c)^9 + 720*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^9*tan(1/2*c)^9 - 720*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^9*tan(1/2*c)^9 - 720*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^9*tan(1/2*c)^9 - 2304*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1

/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 +
 tan(1/2*c)^2 + 1))*tan(1/2*d*x)^9*tan(1/2*c)^9 + 1920*b^3*log(2*(tan(1/2*d
*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c
)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(t
an(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*
x)^9*tan(1/2*c)^9 + 2304*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1
/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1
/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 +
 tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^9*tan(1/2*c)^9 - 1920*b^3
log(2(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1
/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*t
an(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2
 + 1))*tan(1/2*d*x)^9*tan(1/2*c)^9 - 1152*a^2*b*tan(1/2*d*x)^10*tan(1/2*c)^
9 + 960*b^3*tan(1/2*d*x)^10*tan(1/2*c)^9 - 90*pi*a*b^2*sgn(tan(1/2*d*x)^2*t
an(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 +
 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2
*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*
tan(1/2*c)^10 - 90*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x
)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(
1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + ta
n(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^10 - 90*a*b^2*ar
ctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x
)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^10
 - 90*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1
)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^8
*tan(1/2*c)^10 + 90*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) +
tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*
tan(1/2*d*x)^8*tan(1/2*c)^10 + 90*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - t
an(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan
(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^10 + 288*a^2*b*log(2*(tan(1/2*d*x)^
2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2
 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1
/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8
*tan(1/2*c)^10 - 240*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x
)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^
2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1
/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^10 - 288*a^2*b*log
(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d
*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1
/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1
))*tan(1/2*d*x)^8*tan(1/2*c)^10 + 240*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^
2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x
)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan
(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^1
0 - 1152*a^2*b*tan(1/2*d*x)^9*tan(1/2*c)^10 + 960*b^3*tan(1/2*d*x)^9*tan(1/

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2*c)^10 - 90*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*ta
n(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^1
0*tan(1/2*c)^6 - 90*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*
x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2
*d*x)^10*tan(1/2*c)^6 + 720*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(
1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^10*
tan(1/2*c)^6 - 720*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x
)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*
d*x)^9*tan(1/2*c)^7 - 720*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(
1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*t
an(1/2*d*x)^9*tan(1/2*c)^7 + 5760*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*
x)^9*tan(1/2*c)^7 + 765*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/
2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan
(1/2*d*x)^8*tan(1/2*c)^8 + 765*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2
*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) -
 1)*tan(1/2*d*x)^8*tan(1/2*c)^8 - 6120*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*
c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1
/2*d*x)^8*tan(1/2*c)^8 + 128*a^3*tan(1/2*d*x)^10*tan(1/2*c)^8 - 1536*a*b^2*
tan(1/2*d*x)^10*tan(1/2*c)^8 - 720*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*
c) - 1)*tan(1/2*d*x)^7*tan(1/2*c)^9 - 720*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1
/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*t
an(1/2*c) - 1)*tan(1/2*d*x)^7*tan(1/2*c)^9 + 5760*pi*a*b^2*sgn(tan(1/2*d*x)
^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2
 + 1)*tan(1/2*d*x)^7*tan(1/2*c)^9 - 1024*a^3*tan(1/2*d*x)^9*tan(1/2*c)^9 +
12288*a*b^2*tan(1/2*d*x)^9*tan(1/2*c)^9 - 90*pi*a*b^2*sgn(tan(1/2*d*x)^2*ta
n(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 +
2*tan(1/2*c) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^10 - 90*pi*a*b^2*sgn(tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*
c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^10 + 720*pi*a*b^2*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - ta
n(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^10 + 128*a^3*tan(1/2*d*x)^8*tan(1
/2*c)^10 - 1536*a*b^2*tan(1/2*d*x)^8*tan(1/2*c)^10 - 90*pi*a*b^2*sgn(tan(1/
2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(
1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*
x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1
/2*d*x)^10*tan(1/2*c)^4 - 90*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*t
an(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1
)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d
*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^4 + 2
070*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c)
+ tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(
1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*
tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^6 + 2070*pi*a*b^2*sgn(tan(1/2*d
```

```
*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2
*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*
tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*
d*x)^8*tan(1/2*c)^6 + 180*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d
*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c)
+ 1))*tan(1/2*d*x)^10*tan(1/2*c)^6 + 180*a*b^2*arctan((tan(1/2*d*x)*tan(1/2
*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x
) - tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^6 - 180*a*b^2*arctan((tan(1
/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c
) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^6 - 180*a*b^
2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2
*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*
c)^6 - 576*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(
1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*ta
n(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^
2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^10*tan(1/2*c)^6 + 480*b^3*log(2*(tan(1/
2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/
2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)
/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2
*d*x)^10*tan(1/2*c)^6 + 576*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*ta
n(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + ta
n(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^
2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^10*tan(1/2*c)^6 - 480*
b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*ta
n(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) +
2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c
)^2 + 1))*tan(1/2*d*x)^10*tan(1/2*c)^6 - 1440*pi*a*b^2*sgn(tan(1/2*d*x)^2*t
an(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 +
 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2
*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^7*
tan(1/2*c)^7 - 1440*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*
x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan
(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + t
an(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^7*tan(1/2*c)^7 + 1440*a*b^2*
arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d
*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^9*tan(1/2*c)^
7 + 1440*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c)
+ 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x
)^9*tan(1/2*c)^7 - 1440*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x
) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) -
1))*tan(1/2*d*x)^9*tan(1/2*c)^7 - 1440*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c
) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x)
+ tan(1/2*c) - 1))*tan(1/2*d*x)^9*tan(1/2*c)^7 - 4608*a^2*b*log(2*(tan(1/2*
d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*
c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(
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tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d
*x)^9*tan(1/2*c)^7 + 3840*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/
2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/
2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 +
tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^9*tan(1/2*c)^7 + 4608*a^2*
b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(
1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*
tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^
2 + 1))*tan(1/2*d*x)^9*tan(1/2*c)^7 - 3840*b^3*log(2*(tan(1/2*d*x)^2*tan(1/
2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/
2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^
2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^9*tan(1/2
*c)^7 - 1536*a^2*b*tan(1/2*d*x)^10*tan(1/2*c)^7 + 1280*b^3*tan(1/2*d*x)^10*
tan(1/2*c)^7 + 2070*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*
x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan
(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + t
an(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^8 + 2070*pi*a*b
^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*
d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 -
2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*
x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^8 - 1530*a*b^2*arctan((tan(1/2*d*x)*tan(1
/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d
*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 - 1530*a*b^2*arctan((tan
(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2
*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 + 1530*a*
b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1
/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2
*c)^8 + 1530*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2
*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2
*d*x)^8*tan(1/2*c)^8 + 4896*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + ta
n(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^
2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 - 4080*
b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*ta
n(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) -
2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c
)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 - 4896*a^2*b*log(2*(tan(1/2*d*x)^2*ta
n(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + ta
n(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d
*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8*tan
(1/2*c)^8 + 4080*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*
tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 +
2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d
*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 + 5760*a^2*b*tan(1/2
*d*x)^9*tan(1/2*c)^8 - 4800*b^3*tan(1/2*d*x)^9*tan(1/2*c)^8 + 1440*a*b^2*ar

ctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x
)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^7*tan(1/2*c)^9
+ 1440*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) +
1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^
7*tan(1/2*c)^9 - 1440*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x)
+ tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)
)*tan(1/2*d*x)^7*tan(1/2*c)^9 - 1440*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c)
- tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) +
tan(1/2*c) - 1))*tan(1/2*d*x)^7*tan(1/2*c)^9 - 4608*a^2*b*log(2*(tan(1/2*d*
x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)
^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(ta
n(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x
)^7*tan(1/2*c)^9 + 3840*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*
c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + ta
n(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^7*tan(1/2*c)^9 + 4608*a^2*b*
log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/
2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*ta
n(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2
+ 1))*tan(1/2*d*x)^7*tan(1/2*c)^9 - 3840*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*
c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*
d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*
tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^7*tan(1/2*c
)^9 + 5760*a^2*b*tan(1/2*d*x)^8*tan(1/2*c)^9 - 4800*b^3*tan(1/2*d*x)^8*tan(
1/2*c)^9 - 90*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*t
an(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2
*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^10 - 90*pi*a*b^2*sgn(
tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2
- tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(
1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)
*tan(1/2*d*x)^4*tan(1/2*c)^10 + 180*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) +
tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + t
an(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^10 + 180*a*b^2*arctan((tan(1/2*d*
x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + t
an(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^10 - 180*a*b^2*arc
tan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)
*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^10
- 180*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1
)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6
*tan(1/2*c)^10 - 576*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d
*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c
)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan
(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^10 + 480*b^3*log
(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d

*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1
/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1
))*tan(1/2*d*x)^6*tan(1/2*c)^10 + 576*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c
)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d
*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*t
an(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)
^10 - 480*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2
*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1
/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 +
 tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^10 - 1536*a^2*b*tan(1/2*d*x)^
7*tan(1/2*c)^10 + 1280*b^3*tan(1/2*d*x)^7*tan(1/2*c)^10 - 90*pi*a*b^2*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
 tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^4 - 90*pi*a*b^
2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d
*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^4 + 720
*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)
*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^10*tan(1/2*c)^4 + 2070*pi*a*b^
2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d
*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^6 + 2070
*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + t
an(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^
6 - 16560*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan
(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^8*tan(1/2*c)^6 - 1024
*a^3*tan(1/2*d*x)^10*tan(1/2*c)^6 + 3072*a*b^2*tan(1/2*d*x)^10*tan(1/2*c)^6
 - 1440*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2
*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^7*tan(
1/2*c)^7 - 1440*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2
*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x
)^7*tan(1/2*c)^7 + 11520*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2
*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^7*tan(
1/2*c)^7 - 5120*a^3*tan(1/2*d*x)^9*tan(1/2*c)^7 + 24576*a*b^2*tan(1/2*d*x)^
9*tan(1/2*c)^7 + 2070*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1
/2*d*x)^6*tan(1/2*c)^8 + 2070*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*
tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) -
1)*tan(1/2*d*x)^6*tan(1/2*c)^8 - 16560*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*
c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1
/2*d*x)^6*tan(1/2*c)^8 - 2432*a^3*tan(1/2*d*x)^8*tan(1/2*c)^8 - 26112*a*b^2
*tan(1/2*d*x)^8*tan(1/2*c)^8 - 5120*a^3*tan(1/2*d*x)^7*tan(1/2*c)^9 + 24576
*a*b^2*tan(1/2*d*x)^7*tan(1/2*c)^9 - 90*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2
*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan
(1/2*c) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^10 - 90*pi*a*b^2*sgn(tan(1/2*d*x)^2*
tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2
 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^10 + 720*pi*a*b^2*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2

*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^10 - 1024*a^3*tan(1/2*d*x)^6*tan(1/2*c
)^10 + 3072*a*b^2*tan(1/2*d*x)^6*tan(1/2*c)^10 + 45*pi*a*b^2*sgn(tan(1/2*d*
x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*
c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*t
an(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d
*x)^10*tan(1/2*c)^2 + 45*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1
/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sg
n(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^
2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^2 + 720*p
i*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan
(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c
)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1
/2*d*x) - 1)*tan(1/2*d*x)^9*tan(1/2*c)^3 + 720*pi*a*b^2*sgn(tan(1/2*d*x)^2*
tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2
- 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/
2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^9
*tan(1/2*c)^3 + 2070*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d
*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^4 + 2070*pi*a*
b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2
*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
- 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d
*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^4 + 180*a*b^2*arctan((tan(1/2*d*x)*tan(1
/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d
*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^4 + 180*a*b^2*arctan((tan
(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2
*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^4 - 180*a*
b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1
/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/
2*c)^4 - 180*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2
*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2
*d*x)^10*tan(1/2*c)^4 - 576*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + ta
n(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^
2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^10*tan(1/2*c)^4 + 480*
b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*ta
n(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) -
2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c
)^2 + 1))*tan(1/2*d*x)^10*tan(1/2*c)^4 + 576*a^2*b*log(2*(tan(1/2*d*x)^2*ta
n(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + ta
n(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d
*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^10*ta
n(1/2*c)^4 - 480*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*
tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 +

2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^10*tan(1/2*c)^4 + 5376*a^2*b*tan(1/2*d*x)^10*tan(1/2*c)^5 - 1408*b^3*tan(1/2*d*x)^10*tan(1/2*c)^5 + 6660*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 6660*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 - 4140*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^6 - 4140*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^6 + 4140*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^6 + 4140*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^6 + 13248*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^6 - 11040*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^6 - 13248*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^6 + 11040*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^6 + 37632*a^2*b*tan(1/2*d*x)^9*tan(1/2*c)^6 - 16000*b^3*tan(1/2*d*x)^9*tan(1/2*c)^6 + 2880*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^7*tan(1/2*c)^7 + 2880*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^7*tan(1/2*c)^7 - 2880*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^7*tan(1/2*c)^7 - 2880*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^7*tan(1/2*c)^7 - 9216*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2

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 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/
2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^7*tan(1/2*c)^7 +
7680*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) +
 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*
x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(
1/2*c)^2 + 1))*tan(1/2*d*x)^7*tan(1/2*c)^7 + 9216*a^2*b*log(2*(tan(1/2*d*x)
^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2
 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(
1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^
7*tan(1/2*c)^7 - 7680*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*
x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)
^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(
1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^7*tan(1/2*c)^7 + 44544*a^2*b*t
an(1/2*d*x)^8*tan(1/2*c)^7 - 6400*b^3*tan(1/2*d*x)^8*tan(1/2*c)^7 + 2070*pi
*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(
1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)
^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/
2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^8 + 2070*pi*a*b^2*sgn(tan(1/2*d*x)^2*
tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2
 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/
2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4
*tan(1/2*c)^8 - 4140*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) -
 tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))
*tan(1/2*d*x)^6*tan(1/2*c)^8 - 4140*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) -
 tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - t
an(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^8 + 4140*a*b^2*arctan((tan(1/2*d*
x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - t
an(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^8 + 4140*a*b^2*arc
tan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)
*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^8 +
 13248*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*
c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/
2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^8 - 11040*b^3*log(2*(tan(1/2*d
*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c
)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(t
an(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*
x)^6*tan(1/2*c)^8 - 13248*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(
1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(
1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2
 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^8 + 11040*b
^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan
(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2
*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)
^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^8 + 44544*a^2*b*tan(1/2*d*x)^7*tan(1/2*c
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)^8 - 6400*b^3*tan(1/2*d*x)^7*tan(1/2*c)^8 + 720*pi*a*b^2*sgn(tan(1/2*d*x)^
2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^
2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(
1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)
^3*tan(1/2*c)^9 + 720*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*
d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
 tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^3*tan(1/2*c)^9 + 37632*a^2
*b*tan(1/2*d*x)^6*tan(1/2*c)^9 - 16000*b^3*tan(1/2*d*x)^6*tan(1/2*c)^9 + 45
*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + t
an(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2
*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan
(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^10 + 45*pi*a*b^2*sgn(tan(1/2*d*x)^
2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^
2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(
1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)
^2*tan(1/2*c)^10 + 180*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x)
 - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1
))*tan(1/2*d*x)^4*tan(1/2*c)^10 + 180*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c)
 - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) -
 tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^10 - 180*a*b^2*arctan((tan(1/2*
d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) -
 tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^10 - 180*a*b^2*a
rctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*
x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^1
0 - 576*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2
*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1
/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 +
 tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^10 + 480*b^3*log(2*(tan(1/2*d
*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c
)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(t
an(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*
x)^4*tan(1/2*c)^10 + 576*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1
/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1
/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 +
 tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^10 - 480*b^3
log(2(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1
/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*t
an(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2
 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^10 + 5376*a^2*b*tan(1/2*d*x)^5*tan(1/2*c)^
10 - 1408*b^3*tan(1/2*d*x)^5*tan(1/2*c)^10 + 45*pi*a*b^2*sgn(tan(1/2*d*x)^2
*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2
 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^2 + 45*pi*a*b^2*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1
/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^2 - 360*pi*a*b^2*sgn

(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^10*tan(1/2*c)^2 + 720*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^9*tan(1/2*c)^3 + 720*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^9*tan(1/2*c)^3 - 5760*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^9*tan(1/2*c)^3 + 2070*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^4 + 2070*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^4 - 16560*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^8*tan(1/2*c)^4 + 1024*a^3*tan(1/2*d*x)^10*tan(1/2*c)^4 - 3072*a*b^2*tan(1/2*d*x)^10*tan(1/2*c)^4 + 12288*a^3*tan(1/2*d*x)^9*tan(1/2*c)^5 - 36864*a*b^2*tan(1/2*d*x)^9*tan(1/2*c)^5 + 6660*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 6660*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 - 53280*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 43520*a^3*tan(1/2*d*x)^8*tan(1/2*c)^6 - 162816*a*b^2*tan(1/2*d*x)^8*tan(1/2*c)^6 + 49152*a^3*tan(1/2*d*x)^7*tan(1/2*c)^7 - 73728*a*b^2*tan(1/2*d*x)^7*tan(1/2*c)^7 + 2070*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^8 + 2070*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^8 - 16560*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^8 + 43520*a^3*tan(1/2*d*x)^6*tan(1/2*c)^8 - 162816*a*b^2*tan(1/2*d*x)^6*tan(1/2*c)^8 + 720*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^3*tan(1/2*c)^9 + 720*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^3*tan(1/2*c)^9 - 5760*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^3*tan(1/2*c)^9 + 12288*a^3*tan(1/2*d*x)^5*tan(1/2*c)^9 - 36864*a*b^2*tan(1/2*d*x)^5*tan(1/2*c)^9 + 45*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^10 + 45*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^10 - 360*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2*tan(1/2*c)^10 + 1024*a^3*tan(1/2* d*x)^4*tan(1/2*c)^10 - 3072*a*b^2*tan(1/2*d*x)^4*tan(1/2*c)^10 + 45*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10 + 45*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10 + 360*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^9*tan(1/2*c) + 360*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^9*tan(1/2*c) + 765*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^2 + 765*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^2 - 90*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^2 - 90*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^2 + 90*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^2 + 90*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^2 + 288*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^10*tan(1/2*c)^2 - 240*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^10*tan(1/2*c)^2 - 288*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^10*tan(1/2*c)^2 + 240*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2

+ 1))*tan(1/2*d*x)^10*tan(1/2*c)^2 + 1440*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(
1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*
tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)
^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^7*tan
(1/2*c)^3 + 1440*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^
2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/
2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(
1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^7*tan(1/2*c)^3 - 1440*a*b^2*arc
tan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)
*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^9*tan(1/2*c)^3 -
1440*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1
)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^9
*tan(1/2*c)^3 + 1440*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) +
tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))
*tan(1/2*d*x)^9*tan(1/2*c)^3 + 1440*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) -
tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + t
an(1/2*c) - 1))*tan(1/2*d*x)^9*tan(1/2*c)^3 + 4608*a^2*b*log(2*(tan(1/2*d*x
)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^
2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan
(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)
^9*tan(1/2*c)^3 - 3840*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d
*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c
)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan
(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^9*tan(1/2*c)^3 - 4608*a^2*b*l
og(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2
*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan
(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 +
1))*tan(1/2*d*x)^9*tan(1/2*c)^3 + 3840*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c
)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d
*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*t
an(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^9*tan(1/2*c)
^3 - 1536*a^2*b*tan(1/2*d*x)^10*tan(1/2*c)^3 + 1280*b^3*tan(1/2*d*x)^10*tan
(1/2*c)^3 + 6660*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^
2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/
2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(
1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 + 6660*pi*a*b^2*
sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x
)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*
tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x)
- 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 - 4140*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*
c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x)
+ tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^4 - 4140*a*b^2*arctan((tan(1/
2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c)
+ tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^4 + 4140*a*b^2
*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*

d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)
^4 + 4140*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c)
 - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*
x)^8*tan(1/2*c)^4 + 13248*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(
1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(
1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2
 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^4 - 11040*b
^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan
(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2
*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)
^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^4 - 13248*a^2*b*log(2*(tan(1/2*d*x)^2*ta
n(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + ta
n(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d
*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8*tan
(1/2*c)^4 + 11040*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2
*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 +
 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*
d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^4 - 37632*a^2*b*tan(1
/2*d*x)^9*tan(1/2*c)^4 + 16000*b^3*tan(1/2*d*x)^9*tan(1/2*c)^4 - 198912*a^2
*b*tan(1/2*d*x)^8*tan(1/2*c)^5 + 88960*b^3*tan(1/2*d*x)^8*tan(1/2*c)^5 + 66
60*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) +
 tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1
/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*t
an(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 + 6660*pi*a*b^2*sgn(tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*
c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*t
an(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d
*x)^4*tan(1/2*c)^6 - 13320*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*
d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c)
 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 - 13320*a*b^2*arctan((tan(1/2*d*x)*tan(1
/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d
*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 + 13320*a*b^2*arctan((ta
n(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/
2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 + 13320*
a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan
(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1
/2*c)^6 + 42624*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2
*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 -
 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*
d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 - 35520*b^3*log(2*(
tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*
tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c
) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*t
an(1/2*d*x)^6*tan(1/2*c)^6 - 42624*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2
 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)

^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(
1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6
+ 35520*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c
) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2
*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + t
an(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 - 322560*a^2*b*tan(1/2*d*x)^7
*tan(1/2*c)^6 + 115200*b^3*tan(1/2*d*x)^7*tan(1/2*c)^6 + 1440*pi*a*b^2*sgn(
tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2
- tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(
1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)
*tan(1/2*d*x)^3*tan(1/2*c)^7 + 1440*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2
*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - ta
n(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^3*tan(1/2*c)
^7 - 322560*a^2*b*tan(1/2*d*x)^6*tan(1/2*c)^7 + 115200*b^3*tan(1/2*d*x)^6*t
an(1/2*c)^7 + 765*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)
^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1
/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan
(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^8 + 765*pi*a*b^2*
sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x
)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*
tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x)
- 1)*tan(1/2*d*x)^2*tan(1/2*c)^8 - 4140*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*
c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x)
+ tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^8 - 4140*a*b^2*arctan((tan(1/
2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c)
+ tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^8 + 4140*a*b^2
*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*
d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)
^8 + 4140*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c)
- 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*
x)^4*tan(1/2*c)^8 + 13248*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(
1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(
1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2
+ tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^8 - 11040*b
^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan
(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2
*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)
^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^8 - 13248*a^2*b*log(2*(tan(1/2*d*x)^2*ta
n(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + ta
n(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d
*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan
(1/2*c)^8 + 11040*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2
*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 +
2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*

d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^8 - 198912*a^2*b*tan(
1/2*d*x)^5*tan(1/2*c)^8 + 88960*b^3*tan(1/2*d*x)^5*tan(1/2*c)^8 + 360*pi*a*
b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2
*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
+ 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d
*x) - 1)*tan(1/2*d*x)*tan(1/2*c)^9 + 360*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/
2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*ta
n(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2
 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)*tan(1/2
*c)^9 - 1440*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2
*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2
*d*x)^3*tan(1/2*c)^9 - 1440*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2
*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c
) + 1))*tan(1/2*d*x)^3*tan(1/2*c)^9 + 1440*a*b^2*arctan((tan(1/2*d*x)*tan(1
/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d
*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^3*tan(1/2*c)^9 + 1440*a*b^2*arctan((tan
(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2
*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^3*tan(1/2*c)^9 + 4608*a^
2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*ta
n(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) -
2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c
)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c)^9 - 3840*b^3*log(2*(tan(1/2*d*x)^2*tan(
1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(
1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x
)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1
/2*c)^9 - 4608*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*
tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 +
2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d
*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c)^9 + 3840*b^3*log(2*(ta
n(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*ta
n(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c)
+ 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan
(1/2*d*x)^3*tan(1/2*c)^9 - 37632*a^2*b*tan(1/2*d*x)^4*tan(1/2*c)^9 + 16000*
b^3*tan(1/2*d*x)^4*tan(1/2*c)^9 + 45*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)
^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/
2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - t
an(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^10 + 45*pi*a*
b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2
*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
- 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d
*x) - 1)*tan(1/2*c)^10 - 90*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2
*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c
) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^10 - 90*a*b^2*arctan((tan(1/2*d*x)*tan(1/
2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*
x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^10 + 90*a*b^2*arctan((tan(1

/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c
) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^10 + 90*a*b^2
*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*
d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)
^10 + 288*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1
/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan
(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2
 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^10 - 240*b^3*log(2*(tan(1/2
*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2
*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/
(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*
d*x)^2*tan(1/2*c)^10 - 288*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan
(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan
(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2
 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^10 + 240*b
^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan
(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2
*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)
^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^10 - 1536*a^2*b*tan(1/2*d*x)^3*tan(1/2*c
)^10 + 1280*b^3*tan(1/2*d*x)^3*tan(1/2*c)^10 + 45*pi*a*b^2*sgn(tan(1/2*d*x)
^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)
^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^10 + 45*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan
(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2
*tan(1/2*c) - 1)*tan(1/2*d*x)^10 - 360*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*
c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1
/2*d*x)^10 + 360*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^
2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*
x)^9*tan(1/2*c) + 360*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*
d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1
/2*d*x)^9*tan(1/2*c) - 2880*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(
1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^9*t
an(1/2*c) + 765*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2
*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x
)^8*tan(1/2*c)^2 + 765*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2
*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(
1/2*d*x)^8*tan(1/2*c)^2 - 6120*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - t
an(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^
8*tan(1/2*c)^2 - 128*a^3*tan(1/2*d*x)^10*tan(1/2*c)^2 + 1536*a*b^2*tan(1/2*
d*x)^10*tan(1/2*c)^2 + 1440*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)
*tan(1/2*d*x)^7*tan(1/2*c)^3 + 1440*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2
*c) - 1)*tan(1/2*d*x)^7*tan(1/2*c)^3 - 11520*pi*a*b^2*sgn(tan(1/2*d*x)^2*ta
n(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)
*tan(1/2*d*x)^7*tan(1/2*c)^3 - 5120*a^3*tan(1/2*d*x)^9*tan(1/2*c)^3 + 24576

*a*b^2*tan(1/2*d*x)^9*tan(1/2*c)^3 + 6660*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 + 6660*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 - 53280*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 - 43520*a^3*tan(1/2*d*x)^8*tan(1/2*c)^4 + 162816*a*b^2*tan(1/2*d*x)^8*tan(1/2*c)^4 - 137216*a^3*tan(1/2*d*x)^7*tan(1/2*c)^5 + 540672*a*b^2*tan(1/2*d*x)^7*tan(1/2*c)^5 + 6660*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 + 6660*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 - 53280*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 - 170496*a^3*tan(1/2*d*x)^6*tan(1/2*c)^6 + 479232*a*b^2*tan(1/2*d*x)^6*tan(1/2*c)^6 + 1440*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^3*tan(1/2*c)^7 + 1440*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^3*tan(1/2*c)^7 - 11520*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^3*tan(1/2*c)^7 - 137216*a^3*tan(1/2*d*x)^5*tan(1/2*c)^7 + 540672*a*b^2*tan(1/2*d*x)^5*tan(1/2*c)^7 + 765*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^8 + 765*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^8 - 6120*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2*tan(1/2*c)^8 - 43520*a^3*tan(1/2*d*x)^4*tan(1/2*c)^8 + 162816*a*b^2*tan(1/2*d*x)^4*tan(1/2*c)^8 + 360*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)*tan(1/2*c)^9 + 360*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)*tan(1/2*c)^9 - 2880*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)*tan(1/2*c)^9 - 5120*a^3*tan(1/2*d*x)^3*tan(1/2*c)^9 + 24576*a*b^2*tan(1/2*d*x)^3*tan(1/2*c)^9 + 45*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*c)^10 + 45*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*c)^10 - 360*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^10 - 128*a^3*tan(1/2*d*x)^2*tan(1/2*c)^10 + 1536*a*b^2*tan(1/2*d*x)^2*tan(1/2*c)^10 + 45*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2

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*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*
tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2
+ 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8 + 45*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1
/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*t
an(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^
2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8 - 90
*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(ta
n(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^10 - 9
0*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(t
an(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^10 +
90*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(
tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^10 +
 90*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/
(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^10
+ 288*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c
) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2
*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + t
an(1/2*c)^2 + 1))*tan(1/2*d*x)^10 - 240*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c
)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d
*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*t
an(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^10 - 288*a^2
*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan
(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2
*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)
^2 + 1))*tan(1/2*d*x)^10 + 240*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*t
an(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + t
an(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)
^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^10 + 720*pi*a*b^2*sgn
(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2
 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan
(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1
)*tan(1/2*d*x)^7*tan(1/2*c) + 720*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
- 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c
) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(
1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^7*tan(1/2*c) -
 720*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)
/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^9*
tan(1/2*c) - 720*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan
(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan
(1/2*d*x)^9*tan(1/2*c) + 720*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/
2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*
c) - 1))*tan(1/2*d*x)^9*tan(1/2*c) + 720*a*b^2*arctan((tan(1/2*d*x)*tan(1/2
*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x
) + tan(1/2*c) - 1))*tan(1/2*d*x)^9*tan(1/2*c) + 2304*a^2*b*log(2*(tan(1/2*
d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*
```

c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(
tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d
*x)^9*tan(1/2*c) - 1920*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*
c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + ta
n(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^9*tan(1/2*c) - 2304*a^2*b*lo
g(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*
d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(
1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 +
1))*tan(1/2*d*x)^9*tan(1/2*c) + 1920*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2
 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)
^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(
1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^9*tan(1/2*c) -
1152*a^2*b*tan(1/2*d*x)^10*tan(1/2*c) + 960*b^3*tan(1/2*d*x)^10*tan(1/2*c)
+ 2070*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*
c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*t
an(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 +
 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 + 2070*pi*a*b^2*sgn(tan(1/
2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(
1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*
x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1
/2*d*x)^6*tan(1/2*c)^2 - 1530*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1
/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2
*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^2 - 1530*a*b^2*arctan((tan(1/2*d*x)*tan
(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2
*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^2 + 1530*a*b^2*arctan((t
an(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1
/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^2 + 1530*
a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan
(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1
/2*c)^2 + 4896*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*
tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 -
2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d
*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^2 - 4080*b^3*log(2*(ta
n(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*ta
n(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c)
+ 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan
(1/2*d*x)^8*tan(1/2*c)^2 - 4896*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 -
2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2
+ tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2
*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^2 + 4
080*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) -
2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x
) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1
/2*c)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^2 - 5760*a^2*b*tan(1/2*d*x)^9*tan(1

```
/2*c)^2 + 4800*b^3*tan(1/2*d*x)^9*tan(1/2*c)^2 - 2880*a*b^2*arctan((tan(1/2
*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c)
- tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^7*tan(1/2*c)^3 - 2880*a*b^2*
arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d
*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^7*tan(1/2*c)^
3 + 2880*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c)
- 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x
)^7*tan(1/2*c)^3 + 2880*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x
) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) -
1))*tan(1/2*d*x)^7*tan(1/2*c)^3 + 9216*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*
c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*
d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*
tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^7*tan(1/2*c
)^3 - 7680*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/
2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(
1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2
+ tan(1/2*c)^2 + 1))*tan(1/2*d*x)^7*tan(1/2*c)^3 - 9216*a^2*b*log(2*(tan(1/
2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/
2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)
/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2
*d*x)^7*tan(1/2*c)^3 + 7680*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(
1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(
1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2
+ tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^7*tan(1/2*c)^3 + 44544*a
^2*b*tan(1/2*d*x)^8*tan(1/2*c)^3 - 6400*b^3*tan(1/2*d*x)^8*tan(1/2*c)^3 + 6
660*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c)
+ tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(
1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*
tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 + 6660*pi*a*b^2*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2
*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*
tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*
d*x)^4*tan(1/2*c)^4 - 13320*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2
*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c
) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 - 13320*a*b^2*arctan((tan(1/2*d*x)*tan(
1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*
d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 + 13320*a*b^2*arctan((t
an(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1
/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 + 13320
*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(ta
n(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(
1/2*c)^4 + 42624*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^
2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2
- 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2
*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 - 35520*b^3*log(2*
```

(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)
*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*
c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*
tan(1/2*d*x)^6*tan(1/2*c)^4 - 42624*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^
2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x
)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan
(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^4
 + 35520*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*
c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/
2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 + 322560*a^2*b*tan(1/2*d*x)^
7*tan(1/2*c)^4 - 115200*b^3*tan(1/2*d*x)^7*tan(1/2*c)^4 + 569856*a^2*b*tan(
1/2*d*x)^6*tan(1/2*c)^5 - 167680*b^3*tan(1/2*d*x)^6*tan(1/2*c)^5 + 2070*pi*
a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1
/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2
*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^6 + 2070*pi*a*b^2*sgn(tan(1/2*d*x)^2*t
an(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 -
 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2
*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*
tan(1/2*c)^6 - 13320*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) -
 tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))
*tan(1/2*d*x)^4*tan(1/2*c)^6 - 13320*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c)
 - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) -
tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^6 + 13320*a*b^2*arctan((tan(1/2*
d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) -
 tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^6 + 13320*a*b^2*
arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d
*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^
6 + 42624*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1
/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan
(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2
 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^6 - 35520*b^3*log(2*(tan(1/
2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/
2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)
/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2
*d*x)^4*tan(1/2*c)^6 - 42624*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*t
an(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + t
an(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)
^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^6 + 3552
0*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*
tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x)
+ 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2
*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^6 + 569856*a^2*b*tan(1/2*d*x)^5*tan(1
/2*c)^6 - 167680*b^3*tan(1/2*d*x)^5*tan(1/2*c)^6 + 720*pi*a*b^2*sgn(tan(1/2

```
*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1
/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x
)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/
2*d*x)*tan(1/2*c)^7 + 720*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(
1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*s
gn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)
^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)*tan(1/2*c)^7 - 2880*a*
b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1
/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^3*tan(1/2
*c)^7 - 2880*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2
*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2
*d*x)^3*tan(1/2*c)^7 + 2880*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2
*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c
) - 1))*tan(1/2*d*x)^3*tan(1/2*c)^7 + 2880*a*b^2*arctan((tan(1/2*d*x)*tan(1
/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d
*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^3*tan(1/2*c)^7 + 9216*a^2*b*log(2*(tan(
1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(
1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) +
1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1
/2*d*x)^3*tan(1/2*c)^7 - 7680*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + ta
n(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^
2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c)^7 - 9216*
a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*
tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x)
+ 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2
*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c)^7 + 7680*b^3*log(2*(tan(1/2*d*x)^2*ta
n(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + ta
n(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d
*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan
(1/2*c)^7 + 322560*a^2*b*tan(1/2*d*x)^4*tan(1/2*c)^7 - 115200*b^3*tan(1/2*d
*x)^4*tan(1/2*c)^7 + 45*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/
2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn
(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2
 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^8 + 45*pi*a*b^2*sgn(tan(1/
2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(
1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*
x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1
/2*c)^8 - 1530*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1
/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1
/2*d*x)^2*tan(1/2*c)^8 - 1530*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1
/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2
*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^8 + 1530*a*b^2*arctan((tan(1/2*d*x)*tan
(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2
*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^8 + 1530*a*b^2*arctan((t
```

an(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1
/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^8 + 4896*
a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*
tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x)
- 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2
*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^8 - 4080*b^3*log(2*(tan(1/2*d*x)^2*ta
n(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + ta
n(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d
*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan
(1/2*c)^8 - 4896*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^
2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2
+ 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2
*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^8 + 4080*b^3*log(2*(
tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*
tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c
) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*t
an(1/2*d*x)^2*tan(1/2*c)^8 + 44544*a^2*b*tan(1/2*d*x)^3*tan(1/2*c)^8 - 6400
*b^3*tan(1/2*d*x)^3*tan(1/2*c)^8 - 720*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c
) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x)
+ tan(1/2*c) + 1))*tan(1/2*d*x)*tan(1/2*c)^9 - 720*a*b^2*arctan((tan(1/2*d*
x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + t
an(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)*tan(1/2*c)^9 + 720*a*b^2*arctan
((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*ta
n(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)*tan(1/2*c)^9 + 720*
a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan
(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)*tan(1/2
*c)^9 + 2304*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*ta
n(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*
tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x
)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)*tan(1/2*c)^9 - 1920*b^3*log(2*(tan(1/
2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/
2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)
/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2
*d*x)*tan(1/2*c)^9 - 2304*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(
1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(
1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2
+ tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)*tan(1/2*c)^9 + 1920*b^3*
log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/
2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*ta
n(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2
+ 1))*tan(1/2*d*x)*tan(1/2*c)^9 - 5760*a^2*b*tan(1/2*d*x)^2*tan(1/2*c)^9 +
4800*b^3*tan(1/2*d*x)^2*tan(1/2*c)^9 - 90*a*b^2*arctan((tan(1/2*d*x)*tan(1/
2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*
x) + tan(1/2*c) + 1))*tan(1/2*c)^10 - 90*a*b^2*arctan((tan(1/2*d*x)*tan(1/2
*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x

) - tan(1/2*c) + 1))*tan(1/2*c)^10 + 90*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*c)^10 + 90*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*c)^10 + 288*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^10 - 240*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^10 - 288*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^10 + 240*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^10 - 1152*a^2*b*tan(1/2*d*x)*tan(1/2*c)^10 + 960*b^3*tan(1/2*d*x)*tan(1/2*c)^10 + 45*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^8 + 45*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^8 - 360*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^8 - 128*a^3*tan(1/2*d*x)^10 + 1536*a*b^2*tan(1/2*d*x)^10 + 720*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^7*tan(1/2*c) + 720*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^7*tan(1/2*c) - 5760*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^7*tan(1/2*c) - 1024*a^3*tan(1/2*d*x)^9*tan(1/2*c) + 12288*a*b^2*tan(1/2*d*x)^9*tan(1/2*c) + 2070*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 + 2070*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 - 16560*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 + 2432*a^3*tan(1/2*d*x)^8*tan(1/2*c)^2 + 26112*a*b^2*tan(1/2*d*x)^8*tan(1/2*c)^2 + 49152*a^3*tan(1/2*d*x)^7*tan(1/2*c)^3 - 73728*a*b^2*tan(1/2*d*x)^7*tan(1/2*c)^3 + 6660*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 + 6660*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 - 53280*pi

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*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*ta
n(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 + 170496*a^3*tan(1
/2*d*x)^6*tan(1/2*c)^4 - 479232*a*b^2*tan(1/2*d*x)^6*tan(1/2*c)^4 + 217088*
a^3*tan(1/2*d*x)^5*tan(1/2*c)^5 - 393216*a*b^2*tan(1/2*d*x)^5*tan(1/2*c)^5
+ 2070*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*
c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^2*tan(1
/2*c)^6 + 2070*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*
tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)
^2*tan(1/2*c)^6 - 16560*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*
d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2*tan(1
/2*c)^6 + 170496*a^3*tan(1/2*d*x)^4*tan(1/2*c)^6 - 479232*a*b^2*tan(1/2*d*x
)^4*tan(1/2*c)^6 + 720*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2
*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(
1/2*d*x)*tan(1/2*c)^7 + 720*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*ta
n(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)
*tan(1/2*d*x)*tan(1/2*c)^7 - 5760*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
- tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*
x)*tan(1/2*c)^7 + 49152*a^3*tan(1/2*d*x)^3*tan(1/2*c)^7 - 73728*a*b^2*tan(1
/2*d*x)^3*tan(1/2*c)^7 + 45*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)
*tan(1/2*c)^8 + 45*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x
)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*
c)^8 - 360*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*ta
n(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^8 + 2432*a^3*tan(1/2*d
*x)^2*tan(1/2*c)^8 + 26112*a*b^2*tan(1/2*d*x)^2*tan(1/2*c)^8 - 1024*a^3*tan
(1/2*d*x)*tan(1/2*c)^9 + 12288*a*b^2*tan(1/2*d*x)*tan(1/2*c)^9 - 128*a^3*ta
n(1/2*c)^10 + 1536*a*b^2*tan(1/2*c)^10 - 90*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan
(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2
*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c
)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6 -
90*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) +
 tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1
/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*t
an(1/2*d*x) - 1)*tan(1/2*d*x)^6 - 90*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c)
+ tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) +
tan(1/2*c) + 1))*tan(1/2*d*x)^8 - 90*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c)
- tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) -
tan(1/2*c) + 1))*tan(1/2*d*x)^8 + 90*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c)
+ tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) -
tan(1/2*c) - 1))*tan(1/2*d*x)^8 + 90*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c)
- tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) +
tan(1/2*c) - 1))*tan(1/2*d*x)^8 + 288*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c
)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d
*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*t
an(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8 - 240*b^3*
```

log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8 - 288*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8 + 240*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8 + 1152*a^2*b*tan(1/2*d*x)^9 - 960*b^3*tan(1/2*d*x)^9 - 1440*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^7*tan(1/2*c) - 1440*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^7*tan(1/2*c) + 1440*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^7*tan(1/2*c) + 1440*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^7*tan(1/2*c) + 4608*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^7*tan(1/2*c) - 3840*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^7*tan(1/2*c) - 4608*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^7*tan(1/2*c) + 3840*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^7*tan(1/2*c) + 5760*a^2*b*tan(1/2*d*x)^8*tan(1/2*c) - 4800*b^3*tan(1/2*d*x)^8*tan(1/2*c) + 2070*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^2 + 2070*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^2 - 4140*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 - 4140*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c)

- tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 + 4140*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 + 4140*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 + 13248*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 - 11040*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 - 13248*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 + 11040*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 - 44544*a^2*b*tan(1/2*d*x)^7*tan(1/2*c)^2 + 6400*b^3*tan(1/2*d*x)^7*tan(1/2*c)^2 - 1440*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^3*tan(1/2*c)^3 - 1440*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^3*tan(1/2*c)^3 - 322560*a^2*b*tan(1/2*d*x)^6*tan(1/2*c)^3 + 115200*b^3*tan(1/2*d*x)^6*tan(1/2*c)^3 + 2070*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^4 + 2070*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^4 - 13320*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 - 13320*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 + 13320*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 + 13320*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)

```
/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4*
tan(1/2*c)^4 + 42624*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d
*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c
)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan
(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 - 35520*b^3*lo
g(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*
d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(
1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 +
1))*tan(1/2*d*x)^4*tan(1/2*c)^4 - 42624*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2
*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2
*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2
*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*
c)^4 + 35520*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(
1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*ta
n(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^
2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 - 569856*a^2*b*tan(1/2*d
*x)^5*tan(1/2*c)^4 + 167680*b^3*tan(1/2*d*x)^5*tan(1/2*c)^4 - 569856*a^2*b*
tan(1/2*d*x)^4*tan(1/2*c)^5 + 167680*b^3*tan(1/2*d*x)^4*tan(1/2*c)^5 - 90*p
i*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan
(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c
)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1
/2*d*x) - 1)*tan(1/2*c)^6 - 90*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2
*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) -
 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2
*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^6 - 4140*a*b^2*arct
an((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*
tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 -
4140*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)
/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2*
tan(1/2*c)^6 + 4140*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) +
tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*
tan(1/2*d*x)^2*tan(1/2*c)^6 + 4140*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) -
tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + ta
n(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 + 13248*a^2*b*log(2*(tan(1/2*d*x
)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^
2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan
(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)
^2*tan(1/2*c)^6 - 11040*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*
c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + ta
n(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 - 13248*a^2*b
*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1
/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*t
an(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2
 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 + 11040*b^3*log(2*(tan(1/2*d*x)^2*tan(1/
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2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/
2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^
2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2
*c)^6 - 322560*a^2*b*tan(1/2*d*x)^3*tan(1/2*c)^6 + 115200*b^3*tan(1/2*d*x)^
3*tan(1/2*c)^6 - 1440*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x)
 - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)
)*tan(1/2*d*x)*tan(1/2*c)^7 - 1440*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) -
tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - ta
n(1/2*c) + 1))*tan(1/2*d*x)*tan(1/2*c)^7 + 1440*a*b^2*arctan((tan(1/2*d*x)*
tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(
1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)*tan(1/2*c)^7 + 1440*a*b^2*arctan((
tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(
1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)*tan(1/2*c)^7 + 4608*a
^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*t
an(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) -
 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*
c)^2 + 1))*tan(1/2*d*x)*tan(1/2*c)^7 - 3840*b^3*log(2*(tan(1/2*d*x)^2*tan(1
/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1
/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)
^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)*tan(1/2*
c)^7 - 4608*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan
(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*t
an(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)
^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)*tan(1/2*c)^7 + 3840*b^3*log(2*(tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2
*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/
(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*
d*x)*tan(1/2*c)^7 - 44544*a^2*b*tan(1/2*d*x)^2*tan(1/2*c)^7 + 6400*b^3*tan(
1/2*d*x)^2*tan(1/2*c)^7 - 90*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/
2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*
c) + 1))*tan(1/2*c)^8 - 90*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*
d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c)
 + 1))*tan(1/2*c)^8 + 90*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*
x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) -
 1))*tan(1/2*c)^8 + 90*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x)
 - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1
))*tan(1/2*c)^8 + 288*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*
c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + ta
n(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^8 - 240*b^3*log(2*(tan(1/2*d*x
)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^
2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan
(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^8
 - 288*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*
c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/
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2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 1))*tan(1/2*c)^8 + 240*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^
2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x
)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan
(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^8 + 5760*a^2*b*t
an(1/2*d*x)*tan(1/2*c)^8 - 4800*b^3*tan(1/2*d*x)*tan(1/2*c)^8 + 1152*a^2*b*
tan(1/2*c)^9 - 960*b^3*tan(1/2*c)^9 - 90*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/
2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*ta
n(1/2*c) - 1)*tan(1/2*d*x)^6 - 90*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
- 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c
) - 1)*tan(1/2*d*x)^6 + 720*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(
1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6 -
 128*a^3*tan(1/2*d*x)^8 + 1536*a*b^2*tan(1/2*d*x)^8 - 5120*a^3*tan(1/2*d*x)
^7*tan(1/2*c) + 24576*a*b^2*tan(1/2*d*x)^7*tan(1/2*c) + 2070*pi*a*b^2*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
 tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^2 + 2070*pi*a*b
^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*
d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^2 - 165
60*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*
x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^2 - 43520*a^3*t
an(1/2*d*x)^6*tan(1/2*c)^2 + 162816*a*b^2*tan(1/2*d*x)^6*tan(1/2*c)^2 - 144
0*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) +
tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^3*tan(1/2*c)
^3 - 1440*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1
/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^3*ta
n(1/2*c)^3 + 11520*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^
2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^3*tan(1/2*c)
^3 - 137216*a^3*tan(1/2*d*x)^5*tan(1/2*c)^3 + 540672*a*b^2*tan(1/2*d*x)^5*t
an(1/2*c)^3 + 2070*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x
)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*
d*x)^2*tan(1/2*c)^4 + 2070*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan
(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*
tan(1/2*d*x)^2*tan(1/2*c)^4 - 16560*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*
d*x)^2*tan(1/2*c)^4 - 170496*a^3*tan(1/2*d*x)^4*tan(1/2*c)^4 + 479232*a*b^2
*tan(1/2*d*x)^4*tan(1/2*c)^4 - 137216*a^3*tan(1/2*d*x)^3*tan(1/2*c)^5 + 540
672*a*b^2*tan(1/2*d*x)^3*tan(1/2*c)^5 - 90*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(
1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*
tan(1/2*c) - 1)*tan(1/2*c)^6 - 90*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
- 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c
) - 1)*tan(1/2*c)^6 + 720*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/
2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^6 - 435
20*a^3*tan(1/2*d*x)^2*tan(1/2*c)^6 + 162816*a*b^2*tan(1/2*d*x)^2*tan(1/2*c)
^6 - 5120*a^3*tan(1/2*d*x)*tan(1/2*c)^7 + 24576*a*b^2*tan(1/2*d*x)*tan(1/2*
c)^7 - 128*a^3*tan(1/2*c)^8 + 1536*a*b^2*tan(1/2*c)^8 - 90*pi*a*b^2*sgn(tan
```

(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - t
an(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2
*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*ta
n(1/2*d*x)^4 - 90*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)
^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1
/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan
(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4 + 180*a*b^2*arctan((tan(1/2*
d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) -
 tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6 + 180*a*b^2*arctan((tan(1/2
*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c)
+ tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6 - 180*a*b^2*arctan((tan(1/
2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c)
 - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6 - 180*a*b^2*arctan((tan(1
/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c
) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6 - 576*a^2*b*log(2*(tan(1
/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1
/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1
)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/
2*d*x)^6 + 480*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*ta
n(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*
tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x
)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6 + 576*a^2*b*log(2*(tan(1/2*d*x)^2*t
an(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + t
an(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*
d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6 -
480*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) -
2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x
) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1
/2*c)^2 + 1))*tan(1/2*d*x)^6 + 1536*a^2*b*tan(1/2*d*x)^7 - 1280*b^3*tan(1/2
*d*x)^7 - 720*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*t
an(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2
*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^3*tan(1/2*c) - 720*pi*a*b^2*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/
2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*t
an(1/2*d*x)^3*tan(1/2*c) + 37632*a^2*b*tan(1/2*d*x)^6*tan(1/2*c) - 16000*b^
3*tan(1/2*d*x)^6*tan(1/2*c) + 765*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
+ 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c
) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(
1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^2
 + 765*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*
c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*t
an(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 -
 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4140*a*b^2*arctan((tan(1

/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c
) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^2 - 4140*a*b^
2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2
*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c
)^2 + 4140*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c
) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d
*x)^4*tan(1/2*c)^2 + 4140*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d
*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c)
- 1))*tan(1/2*d*x)^4*tan(1/2*c)^2 + 13248*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1
/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1
/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)
^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/
2*c)^2 - 11040*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*ta
n(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*
tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x
)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^2 - 13248*a^2*b*log(2*(t
an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*t
an(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c)
 + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*ta
n(1/2*d*x)^4*tan(1/2*c)^2 + 11040*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 -
2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2
+ tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2
*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^2 + 1
98912*a^2*b*tan(1/2*d*x)^5*tan(1/2*c)^2 - 88960*b^3*tan(1/2*d*x)^5*tan(1/2*
c)^2 - 720*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(
1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)
^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)
^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)*tan(1/2*c)^3 - 720*pi*a*b^2*sgn(tan(1
/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan
(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d
*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(
1/2*d*x)*tan(1/2*c)^3 + 2880*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/
2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*
c) + 1))*tan(1/2*d*x)^3*tan(1/2*c)^3 + 2880*a*b^2*arctan((tan(1/2*d*x)*tan(
1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*
d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^3*tan(1/2*c)^3 - 2880*a*b^2*arctan((ta
n(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/
2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^3*tan(1/2*c)^3 - 2880*a
*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(
1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^3*tan(1/
2*c)^3 - 9216*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*t
an(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2
*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*
x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c)^3 + 7680*b^3*log(2*(tan
(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan

```
(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) +
 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(
1/2*d*x)^3*tan(1/2*c)^3 + 9216*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2
*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 +
 tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*
c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c)^3 - 76
80*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2
*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x)
 + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/
2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c)^3 + 322560*a^2*b*tan(1/2*d*x)^4*tan(
1/2*c)^3 - 115200*b^3*tan(1/2*d*x)^4*tan(1/2*c)^3 - 90*pi*a*b^2*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1
/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x
)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/
2*c)^4 - 90*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan
(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x
)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c
)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^4 - 4140*a*b^2*arctan((tan(1/2*d*x)*ta
n(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/
2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^4 - 4140*a*b^2*arctan((
tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(
1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^4 + 4140
*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(ta
n(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(
1/2*c)^4 + 4140*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(
1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(
1/2*d*x)^2*tan(1/2*c)^4 + 13248*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 +
2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2
+ tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2
*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^4 - 1
1040*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) +
 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*
x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(
1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^4 - 13248*a^2*b*log(2*(tan(1/2*d*x
)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^
2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan
(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)
^2*tan(1/2*c)^4 + 11040*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*
d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*
c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + ta
n(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^4 + 322560*a^2*
b*tan(1/2*d*x)^3*tan(1/2*c)^4 - 115200*b^3*tan(1/2*d*x)^3*tan(1/2*c)^4 + 19
8912*a^2*b*tan(1/2*d*x)^2*tan(1/2*c)^5 - 88960*b^3*tan(1/2*d*x)^2*tan(1/2*c
)^5 + 180*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c)
 + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*c)
```

```
^6 + 180*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c)
+ 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*c)^
6 - 180*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) -
 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*c)^6
 - 180*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) -
1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*c)^6
- 576*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c
) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2
*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + t
an(1/2*c)^2 + 1))*tan(1/2*c)^6 + 480*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2
 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)
^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(
1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^6 + 576*a^2*b*log
(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d
*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1
/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1
))*tan(1/2*c)^6 - 480*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*
x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)
^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(
1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^6 + 37632*a^2*b*tan(1/2*d*x)*tan
(1/2*c)^6 - 16000*b^3*tan(1/2*d*x)*tan(1/2*c)^6 + 1536*a^2*b*tan(1/2*c)^7 -
 1280*b^3*tan(1/2*c)^7 - 90*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)
*tan(1/2*d*x)^4 - 90*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d
*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/
2*d*x)^4 + 720*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 -
4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4 + 1024*a^3*tan
(1/2*d*x)^6 - 3072*a*b^2*tan(1/2*d*x)^6 - 720*pi*a*b^2*sgn(tan(1/2*d*x)^2*t
an(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 +
 2*tan(1/2*c) - 1)*tan(1/2*d*x)^3*tan(1/2*c) - 720*pi*a*b^2*sgn(tan(1/2*d*x
)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c
)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^3*tan(1/2*c) + 5760*pi*a*b^2*sgn(tan(1
/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1
/2*c)^2 + 1)*tan(1/2*d*x)^3*tan(1/2*c) + 12288*a^3*tan(1/2*d*x)^5*tan(1/2*c
) - 36864*a*b^2*tan(1/2*d*x)^5*tan(1/2*c) + 765*pi*a*b^2*sgn(tan(1/2*d*x)^2
*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2
 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 765*pi*a*b^2*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1
/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 6120*pi*a*b^2*sgn
(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) -
 tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 43520*a^3*tan(1/2*d*x)^4*t
an(1/2*c)^2 - 162816*a*b^2*tan(1/2*d*x)^4*tan(1/2*c)^2 - 720*pi*a*b^2*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
 tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)*tan(1/2*c)^3 - 720*pi*a*b^2*
sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x
```

)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)*tan(1/2*c)^3 + 5760*pi*
a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan
(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)*tan(1/2*c)^3 + 49152*a^3*tan(1/2*d
*x)^3*tan(1/2*c)^3 - 73728*a*b^2*tan(1/2*d*x)^3*tan(1/2*c)^3 - 90*pi*a*b^2*
sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x
)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*c)^4 - 90*pi*a*b^2*sgn(tan(1
/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan
(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*c)^4 + 720*pi*a*b^2*sgn(tan(1/2*d*x)^
2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2
+ 1)*tan(1/2*c)^4 + 43520*a^3*tan(1/2*d*x)^2*tan(1/2*c)^4 - 162816*a*b^2*ta
n(1/2*d*x)^2*tan(1/2*c)^4 + 12288*a^3*tan(1/2*d*x)*tan(1/2*c)^5 - 36864*a*b
^2*tan(1/2*d*x)*tan(1/2*c)^5 + 1024*a^3*tan(1/2*c)^6 - 3072*a*b^2*tan(1/2*c
)^6 + 45*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/
2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2
*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2
+ 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2 + 45*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(
1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*
tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)
^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2 + 1
80*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(
tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^4 +
180*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/
(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^4 -
180*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)
/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4
- 180*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1
)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4
- 576*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*
c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/
2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4 + 480*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c
)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d
*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*t
an(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4 + 576*a^2*
b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(
1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*
tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^
2 + 1))*tan(1/2*d*x)^4 - 480*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan
(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan
(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2
+ tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4 - 5376*a^2*b*tan(1/2*
d*x)^5 + 1408*b^3*tan(1/2*d*x)^5 - 360*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*
c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(
1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 -
tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)*tan(1/2*c

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) - 360*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2
*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*
tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2
- 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)*tan(1/2*c) + 1440*a*b^2*arctan((tan(1/2*
d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) -
 tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^3*tan(1/2*c) + 1440*a*b^2*arc
tan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)
*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^3*tan(1/2*c) - 1
440*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/
(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^3*t
an(1/2*c) - 1440*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan
(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan
(1/2*d*x)^3*tan(1/2*c) - 4608*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*
tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 +
tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c
)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c) + 3840*
b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*ta
n(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) -
2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c
)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c) + 4608*a^2*b*log(2*(tan(1/2*d*x)^2*tan(
1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(
1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x
)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1
/2*c) - 3840*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(
1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*ta
n(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^
2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c) - 37632*a^2*b*tan(1/2*d*x)
^4*tan(1/2*c) + 16000*b^3*tan(1/2*d*x)^4*tan(1/2*c) + 45*pi*a*b^2*sgn(tan(1
/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan
(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d
*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(
1/2*c)^2 + 45*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*t
an(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2
*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^2 - 1530*a*b^2*arctan((tan(1/2*d*x)*
tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(
1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 1530*a*b^2*arctan
((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*ta
n(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 15
30*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(
tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2*ta
n(1/2*c)^2 + 1530*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - ta
n(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*ta
n(1/2*d*x)^2*tan(1/2*c)^2 + 4896*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 +
 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2
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+ tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/
2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 -
4080*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) +
2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*
x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(
1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4896*a^2*b*log(2*(tan(1/2*d*x)
^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2
+ tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(
1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^
2*tan(1/2*c)^2 + 4080*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*
x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)
^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(
1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 44544*a^2*b*t
an(1/2*d*x)^3*tan(1/2*c)^2 + 6400*b^3*tan(1/2*d*x)^3*tan(1/2*c)^2 + 1440*a*
b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1
/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)*tan(1/2*c
)^3 + 1440*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c
) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d
*x)*tan(1/2*c)^3 - 1440*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x
) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) -
1))*tan(1/2*d*x)*tan(1/2*c)^3 - 1440*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c)
- tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) +
tan(1/2*c) - 1))*tan(1/2*d*x)*tan(1/2*c)^3 - 4608*a^2*b*log(2*(tan(1/2*d*x)
^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2
+ tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(
1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)*
tan(1/2*c)^3 + 3840*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)
^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2
- 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/
2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)*tan(1/2*c)^3 + 4608*a^2*b*log(2*
(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)
*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*
c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*
tan(1/2*d*x)*tan(1/2*c)^3 - 3840*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2
*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*
c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)*tan(1/2*c)^3 - 4454
4*a^2*b*tan(1/2*d*x)^2*tan(1/2*c)^3 + 6400*b^3*tan(1/2*d*x)^2*tan(1/2*c)^3
+ 180*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1
)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*c)^4 +
180*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)
/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*c)^4 -
180*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/
(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*c)^4 - 1
80*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(

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tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*c)^4 - 57
6*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) +
2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x
) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1
/2*c)^2 + 1))*tan(1/2*c)^4 + 480*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2
*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 +
 tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*
c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^4 + 576*a^2*b*log(2*(
tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*
tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c
) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*t
an(1/2*c)^4 - 480*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2
*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 +
 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*
d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^4 - 37632*a^2*b*tan(1/2*d*x)*tan(1/2
*c)^4 + 16000*b^3*tan(1/2*d*x)*tan(1/2*c)^4 - 5376*a^2*b*tan(1/2*c)^5 + 140
8*b^3*tan(1/2*c)^5 + 45*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/
2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan
(1/2*d*x)^2 + 45*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^
2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*
x)^2 - 360*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*ta
n(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2 - 1024*a^3*tan(1/2
*d*x)^4 + 3072*a*b^2*tan(1/2*d*x)^4 - 360*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1
/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*t
an(1/2*c) - 1)*tan(1/2*d*x)*tan(1/2*c) - 360*pi*a*b^2*sgn(tan(1/2*d*x)^2*ta
n(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 -
2*tan(1/2*c) - 1)*tan(1/2*d*x)*tan(1/2*c) + 2880*pi*a*b^2*sgn(tan(1/2*d*x)^
2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2
+ 1)*tan(1/2*d*x)*tan(1/2*c) - 5120*a^3*tan(1/2*d*x)^3*tan(1/2*c) + 24576*a
*b^2*tan(1/2*d*x)^3*tan(1/2*c) + 45*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2
*c) - 1)*tan(1/2*c)^2 + 45*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan
(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*
tan(1/2*c)^2 - 360*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^
2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^2 - 2432*a^3*t
an(1/2*d*x)^2*tan(1/2*c)^2 - 26112*a*b^2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 5120
*a^3*tan(1/2*d*x)*tan(1/2*c)^3 + 24576*a*b^2*tan(1/2*d*x)*tan(1/2*c)^3 - 10
24*a^3*tan(1/2*c)^4 + 3072*a*b^2*tan(1/2*c)^4 + 45*pi*a*b^2*sgn(tan(1/2*d*x
)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c
)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*ta
n(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1) + 45*pi*a*
b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2
*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
- 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d
*x) - 1) - 90*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/
```

```
2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/
2*d*x)^2 - 90*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/
2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/
2*d*x)^2 + 90*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/
2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/
2*d*x)^2 + 90*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/
2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/
2*d*x)^2 + 288*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*
tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 -
2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d
*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2 - 240*b^3*log(2*(tan(1/2*d*x)^2*t
an(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + t
an(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*
d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2 -
288*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c)
- 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d
*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan
(1/2*c)^2 + 1))*tan(1/2*d*x)^2 + 240*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2
 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)
^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(
1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2 + 1536*a^2*b*
tan(1/2*d*x)^3 - 1280*b^3*tan(1/2*d*x)^3 + 720*a*b^2*arctan((tan(1/2*d*x)*t
an(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1
/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)*tan(1/2*c) + 720*a*b^2*arctan((tan(
1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*
c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)*tan(1/2*c) - 720*a*b^2*ar
ctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x
)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)*tan(1/2*c) - 72
0*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(t
an(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)*tan(1
/2*c) - 2304*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*ta
n(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*
tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x
)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)*tan(1/2*c) + 1920*b^3*log(2*(tan(1/2*
d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*
c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(
tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d
*x)*tan(1/2*c) + 2304*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*
d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*
c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + ta
n(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)*tan(1/2*c) - 1920*b^3*log(2*
(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)
*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*
c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*
tan(1/2*d*x)*tan(1/2*c) - 5760*a^2*b*tan(1/2*d*x)^2*tan(1/2*c) + 4800*b^3*t
```

an(1/2*d*x)^2*tan(1/2*c) - 90*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1
/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2
*c) + 1))*tan(1/2*c)^2 - 90*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2
*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c
) + 1))*tan(1/2*c)^2 + 90*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d
*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c)
- 1))*tan(1/2*c)^2 + 90*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x
) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) -
1))*tan(1/2*c)^2 + 288*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2
*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2
*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + t
an(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^2 - 240*b^3*log(2*(tan(1/2*d*
x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)
^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(ta
n(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^
2 - 288*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2
*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1
/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 1))*tan(1/2*c)^2 + 240*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)
^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*
x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*ta
n(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^2 - 5760*a^2*b*
tan(1/2*d*x)*tan(1/2*c)^2 + 4800*b^3*tan(1/2*d*x)*tan(1/2*c)^2 + 1536*a^2*b
*tan(1/2*c)^3 - 1280*b^3*tan(1/2*c)^3 + 45*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(
1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*
tan(1/2*c) - 1) + 45*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d
*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1) - 360*
pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*
tan(1/2*c) - tan(1/2*c)^2 + 1) + 128*a^3*tan(1/2*d*x)^2 - 1536*a*b^2*tan(1/
2*d*x)^2 - 1024*a^3*tan(1/2*d*x)*tan(1/2*c) + 12288*a*b^2*tan(1/2*d*x)*tan(
1/2*c) + 128*a^3*tan(1/2*c)^2 - 1536*a*b^2*tan(1/2*c)^2 - 90*a*b^2*arctan((
tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(
1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)) - 90*a*b^2*arctan((tan(1/2*d*x)*ta
n(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/
2*d*x) - tan(1/2*c) + 1)) + 90*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(
1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/
2*c) - 1)) + 90*a*b^2*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(
1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)) + 28
8*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) +
2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x
) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1
/2*c)^2 + 1)) - 240*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)
^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2
- 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/
2*d*x)^2 + tan(1/2*c)^2 + 1)) - 288*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^

```
2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x
)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan
(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1)) + 240*b^3*log(2*(tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)
^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(ta
n(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1)) + 1152*a^2*
b*tan(1/2*d*x) - 960*b^3*tan(1/2*d*x) + 1152*a^2*b*tan(1/2*c) - 960*b^3*tan
(1/2*c) + 128*a^3 - 1536*a*b^2)/(d*tan(1/2*d*x)^10*tan(1/2*c)^10 + d*tan(1/
2*d*x)^10*tan(1/2*c)^8 - 8*d*tan(1/2*d*x)^9*tan(1/2*c)^9 + d*tan(1/2*d*x)^8
*tan(1/2*c)^10 - 2*d*tan(1/2*d*x)^10*tan(1/2*c)^6 - 16*d*tan(1/2*d*x)^9*tan
(1/2*c)^7 + 17*d*tan(1/2*d*x)^8*tan(1/2*c)^8 - 16*d*tan(1/2*d*x)^7*tan(1/2*
c)^9 - 2*d*tan(1/2*d*x)^6*tan(1/2*c)^10 - 2*d*tan(1/2*d*x)^10*tan(1/2*c)^4
+ 46*d*tan(1/2*d*x)^8*tan(1/2*c)^6 - 32*d*tan(1/2*d*x)^7*tan(1/2*c)^7 + 46*
d*tan(1/2*d*x)^6*tan(1/2*c)^8 - 2*d*tan(1/2*d*x)^4*tan(1/2*c)^10 + d*tan(1/
2*d*x)^10*tan(1/2*c)^2 + 16*d*tan(1/2*d*x)^9*tan(1/2*c)^3 + 46*d*tan(1/2*d*
x)^8*tan(1/2*c)^4 + 148*d*tan(1/2*d*x)^6*tan(1/2*c)^6 + 46*d*tan(1/2*d*x)^4
*tan(1/2*c)^8 + 16*d*tan(1/2*d*x)^3*tan(1/2*c)^9 + d*tan(1/2*d*x)^2*tan(1/2
*c)^10 + d*tan(1/2*d*x)^10 + 8*d*tan(1/2*d*x)^9*tan(1/2*c) + 17*d*tan(1/2*d
*x)^8*tan(1/2*c)^2 + 32*d*tan(1/2*d*x)^7*tan(1/2*c)^3 + 148*d*tan(1/2*d*x)^
6*tan(1/2*c)^4 + 148*d*tan(1/2*d*x)^4*tan(1/2*c)^6 + 32*d*tan(1/2*d*x)^3*ta
n(1/2*c)^7 + 17*d*tan(1/2*d*x)^2*tan(1/2*c)^8 + 8*d*tan(1/2*d*x)*tan(1/2*c)
^9 + d*tan(1/2*c)^10 + d*tan(1/2*d*x)^8 + 16*d*tan(1/2*d*x)^7*tan(1/2*c) +
46*d*tan(1/2*d*x)^6*tan(1/2*c)^2 + 148*d*tan(1/2*d*x)^4*tan(1/2*c)^4 + 46*d
*tan(1/2*d*x)^2*tan(1/2*c)^6 + 16*d*tan(1/2*d*x)*tan(1/2*c)^7 + d*tan(1/2*c
)^8 - 2*d*tan(1/2*d*x)^6 + 46*d*tan(1/2*d*x)^4*tan(1/2*c)^2 - 32*d*tan(1/2*
d*x)^3*tan(1/2*c)^3 + 46*d*tan(1/2*d*x)^2*tan(1/2*c)^4 - 2*d*tan(1/2*c)^6 -
 2*d*tan(1/2*d*x)^4 - 16*d*tan(1/2*d*x)^3*tan(1/2*c) + 17*d*tan(1/2*d*x)^2*
tan(1/2*c)^2 - 16*d*tan(1/2*d*x)*tan(1/2*c)^3 - 2*d*tan(1/2*c)^4 + d*tan(1/
2*d*x)^2 - 8*d*tan(1/2*d*x)*tan(1/2*c) + d*tan(1/2*c)^2 + d)
```

Mupad [B] (verification not implemented)

Time = 7.57 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.42

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,\left(6\,a^2\,b-5\,b^3\right)}{d}$$
$$-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(6\,a^2\,b-5\,b^3\right)+4\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-16\,a\,b^2-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(16\,a\,b^2-\frac{4\,a^3}{3}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+\tan\right.}$$

```
[In] int(sin(c + d*x)^3*(a + b*tan(c + d*x))^3,x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*(6*a^2*b - 5*b^3))/d - (tan(c/2 + (d*x)/2)*(6*a^
2*b - 5*b^3) + 4*a^3*tan(c/2 + (d*x)/2)^6 - 16*a*b^2 - tan(c/2 + (d*x)/2)^2
```

```
*(16*a*b^2 - (4*a^3)/3) + tan(c/2 + (d*x)/2)^4*(32*a*b^2 - (20*a^3)/3) + ta
n(c/2 + (d*x)/2)^9*(6*a^2*b - 5*b^3) + tan(c/2 + (d*x)/2)^3*(8*a^2*b - (20*
b^3)/3) + tan(c/2 + (d*x)/2)^7*(8*a^2*b - (20*b^3)/3) - tan(c/2 + (d*x)/2)^
5*(28*a^2*b - (22*b^3)/3) + (4*a^3)/3)/(d*(tan(c/2 + (d*x)/2)^2 - 2*tan(c/2
 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + tan(c/2 + (
d*x)/2)^10 + 1))
```

3.33 $\int \sin^2(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 103

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{1}{2}a(a^2-9b^2)\,x - \frac{b(3a^2-2b^2)\log(\cos(c+dx))}{d} + \frac{9ab^2\tan(c+dx)}{2d} + \frac{b^3\tan^2(c+dx)}{d} - \frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^3}{2d}$$

```
[Out] 1/2*a*(a^2-9*b^2)*x-b*(3*a^2-2*b^2)*ln(cos(d*x+c))/d+9/2*a*b^2*tan(d*x+c)/d
+b^3*tan(d*x+c)^2/d-1/2*cos(d*x+c)*sin(d*x+c)*(a+b*tan(d*x+c))^3/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3597, 1659, 815, 649, 209, 266}

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{b(3a^2-2b^2)\log(\cos(c+dx))}{d} + \frac{1}{2}ax(a^2-9b^2) + \frac{9ab^2\tan(c+dx)}{2d} - \frac{\sin(c+dx)\cos(c+dx)(a+b\tan(c+dx))^3}{2d} + \frac{b^3\tan^2(c+dx)}{d}$$

```
[In] Int[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (a*(a^2 - 9*b^2)*x)/2 - (b*(3*a^2 - 2*b^2)*Log[Cos[c + d*x]])/d + (9*a*b^2*
Tan[c + d*x])/(2*d) + (b^3*Tan[c + d*x]^2)/d - (Cos[c + d*x]*Sin[c + d*x]*(
a + b*Tan[c + d*x])^3)/(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{b\text{Subst}\left(\int \frac{x^2(a+x)^3}{(b^2+x^2)^2}\,dx, x, b\tan(c+dx)\right)}{d}$$
$$= -\frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^3}{2d} - \frac{\text{Subst}\left(\int \frac{(a+x)^2(-ab^2-4b^2x)}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{2bd}$$
$$= -\frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^3}{2d} - \frac{\text{Subst}\left(\int \left(-9ab^2-4b^2x-\frac{ab^2(a^2-9b^2)+2b^2(3a^2-2b^2)x}{b^2+x^2}\right)\,dx, x, b\tan(c+dx)\right)}{2bd}$$
$$= \frac{9ab^2\tan(c+dx)}{2d} + \frac{b^3\tan^2(c+dx)}{d} - \frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^3}{2d} + \frac{\text{Subst}\left(\int \frac{ab^2(a^2-9b^2)+2b^2(3a^2-2b^2)x}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{2bd}$$
$$= \frac{9ab^2\tan(c+dx)}{2d} + \frac{b^3\tan^2(c+dx)}{d} - \frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^3}{2d} + \frac{(ab(a^2-9b^2))\,\text{Subst}\left(\int \frac{1}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{2d} + \frac{(b(3a^2-2b^2))\,\text{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{d}$$
$$= \frac{1}{2}a\left(a^2-9b^2\right)x - \frac{b(3a^2-2b^2)\log(\cos(c+dx))}{d} + \frac{9ab^2\tan(c+dx)}{2d} + \frac{b^3\tan^2(c+dx)}{d} - \frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^3}{2d}$$

Mathematica [A] (verified)

Time = 4.79 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.97

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{b\left(-\frac{a(a^2-3b^2)\arctan(\tan(c+dx))}{b} + (3a^2-b^2)\cos^2(c+dx) + \left(3a^2-2b^2+\frac{a^3-6ab^2}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)\right.}{}$$

```
[In] Integrate[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (b*(-((a*(a^2 - 3*b^2)*ArcTan[Tan[c + d*x]])/b) + (3*a^2 - b^2)*Cos[c + d*x
]^2 + (3*a^2 - 2*b^2 + (a^3 - 6*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c
```

```
 + d*x]] + (3*a^2 - 2*b^2 + (-a^3 + 6*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b
*Tan[c + d*x]] - (a*(a^2 - 3*b^2)*Sin[2*(c + d*x)])/(2*b) + 6*a*b*Tan[c + d
*x] + b^2*Tan[c + d*x]^2))/(2*d)
```

Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.58

method	result
derivativedivides	$\frac{a^3\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^2b\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+3a\,b^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+}{2}\right.\right.}{d}$
default	$\frac{a^3\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^2b\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+3a\,b^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+}{2}\right.\right.}{d}$
risch	$\frac{3ie^{-2i(dx+c)}a\,b^2}{8d}-\frac{4ib^3c}{d}+\frac{a^3x}{2}-\frac{9xa\,b^2}{2}+\frac{3\,e^{2i(dx+c)}b\,a^2}{8d}-\frac{e^{2i(dx+c)}b^3}{8d}+3ixb\,a^2-\frac{ie^{-2i(dx+c)}a^3}{8d}+3$

```
[In] int(sin(d*x+c)^2*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+3*a^2*b*(-1/2*sin(d*x+c
)^2-ln(cos(d*x+c)))+3*a*b^2*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(
d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c)+b^3*(1/2*sin(d*x+c)^6/cos(d*x+c)^2+1/2*si
n(d*x+c)^4+sin(d*x+c)^2+2*ln(cos(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.45

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$=\frac{2\,(3\,a^2b-b^3)\cos{(dx+c)}^4-4\,(3\,a^2b-2\,b^3)\cos{(dx+c)}^2\log{(-\cos{(dx+c)})}+2\,b^3-(3\,a^2b-b^3-2\,(a^3}{4\,d\cos{(dx+c)}^2}$$

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(3*a^2*b - b^3)*cos(d*x + c)^4 - 4*(3*a^2*b - 2*b^3)*cos(d*x + c)^2*
log(-cos(d*x + c)) + 2*b^3 - (3*a^2*b - b^3 - 2*(a^3 - 9*a*b^2)*d*x)*cos(d*
x + c)^2 + 2*(6*a*b^2*cos(d*x + c) - (a^3 - 3*a*b^2)*cos(d*x + c)^3)*sin(d*
x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^3\,dx = \int (a+b\tan(c+dx))^3 \sin^2(c+dx)\,dx$$

```
[In] integrate(sin(d*x+c)**2*(a+b*tan(d*x+c))**3,x)
[Out] Integral((a + b*tan(c + d*x))**3*sin(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.10

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{b^3\tan(dx+c)^2 + 6\,ab^2\tan(dx+c) + (a^3-9\,ab^2)(dx+c) + (3\,a^2b-2\,b^3)\log\left(\tan(dx+c)^2+1\right) + \frac{3\,a^2b-b}{}}{2\,d}$$

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
[Out] 1/2*(b^3*tan(d*x + c)^2 + 6*a*b^2*tan(d*x + c) + (a^3 - 9*a*b^2)*(d*x + c)
+ (3*a^2*b - 2*b^3)*log(tan(d*x + c)^2 + 1) + (3*a^2*b - b^3 - (a^3 - 3*a*b
^2)*tan(d*x + c))/(tan(d*x + c)^2 + 1))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2370 vs. 2(97) = 194.

Time = 1.18 (sec) , antiderivative size = 2370, normalized size of antiderivative = 23.01

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^3\,dx = \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="giac")
[Out] 1/4*(2*a^3*d*x*tan(d*x)^4*tan(c)^4 - 18*a*b^2*d*x*tan(d*x)^4*tan(c)^4 - 6*a
^2*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)
^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4 + 4*b^3*log(4*(tan(d*x
)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + t
an(c)^2 + 1))*tan(d*x)^4*tan(c)^4 + 2*a^3*d*x*tan(d*x)^4*tan(c)^2 - 18*a*b^
2*d*x*tan(d*x)^4*tan(c)^2 - 4*a^3*d*x*tan(d*x)^3*tan(c)^3 + 36*a*b^2*d*x*ta
n(d*x)^3*tan(c)^3 + 2*a^3*d*x*tan(d*x)^2*tan(c)^4 - 18*a*b^2*d*x*tan(d*x)^2
*tan(c)^4 + 3*a^2*b*tan(d*x)^4*tan(c)^4 + b^3*tan(d*x)^4*tan(c)^4 - 6*a^2*b
```

```
*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 +
 tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^4*tan(c)^2 + 4*b^3*log(4*(tan(d*x)^2*
tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c
)^2 + 1))*tan(d*x)^4*tan(c)^2 + 12*a^2*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan
(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*
x)^3*tan(c)^3 - 8*b^3*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(
tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 + 2*a
^3*tan(d*x)^4*tan(c)^3 - 18*a*b^2*tan(d*x)^4*tan(c)^3 - 6*a^2*b*log(4*(tan(
d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2
+ tan(c)^2 + 1))*tan(d*x)^2*tan(c)^4 + 4*b^3*log(4*(tan(d*x)^2*tan(c)^2 - 2
*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*ta
n(d*x)^2*tan(c)^4 + 2*a^3*tan(d*x)^3*tan(c)^4 - 18*a*b^2*tan(d*x)^3*tan(c)^
4 - 4*a^3*d*x*tan(d*x)^3*tan(c) + 36*a*b^2*d*x*tan(d*x)^3*tan(c) + 4*a^3*d*
x*tan(d*x)^2*tan(c)^2 - 36*a*b^2*d*x*tan(d*x)^2*tan(c)^2 - 3*a^2*b*tan(d*x)
^4*tan(c)^2 + 5*b^3*tan(d*x)^4*tan(c)^2 - 4*a^3*d*x*tan(d*x)*tan(c)^3 + 36*
a*b^2*d*x*tan(d*x)*tan(c)^3 - 18*a^2*b*tan(d*x)^3*tan(c)^3 + 6*b^3*tan(d*x)
^3*tan(c)^3 - 3*a^2*b*tan(d*x)^2*tan(c)^4 + 5*b^3*tan(d*x)^2*tan(c)^4 + 12*
a^2*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c
)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^3*tan(c) - 8*b^3*log(4*(tan(d*x)
^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + ta
n(c)^2 + 1))*tan(d*x)^3*tan(c) - 12*a*b^2*tan(d*x)^4*tan(c) - 12*a^2*b*log(
4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(
d*x)^2 + tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 + 8*b^3*log(4*(tan(d*x)^2*tan(c
)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 +
 1))*tan(d*x)^2*tan(c)^2 - 6*a^3*tan(d*x)^3*tan(c)^2 + 18*a*b^2*tan(d*x)^3*
tan(c)^2 + 12*a^2*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(ta
n(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)*tan(c)^3 - 8*b^3*l
og(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + t
an(d*x)^2 + tan(c)^2 + 1))*tan(d*x)*tan(c)^3 - 6*a^3*tan(d*x)^2*tan(c)^3 +
18*a*b^2*tan(d*x)^2*tan(c)^3 - 12*a*b^2*tan(d*x)*tan(c)^4 + 2*a^3*d*x*tan(d
*x)^2 - 18*a*b^2*d*x*tan(d*x)^2 + 2*b^3*tan(d*x)^4 - 4*a^3*d*x*tan(d*x)*tan
(c) + 36*a*b^2*d*x*tan(d*x)*tan(c) + 6*a^2*b*tan(d*x)^3*tan(c) - 2*b^3*tan(
d*x)^3*tan(c) + 2*a^3*d*x*tan(c)^2 - 18*a*b^2*d*x*tan(c)^2 + 30*a^2*b*tan(d
*x)^2*tan(c)^2 - 2*b^3*tan(d*x)^2*tan(c)^2 + 6*a^2*b*tan(d*x)*tan(c)^3 - 2*
b^3*tan(d*x)*tan(c)^3 + 2*b^3*tan(c)^4 - 6*a^2*b*log(4*(tan(d*x)^2*tan(c)^2
 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1)
)*tan(d*x)^2 + 4*b^3*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(t
an(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^2 + 12*a*b^2*tan(
d*x)^3 + 12*a^2*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(
d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)*tan(c) - 8*b^3*log(4
*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d
*x)^2 + tan(c)^2 + 1))*tan(d*x)*tan(c) + 6*a^3*tan(d*x)^2*tan(c) - 18*a*b^2
*tan(d*x)^2*tan(c) - 6*a^2*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c)
 + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(c)^2 + 4*b^3*l
og(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + t
```

```
an(d*x)^2 + tan(c)^2 + 1))*tan(c)^2 + 6*a^3*tan(d*x)*tan(c)^2 - 18*a*b^2*ta
n(d*x)*tan(c)^2 + 12*a*b^2*tan(c)^3 + 2*a^3*d*x - 18*a*b^2*d*x - 3*a^2*b*ta
n(d*x)^2 + 5*b^3*tan(d*x)^2 - 18*a^2*b*tan(d*x)*tan(c) + 6*b^3*tan(d*x)*tan
(c) - 3*a^2*b*tan(c)^2 + 5*b^3*tan(c)^2 - 6*a^2*b*log(4*(tan(d*x)^2*tan(c)^
2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1
)) + 4*b^3*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*
tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1)) - 2*a^3*tan(d*x) + 18*a*b^2*tan(d*x)
 - 2*a^3*tan(c) + 18*a*b^2*tan(c) + 3*a^2*b + b^3)/(d*tan(d*x)^4*tan(c)^4 +
 d*tan(d*x)^4*tan(c)^2 - 2*d*tan(d*x)^3*tan(c)^3 + d*tan(d*x)^2*tan(c)^4 -
2*d*tan(d*x)^3*tan(c) + 2*d*tan(d*x)^2*tan(c)^2 - 2*d*tan(d*x)*tan(c)^3 + d
*tan(d*x)^2 - 2*d*tan(d*x)*tan(c) + d*tan(c)^2 + d)
```

Mupad [B] (verification not implemented)

Time = 4.68 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.47

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{b^3\tan(c+d\,x)^2}{2\,d} + \frac{\cos(c+d\,x)^2\left(\frac{3\,a^2\,b}{2}-\frac{b^3}{2}+\tan(c+d\,x)\,\left(\frac{3\,a\,b^2}{2}-\frac{a^3}{2}\right)\right)}{d} + \frac{\ln\left(\tan(c+d\,x)^2+1\right)\,\left(\frac{3\,a^2\,b}{2}-b^3\right)}{d} + \frac{3\,a\,b^2\tan(c+d\,x)}{d} - \frac{a\,\mathrm{atan}\left(\frac{a\tan(c+d\,x)\,(a-3\,b)\,(a+3\,b)}{2\,\left(\frac{9\,a\,b^2}{2}-\frac{a^3}{2}\right)}\right)\,(a-3\,b)\,(a+3\,b)}{2\,d}$$

```
[In] int(sin(c + d*x)^2*(a + b*tan(c + d*x))^3,x)
```

```
[Out] (b^3*tan(c + d*x)^2)/(2*d) + (cos(c + d*x)^2*((3*a^2*b)/2 - b^3/2 + tan(c +
 d*x)*((3*a*b^2)/2 - a^3/2)))/d + (log(tan(c + d*x)^2 + 1)*((3*a^2*b)/2 - b
^3))/d + (3*a*b^2*tan(c + d*x))/d - (a*atan((a*tan(c + d*x)*(a - 3*b)*(a +
3*b))/(2*((9*a*b^2)/2 - a^3/2)))*(a - 3*b)*(a + 3*b))/(2*d)
```

3.34 $\int \sin(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 19, antiderivative size = 133

$$\int \sin(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{3a^2b\operatorname{arctanh}(\sin(c+dx))}{d}-\frac{3b^3\operatorname{arctanh}(\sin(c+dx))}{2d}-\frac{a^3\cos(c+dx)}{d}+\frac{3ab^2\cos(c+dx)}{d}+\frac{3ab^2\sec(c+dx)}{d}-\frac{3a^2b\sin(c+dx)}{d}+\frac{3b^3\sin(c+dx)}{2d}+\frac{b^3\sin(c+dx)\tan^2(c+dx)}{2d}$$

```
[Out] 3*a^2*b*arctanh(sin(d*x+c))/d-3/2*b^3*arctanh(sin(d*x+c))/d-a^3*cos(d*x+c)/
d+3*a*b^2*cos(d*x+c)/d+3*a*b^2*sec(d*x+c)/d-3*a^2*b*sin(d*x+c)/d+3/2*b^3*si
n(d*x+c)/d+1/2*b^3*sin(d*x+c)*tan(d*x+c)^2/d
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {3598, 2718, 2672, 327, 212, 2670, 14, 294}

$$\int \sin(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{a^3\cos(c+dx)}{d}+\frac{3a^2b\operatorname{arctanh}(\sin(c+dx))}{d}-\frac{3a^2b\sin(c+dx)}{d}+\frac{3ab^2\cos(c+dx)}{d}+\frac{3ab^2\sec(c+dx)}{d}-\frac{3b^3\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{3b^3\sin(c+dx)}{2d}+\frac{b^3\sin(c+dx)\tan^2(c+dx)}{2d}$$

```
[In] Int[Sin[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a^2*b*ArcTanh[Sin[c + d*x]])/d - (3*b^3*ArcTanh[Sin[c + d*x]])/(2*d) - (
a^3*Cos[c + d*x])/d + (3*a*b^2*Cos[c + d*x])/d + (3*a*b^2*Sec[c + d*x])/d -
 (3*a^2*b*Sin[c + d*x])/d + (3*b^3*Sin[c + d*x])/(2*d) + (b^3*Sin[c + d*x]*
Tan[c + d*x]^2)/(2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a^3\sin(c+dx)+3a^2b\sin(c+dx)\tan(c+dx)+3ab^2\sin(c+dx)\tan^2(c+dx)\right.\\
&\qquad\left.+b^3\sin(c+dx)\tan^3(c+dx)\right)\,dx\\
&= a^3\int \sin(c+dx)\,dx+\left(3a^2b\right)\int \sin(c+dx)\tan(c+dx)\,dx\\
&\quad+\left(3ab^2\right)\int \sin(c+dx)\tan^2(c+dx)\,dx+b^3\int \sin(c+dx)\tan^3(c+dx)\,dx\\
&= -\frac{a^3\cos(c+dx)}{d}+\frac{\left(3a^2b\right)\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx,x,\sin(c+dx)\right)}{d}\\
&\quad-\frac{\left(3ab^2\right)\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx,x,\cos(c+dx)\right)}{d}+\frac{b^3\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx,x,\sin(c+dx)\right)}{d}\\
&= -\frac{a^3\cos(c+dx)}{d}-\frac{3a^2b\sin(c+dx)}{d}+\frac{b^3\sin(c+dx)\tan^2(c+dx)}{2d}\\
&\quad+\frac{\left(3a^2b\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sin(c+dx)\right)}{d}\\
&\quad-\frac{\left(3ab^2\right)\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)\,dx,x,\cos(c+dx)\right)}{d}\\
&\quad-\frac{\left(3b^3\right)\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx,x,\sin(c+dx)\right)}{2d}\\
&= \frac{3a^2b\,\text{arctanh}(\sin(c+dx))}{d}-\frac{a^3\cos(c+dx)}{d}+\frac{3ab^2\cos(c+dx)}{d}\\
&\quad+\frac{3ab^2\sec(c+dx)}{d}-\frac{3a^2b\sin(c+dx)}{d}+\frac{3b^3\sin(c+dx)}{2d}\\
&\quad+\frac{b^3\sin(c+dx)\tan^2(c+dx)}{2d}-\frac{\left(3b^3\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sin(c+dx)\right)}{2d}\\
&= \frac{3a^2b\,\text{arctanh}(\sin(c+dx))}{d}-\frac{3b^3\text{arctanh}(\sin(c+dx))}{2d}-\frac{a^3\cos(c+dx)}{d}+\frac{3ab^2\cos(c+dx)}{d}\\
&\quad+\frac{3ab^2\sec(c+dx)}{d}-\frac{3a^2b\sin(c+dx)}{d}+\frac{3b^3\sin(c+dx)}{2d}+\frac{b^3\sin(c+dx)\tan^2(c+dx)}{2d}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 637 vs. 2(133) = 266.

Time = 6.79 (sec) , antiderivative size = 637, normalized size of antiderivative = 4.79

$$\int \sin(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{3ab^2\cos^3(c+dx)(a+b\tan(c+dx))^3}{d(a\cos(c+dx)+b\sin(c+dx))^3} - \frac{a(a^2-3b^2)\cos^4(c+dx)(a+b\tan(c+dx))^3}{d(a\cos(c+dx)+b\sin(c+dx))^3}$$
$$- \frac{3(2a^2b-b^3)\cos^3(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^3}{2d(a\cos(c+dx)+b\sin(c+dx))^3}$$
$$+ \frac{3(2a^2b-b^3)\cos^3(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^3}{2d(a\cos(c+dx)+b\sin(c+dx))^3}$$
$$+ \frac{b^3\cos^3(c+dx)(a+b\tan(c+dx))^3}{4d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a\cos(c+dx)+b\sin(c+dx))^3}$$
$$+ \frac{3ab^2\cos^3(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^3}$$
$$- \frac{b^3\cos^3(c+dx)(a+b\tan(c+dx))^3}{4d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a\cos(c+dx)+b\sin(c+dx))^3}$$
$$- \frac{3ab^2\cos^3(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^3}$$
$$- \frac{b(3a^2-b^2)\cos^3(c+dx)\sin(c+dx)(a+b\tan(c+dx))^3}{d(a\cos(c+dx)+b\sin(c+dx))^3}$$

```
[In] Integrate[Sin[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a*b^2*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(d*(a*Cos[c + d*x] + b*Sin[
c + d*x])^3) - (a*(a^2 - 3*b^2)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^3)/(d*(
a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (3*(2*a^2*b - b^3)*Cos[c + d*x]^3*Log
[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3)/(2*d*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^3) + (3*(2*a^2*b - b^3)*Cos[c + d*x]^3*Log[Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3)/(2*d*(a*Cos[c + d*x]
 + b*Sin[c + d*x])^3) + (b^3*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(4*d*(C
os[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)
+ (3*a*b^2*Cos[c + d*x]^3*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^3)/(d*(Cos[
(c + d*x)/2] - Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (b^
3*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(4*d*(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (3*a*b^2*Cos[c + d*x]^3*S
in[(c + d*x)/2]*(a + b*Tan[c + d*x])^3)/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x
)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (b*(3*a^2 - b^2)*Cos[c + d*x]^
3*Sin[c + d*x]*(a + b*Tan[c + d*x])^3)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])
^3)
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{-a^3\cos(dx+c)+3a^2b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+3a\,b^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+b^3\left(\right.}{d}$
default	$\frac{-a^3\cos(dx+c)+3a^2b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+3a\,b^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+b^3\left(\right.}{d}$
risch	$\frac{3ie^{i(dx+c)}b\,a^2}{2d}-\frac{ie^{i(dx+c)}b^3}{2d}-\frac{e^{i(dx+c)}a^3}{2d}+\frac{3\,e^{i(dx+c)}a\,b^2}{2d}-\frac{3ie^{-i(dx+c)}b\,a^2}{2d}+\frac{ie^{-i(dx+c)}b^3}{2d}-\frac{e^{-i(dx+c)}a^3}{2d}$

```
[In] int(sin(d*x+c)*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^3*cos(d*x+c)+3*a^2*b*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+3*a*b^
2*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+b^3*(1/2*sin(d*x+c)
^5/cos(d*x+c)^2+1/2*sin(d*x+c)^3+3/2*sin(d*x+c)-3/2*ln(sec(d*x+c)+tan(d*x+c
))))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.08

$$\int \sin(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$=\frac{12\,ab^2\cos{(dx+c)}-4\,(a^3-3\,ab^2)\cos{(dx+c)}^3+3\,(2\,a^2b-b^3)\cos{(dx+c)}^2\log{(\sin{(dx+c)}+1)}-3\,(2}{4\,d\cos{(dx+}}$$

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(12*a*b^2*cos(d*x + c) - 4*(a^3 - 3*a*b^2)*cos(d*x + c)^3 + 3*(2*a^2*b
- b^3)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - 3*(2*a^2*b - b^3)*cos(d*x + c
)^2*log(-sin(d*x + c) + 1) + 2*(b^3 - 2*(3*a^2*b - b^3)*cos(d*x + c)^2)*sin
(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \sin(c+dx)(a+b\tan(c+dx))^3\,dx = \int (a+b\tan{(c+dx)})^3\sin{(c+dx)}\,dx$$

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*sin(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.96

$$\int \sin(c+dx)(a+b\tan(c+dx))^3\,dx =$$
$$-\frac{b^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}+3\log(\sin(dx+c)+1)-3\log(\sin(dx+c)-1)-4\sin(dx+c)\right)-12ab^2\left(\frac{1}{\cos(dx+c)}+\cdots\right.}{4\cdots}$$

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(d*x + c) + 1) -
3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)) - 12*a*b^2*(1/cos(d*x + c) + cos(
d*x + c)) - 6*a^2*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(
d*x + c)) + 4*a^3*cos(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12476 vs. 2(127) = 254.

Time = 8.35 (sec) , antiderivative size = 12476, normalized size of antiderivative = 93.80

$$\int \sin(c+dx)(a+b\tan(c+dx))^3\,dx = \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4*(9*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/
2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 - 6*a^2*b
*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1
/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*t
an(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2
 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 + 3*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)
^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*
```

```
x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*ta
n(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^
6 + 6*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c
) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2
*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + t
an(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 - 3*b^3*log(2*(tan(1/2*d*x)^2
*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 +
 tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/
2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*
tan(1/2*c)^6 - 9*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2
- 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^4
 - 72*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2
*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^5*tan(1/2*c)^5 - 9*pi*a*b
^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/
2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 - 4*a^3*tan(1/2*d*x)^6
*tan(1/2*c)^6 + 24*a*b^2*tan(1/2*d*x)^6*tan(1/2*c)^6 + 6*a^2*b*log(2*(tan(1
/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1
/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1
)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/
2*d*x)^6*tan(1/2*c)^4 - 3*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/
2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/
2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 +
tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 - 6*a^2*b*l
og(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2
*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan
(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 +
 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 + 3*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2
 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)
^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(
1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^4
+ 48*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c)
 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*
d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + ta
n(1/2*c)^2 + 1))*tan(1/2*d*x)^5*tan(1/2*c)^5 - 24*b^3*log(2*(tan(1/2*d*x)^2
*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 +
 tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/
2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^5*
tan(1/2*c)^5 - 48*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)
^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2
 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/
2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^5*tan(1/2*c)^5 + 24*b^3*log(2*(t
an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*t
an(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c)
 + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*ta
n(1/2*d*x)^5*tan(1/2*c)^5 + 24*a^2*b*tan(1/2*d*x)^6*tan(1/2*c)^5 - 12*b^3*t
```

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an(1/2*d*x)^6*tan(1/2*c)^5 + 6*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2
*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 +
 tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*
c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^6 - 3*
b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*ta
n(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) -
2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c
)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^6 - 6*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1
/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1
/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)
^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/
2*c)^6 + 3*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/
2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(
1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2
+ tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^6 + 24*a^2*b*tan(1/2*d*x)^5*
tan(1/2*c)^6 - 12*b^3*tan(1/2*d*x)^5*tan(1/2*c)^6 - 9*pi*a*b^2*sgn(tan(1/2*
d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*
c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 + 153*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan
(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*
tan(1/2*d*x)^4*tan(1/2*c)^4 + 12*a^3*tan(1/2*d*x)^6*tan(1/2*c)^4 - 24*a*b^2
*tan(1/2*d*x)^6*tan(1/2*c)^4 + 48*a^3*tan(1/2*d*x)^5*tan(1/2*c)^5 - 192*a*b
^2*tan(1/2*d*x)^5*tan(1/2*c)^5 - 9*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d
*x)^2*tan(1/2*c)^6 + 12*a^3*tan(1/2*d*x)^4*tan(1/2*c)^6 - 24*a*b^2*tan(1/2*
d*x)^4*tan(1/2*c)^6 + 6*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/
2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/
2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 +
tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 - 3*b^3*log
(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d
*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1
/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1
))*tan(1/2*d*x)^6*tan(1/2*c)^2 - 6*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2
 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)
^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(
1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^2
+ 3*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) -
2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x
) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1
/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 - 48*a^2*b*tan(1/2*d*x)^6*tan(1/2
*c)^3 + 8*b^3*tan(1/2*d*x)^6*tan(1/2*c)^3 - 102*a^2*b*log(2*(tan(1/2*d*x)^2
*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 +
 tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/
2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*
tan(1/2*c)^4 + 51*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2
*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 -
```

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 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*
d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 + 102*a^2*b*log(2*(
tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*
tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c
) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*t
an(1/2*d*x)^4*tan(1/2*c)^4 - 51*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*
tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c
)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 - 264
*a^2*b*tan(1/2*d*x)^5*tan(1/2*c)^4 + 84*b^3*tan(1/2*d*x)^5*tan(1/2*c)^4 - 2
64*a^2*b*tan(1/2*d*x)^4*tan(1/2*c)^5 + 84*b^3*tan(1/2*d*x)^4*tan(1/2*c)^5 +
 6*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) +
 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*
x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(
1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 - 3*b^3*log(2*(tan(1/2*d*x)^2*ta
n(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + ta
n(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d
*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan
(1/2*c)^6 - 6*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*t
an(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2
*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*
x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 + 3*b^3*log(2*(tan(1/
2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/
2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)
/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2
*d*x)^2*tan(1/2*c)^6 - 48*a^2*b*tan(1/2*d*x)^3*tan(1/2*c)^6 + 8*b^3*tan(1/2
*d*x)^3*tan(1/2*c)^6 + 9*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2
*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6 + 72
*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)
*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^5*tan(1/2*c) + 153*pi*a*b^2*sg
n(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c)
- tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^2 - 12*a^3*tan(1/2*d*x)^6*tan
(1/2*c)^2 + 24*a*b^2*tan(1/2*d*x)^6*tan(1/2*c)^2 - 96*a^3*tan(1/2*d*x)^5*ta
n(1/2*c)^3 + 192*a*b^2*tan(1/2*d*x)^5*tan(1/2*c)^3 + 153*pi*a*b^2*sgn(tan(1
/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1
/2*c)^2 + 1)*tan(1/2*d*x)^2*tan(1/2*c)^4 - 228*a^3*tan(1/2*d*x)^4*tan(1/2*c
)^4 + 696*a*b^2*tan(1/2*d*x)^4*tan(1/2*c)^4 + 72*pi*a*b^2*sgn(tan(1/2*d*x)^
2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2
+ 1)*tan(1/2*d*x)*tan(1/2*c)^5 - 96*a^3*tan(1/2*d*x)^3*tan(1/2*c)^5 + 192*a
*b^2*tan(1/2*d*x)^3*tan(1/2*c)^5 + 9*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)
^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2
*c)^6 - 12*a^3*tan(1/2*d*x)^2*tan(1/2*c)^6 + 24*a*b^2*tan(1/2*d*x)^2*tan(1/
2*c)^6 - 6*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(
1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*ta
n(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^
```

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2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6 + 3*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2
*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2
*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2
*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6 + 6*a^2*
b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(
1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*
tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^
2 + 1))*tan(1/2*d*x)^6 - 3*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1
/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1
/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 +
 tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6 - 48*a^2*b*log(2*(tan(1
/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1
/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1
)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/
2*d*x)^5*tan(1/2*c) + 24*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2
*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2
*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + t
an(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^5*tan(1/2*c) + 48*a^2*b*log
(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d
*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1
/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1
))*tan(1/2*d*x)^5*tan(1/2*c) - 24*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 -
2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2
+ tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2
*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^5*tan(1/2*c) + 24*
a^2*b*tan(1/2*d*x)^6*tan(1/2*c) - 12*b^3*tan(1/2*d*x)^6*tan(1/2*c) - 102*a^
2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*ta
n(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) -
2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c
)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^2 + 51*b^3*log(2*(tan(1/2*d*x)^2*tan(1/
2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/
2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^
2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2
*c)^2 + 102*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan
(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*t
an(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)
^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^2 - 51*b^3*log(2*(tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2
*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/
(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*
d*x)^4*tan(1/2*c)^2 + 264*a^2*b*tan(1/2*d*x)^5*tan(1/2*c)^2 - 84*b^3*tan(1/
2*d*x)^5*tan(1/2*c)^2 + 912*a^2*b*tan(1/2*d*x)^4*tan(1/2*c)^3 - 312*b^3*tan
(1/2*d*x)^4*tan(1/2*c)^3 - 102*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2
*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 +
 tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*
```

c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^4 + 51
*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*t
an(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) -
 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*
c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^4 + 102*a^2*b*log(2*(tan(1/2*d*x)^2*ta
n(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + ta
n(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d
*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan
(1/2*c)^4 - 51*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*ta
n(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*
tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x
)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^4 + 912*a^2*b*tan(1/2*d*
x)^3*tan(1/2*c)^4 - 312*b^3*tan(1/2*d*x)^3*tan(1/2*c)^4 - 48*a^2*b*log(2*(t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*t
an(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c)
 + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*ta
n(1/2*d*x)*tan(1/2*c)^5 + 24*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan
(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan
(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2
 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)*tan(1/2*c)^5 + 48*a^2*b
log(2(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1
/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*t
an(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2
 + 1))*tan(1/2*d*x)*tan(1/2*c)^5 - 24*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^
2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x
)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan
(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)*tan(1/2*c)^5 +
 264*a^2*b*tan(1/2*d*x)^2*tan(1/2*c)^5 - 84*b^3*tan(1/2*d*x)^2*tan(1/2*c)^5
 - 6*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c)
 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*
d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + ta
n(1/2*c)^2 + 1))*tan(1/2*c)^6 + 3*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 +
2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2
+ tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2
*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^6 + 6*a^2*b*log(2*(t
an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*t
an(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c)
 + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*ta
n(1/2*c)^6 - 3*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*ta
n(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*
tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x
)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^6 + 24*a^2*b*tan(1/2*d*x)*tan(1/2*c)^6
- 12*b^3*tan(1/2*d*x)*tan(1/2*c)^6 - 9*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*
c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1
/2*d*x)^4 + 4*a^3*tan(1/2*d*x)^6 - 24*a*b^2*tan(1/2*d*x)^6 + 48*a^3*tan(1/2

*d*x)^5*tan(1/2*c) - 192*a*b^2*tan(1/2*d*x)^5*tan(1/2*c) + 153*pi*a*b^2*sgn
(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) -
tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 228*a^3*tan(1/2*d*x)^4*tan
(1/2*c)^2 - 696*a*b^2*tan(1/2*d*x)^4*tan(1/2*c)^2 + 448*a^3*tan(1/2*d*x)^3*
tan(1/2*c)^3 - 1536*a*b^2*tan(1/2*d*x)^3*tan(1/2*c)^3 - 9*pi*a*b^2*sgn(tan(
1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(
1/2*c)^2 + 1)*tan(1/2*c)^4 + 228*a^3*tan(1/2*d*x)^2*tan(1/2*c)^4 - 696*a*b^
2*tan(1/2*d*x)^2*tan(1/2*c)^4 + 48*a^3*tan(1/2*d*x)*tan(1/2*c)^5 - 192*a*b^
2*tan(1/2*d*x)*tan(1/2*c)^5 + 4*a^3*tan(1/2*c)^6 - 24*a*b^2*tan(1/2*c)^6 +
6*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) +
2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x
) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1
/2*c)^2 + 1))*tan(1/2*d*x)^4 - 3*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2
*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 +
tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*
c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4 - 6*a^2*b*log(2*(
tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*
tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c
) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*t
an(1/2*d*x)^4 + 3*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2
*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 +
2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*
d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4 - 24*a^2*b*tan(1/2*d*x)^5 + 12*b
^3*tan(1/2*d*x)^5 - 264*a^2*b*tan(1/2*d*x)^4*tan(1/2*c) + 84*b^3*tan(1/2*d*
x)^4*tan(1/2*c) - 102*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*
c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + ta
n(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 51*b^3*log(
2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*
x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/
2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1)
)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 102*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^
2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x
)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan
(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2
- 51*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c)
- 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d
*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan
(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 912*a^2*b*tan(1/2*d*x)^3*tan(
1/2*c)^2 + 312*b^3*tan(1/2*d*x)^3*tan(1/2*c)^2 - 912*a^2*b*tan(1/2*d*x)^2*t
an(1/2*c)^3 + 312*b^3*tan(1/2*d*x)^2*tan(1/2*c)^3 + 6*a^2*b*log(2*(tan(1/2*
d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*
c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(
tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c
)^4 - 3*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c

) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^4 - 6*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^4 + 3*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^4 - 264*a^2*b*tan(1/2*d*x)*tan(1/2*c)^4 + 84*b^3*tan(1/2*d*x)*tan(1/2*c)^4 - 24*a^2*b*tan(1/2*c)^5 + 12*b^3*tan(1/2*c)^5 - 9*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2 - 12*a^3*tan(1/2*d*x)^4 + 24*a*b^2*tan(1/2*d*x)^4 - 72*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)*tan(1/2*c) - 96*a^3*tan(1/2*d*x)^3*tan(1/2*c) + 192*a*b^2*tan(1/2*d*x)^3*tan(1/2*c) - 9*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^2 - 228*a^3*tan(1/2*d*x)^2*tan(1/2*c)^2 + 696*a*b^2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 96*a^3*tan(1/2*d*x)*tan(1/2*c)^3 + 192*a*b^2*tan(1/2*d*x)*tan(1/2*c)^3 - 12*a^3*tan(1/2*c)^4 + 24*a*b^2*tan(1/2*c)^4 + 6*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2 - 3*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2 - 6*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2 + 3*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2 + 48*a^2*b*tan(1/2*d*x)^3 - 8*b^3*tan(1/2*d*x)^3 + 48*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)*tan(1/2*c) - 24*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)*tan(1/2*c) - 48*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)*tan(1/2*c) +

```
 24*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) -
2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x
) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1
/2*c)^2 + 1))*tan(1/2*d*x)*tan(1/2*c) + 264*a^2*b*tan(1/2*d*x)^2*tan(1/2*c)
 - 84*b^3*tan(1/2*d*x)^2*tan(1/2*c) + 6*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2
*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2
*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2
*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^2 - 3*b^3*lo
g(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*
d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(
1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 +
1))*tan(1/2*c)^2 - 6*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d
*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c
)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan
(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^2 + 3*b^3*log(2*(tan(1/2*d*x)^2
*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 +
 tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/
2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^2 +
264*a^2*b*tan(1/2*d*x)*tan(1/2*c)^2 - 84*b^3*tan(1/2*d*x)*tan(1/2*c)^2 + 48
*a^2*b*tan(1/2*c)^3 - 8*b^3*tan(1/2*c)^3 + 9*pi*a*b^2*sgn(tan(1/2*d*x)^2*ta
n(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)
 + 12*a^3*tan(1/2*d*x)^2 - 24*a*b^2*tan(1/2*d*x)^2 + 48*a^3*tan(1/2*d*x)*ta
n(1/2*c) - 192*a*b^2*tan(1/2*d*x)*tan(1/2*c) + 12*a^3*tan(1/2*c)^2 - 24*a*b
^2*tan(1/2*c)^2 - 6*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*
x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)
^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(
1/2*d*x)^2 + tan(1/2*c)^2 + 1)) + 3*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2
+ 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^
2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1
/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1)) + 6*a^2*b*log(2*(tan(1/2*d*x)
^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2
 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(
1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1)) - 3*b^3*log(2
*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x
)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2
*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))
 - 24*a^2*b*tan(1/2*d*x) + 12*b^3*tan(1/2*d*x) - 24*a^2*b*tan(1/2*c) + 12*b
^3*tan(1/2*c) - 4*a^3 + 24*a*b^2)/(d*tan(1/2*d*x)^6*tan(1/2*c)^6 - d*tan(1/
2*d*x)^6*tan(1/2*c)^4 - 8*d*tan(1/2*d*x)^5*tan(1/2*c)^5 - d*tan(1/2*d*x)^4*
tan(1/2*c)^6 - d*tan(1/2*d*x)^6*tan(1/2*c)^2 + 17*d*tan(1/2*d*x)^4*tan(1/2*
c)^4 - d*tan(1/2*d*x)^2*tan(1/2*c)^6 + d*tan(1/2*d*x)^6 + 8*d*tan(1/2*d*x)^
5*tan(1/2*c) + 17*d*tan(1/2*d*x)^4*tan(1/2*c)^2 + 17*d*tan(1/2*d*x)^2*tan(1
/2*c)^4 + 8*d*tan(1/2*d*x)*tan(1/2*c)^5 + d*tan(1/2*c)^6 - d*tan(1/2*d*x)^4
 + 17*d*tan(1/2*d*x)^2*tan(1/2*c)^2 - d*tan(1/2*c)^4 - d*tan(1/2*d*x)^2 - 8
*d*tan(1/2*d*x)*tan(1/2*c) - d*tan(1/2*c)^2 + d)
```

Mupad [B] (verification not implemented)

Time = 6.43 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.45

$$\int \sin(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$=\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,(6\,a^2\,b-3\,b^3)+2\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-12\,a\,b^2+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(12\,a\,b^2-4\,a^3)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{d\,\left(-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-\right.}$$
$$+\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,(6\,a^2\,b-3\,b^3)}{d}$$

```
[In] int(sin(c + d*x)*(a + b*tan(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(6*a^2*b - 3*b^3) + 2*a^3*tan(c/2 + (d*x)/2)^4 - 12*a*b
^2 + tan(c/2 + (d*x)/2)^2*(12*a*b^2 - 4*a^3) + tan(c/2 + (d*x)/2)^5*(6*a^2*
b - 3*b^3) - tan(c/2 + (d*x)/2)^3*(12*a^2*b - 2*b^3) + 2*a^3)/(d*(tan(c/2 +
 (d*x)/2)^2 + tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^6 - 1)) + (atanh(ta
n(c/2 + (d*x)/2))*(6*a^2*b - 3*b^3))/d
```

3.35 $\int \csc(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 19, antiderivative size = 86

$$\int \csc(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{a^3\text{arctanh}(\cos(c+dx))}{d} + \frac{3a^2b\text{arctanh}(\sin(c+dx))}{d} - \frac{b^3\text{arctanh}(\sin(c+dx))}{2d} + \frac{3ab^2\sec(c+dx)}{d} + \frac{b^3\sec(c+dx)\tan(c+dx)}{2d}$$

```
[Out] -a^3*arctanh(cos(d*x+c))/d+3*a^2*b*arctanh(sin(d*x+c))/d-1/2*b^3*arctanh(si
n(d*x+c))/d+3*a*b^2*sec(d*x+c)/d+1/2*b^3*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3598, 3855, 2686, 8, 2691}

$$\int \csc(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{a^3\text{arctanh}(\cos(c+dx))}{d} + \frac{3a^2b\text{arctanh}(\sin(c+dx))}{d} + \frac{3ab^2\sec(c+dx)}{d} - \frac{b^3\text{arctanh}(\sin(c+dx))}{2d} + \frac{b^3\tan(c+dx)\sec(c+dx)}{2d}$$

```
[In] Int[Csc[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -((a^3*ArcTanh[Cos[c + d*x]])/d) + (3*a^2*b*ArcTanh[Sin[c + d*x]])/d - (b^3
*ArcTanh[Sin[c + d*x]])/(2*d) + (3*a*b^2*Sec[c + d*x])/d + (b^3*Sec[c + d*x
]*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^3\csc(c+dx)+3a^2b\sec(c+dx)+3ab^2\sec(c+dx)\tan(c+dx)\right. \\
&\qquad \left.+b^3\sec(c+dx)\tan^2(c+dx)\right)\,dx \\
&= a^3\int\csc(c+dx)\,dx+\left(3a^2b\right)\int\sec(c+dx)\,dx \\
&\quad +\left(3ab^2\right)\int\sec(c+dx)\tan(c+dx)\,dx+b^3\int\sec(c+dx)\tan^2(c+dx)\,dx \\
&= -\frac{a^3\text{arctanh}(\cos(c+dx))}{d}+\frac{3a^2b\text{arctanh}(\sin(c+dx))}{d}+\frac{b^3\sec(c+dx)\tan(c+dx)}{2d} \\
&\quad -\frac{1}{2}b^3\int\sec(c+dx)\,dx+\frac{\left(3ab^2\right)\text{Subst}\left(\int 1\,dx,x,\sec(c+dx)\right)}{d}
\end{aligned}$$

$$= -\frac{a^3\operatorname{arctanh}(\cos(c+dx))}{d} + \frac{3a^2b\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{b^3\operatorname{arctanh}(\sin(c+dx))}{2d} + \frac{3ab^2\sec(c+dx)}{d} + \frac{b^3\sec(c+dx)\tan(c+dx)}{2d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 241 vs. $2(86) = 172$.

Time = 3.87 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.80

$$\int \csc(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$= \frac{12ab^2 - 4a^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - 12a^2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) + 2b^3\log\left(\cos\left(\frac{1}{2}(c+dx\right.\right.}{}$$

```
[In] Integrate[Csc[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (12*a*b^2 - 4*a^3*Log[Cos[(c + d*x)/2]] - 12*a^2*b*Log[Cos[(c + d*x)/2] - S
in[(c + d*x)/2]] + 2*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4*a^3*L
og[Sin[(c + d*x)/2]] + 12*a^2*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] -
2*b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + b^3/(Cos[(c + d*x)/2] - Si
n[(c + d*x)/2])^2 + 24*a*b^2*Sec[c + d*x]*Sin[(c + d*x)/2]^2 - b^3/(Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2])^2)/(4*d)
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.24

method	result
derivativedivides	$\frac{b^3\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{3a\,b^2}{\cos(dx+c)}+3a^2b\ln(\sec(dx+c)+\tan(dx+c))+a^3\ln(\csc(dx+c)-\mathrm{co}}{d}$
default	$\frac{b^3\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{3a\,b^2}{\cos(dx+c)}+3a^2b\ln(\sec(dx+c)+\tan(dx+c))+a^3\ln(\csc(dx+c)-\mathrm{co}}{d}$
risch	$-\frac{ib^2\mathrm{e}^{i(dx+c)}\left(6ia\,\mathrm{e}^{2i(dx+c)}+b\,\mathrm{e}^{2i(dx+c)}+6ia-b\right)}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}-\frac{3b\ln\left(\mathrm{e}^{i(dx+c)}-i\right)a^2}{d}+\frac{b^3\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2d}+\frac{3b\ln\left(\mathrm{e}^{i(dx+c)}+i\right)a^2}{d}$

```
[In] int(csc(d*x+c)*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^3*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+ta
n(d*x+c)))+3*a*b^2/cos(d*x+c)+3*a^2*b*ln(sec(d*x+c)+tan(d*x+c))+a^3*ln(csc(
d*x+c)-cot(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.72

$$\int \csc(c+dx)(a+b\tan(c+dx))^3\,dx =$$

$$-\frac{2\,a^3\cos\left(dx+c\right)^2\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-2\,a^3\cos\left(dx+c\right)^2\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-12\,ab^2\cos\left(d\right.}{}$$

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^3*cos(d*x + c)^2*log(1/2*cos(d*x + c) + 1/2) - 2*a^3*cos(d*x + c)
^2*log(-1/2*cos(d*x + c) + 1/2) - 12*a*b^2*cos(d*x + c) - (6*a^2*b - b^3)*c
os(d*x + c)^2*log(sin(d*x + c) + 1) + (6*a^2*b - b^3)*cos(d*x + c)^2*log(-s
in(d*x + c) + 1) - 2*b^3*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \csc(c+dx)(a+b\tan(c+dx))^3\,dx = \int \left(a+b\tan\left(c+dx\right)\right)^3\csc\left(c+dx\right)dx$$

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*csc(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.29

$$\int \csc(c+dx)(a+b\tan(c+dx))^3\,dx =$$

$$-\frac{b^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)-6\,a^2b(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\right.}{4\,d}$$

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - lo
g(sin(d*x + c) - 1)) - 6*a^2*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) -
1)) + 4*a^3*log(cot(d*x + c) + csc(d*x + c)) - 12*a*b^2/cos(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.68 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.67

$$\int \csc(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$=\frac{2\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+\left(6\,a^2b-b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(6\,a^2b-b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.\right.}{2\,d}$$

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + (6*a^2*b - b^3)*log(abs(tan(1/2
*d*x + 1/2*c) + 1)) - (6*a^2*b - b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) +
2*(b^3*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^2*tan(1/2*d*x + 1/2*c)^2 + b^3*tan(1/
2*d*x + 1/2*c) + 6*a*b^2)/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 5.64 (sec) , antiderivative size = 278, normalized size of antiderivative = 3.23

$$\int \csc(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$=\frac{2\left(\frac{a^3\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2}-\frac{b^3\,\mathrm{atan}\left(\frac{2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^3-6\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^3}{2\mathrm{i}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^3-6\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b+1\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^3}\right)\,1\mathrm{i}}{2}+a^2\,b\,\mathrm{atan}\left(\frac{2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^3-6\,\cos\left(\frac{c}{2}\right.}{2\mathrm{i}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^3-6\mathrm{i}\,\sin\left(\frac{c}{2}+\right.}\right.\right.}{d}$$

$$+\frac{\frac{\sin(c+d\,x)\,b^3}{2}+3\,a\,\cos\left(c+d\,x\right)\,b^2}{d\,\left(\frac{\cos(2\,c+2\,d\,x)}{2}+\frac{1}{2}\right)}$$

```
[In] int((a + b*tan(c + d*x))^3/sin(c + d*x),x)
```

```
[Out] (2*((a^3*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 - (b^3*atan((b^3*cos
(c/2 + (d*x)/2) + 2*a^3*sin(c/2 + (d*x)/2) - 6*a^2*b*cos(c/2 + (d*x)/2))/(a
^3*cos(c/2 + (d*x)/2)*2i + b^3*sin(c/2 + (d*x)/2)*1i - a^2*b*sin(c/2 + (d*x
)/2)*6i))*1i)/2 + a^2*b*atan((b^3*cos(c/2 + (d*x)/2) + 2*a^3*sin(c/2 + (d*x
)/2) - 6*a^2*b*cos(c/2 + (d*x)/2))/(a^3*cos(c/2 + (d*x)/2)*2i + b^3*sin(c/2
 + (d*x)/2)*1i - a^2*b*sin(c/2 + (d*x)/2)*6i))*3i))/d + ((b^3*sin(c + d*x))
/2 + 3*a*b^2*cos(c + d*x))/(d*(cos(2*c + 2*d*x)/2 + 1/2))
```

3.36 $\int \csc^2(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 64

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{a^3\cot(c+dx)}{d}+\frac{3a^2b\log(\tan(c+dx))}{d}+\frac{3ab^2\tan(c+dx)}{d}+\frac{b^3\tan^2(c+dx)}{2d}$$

```
[Out] -a^3*cot(d*x+c)/d+3*a^2*b*ln(tan(d*x+c))/d+3*a*b^2*tan(d*x+c)/d+1/2*b^3*tan
(d*x+c)^2/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3597, 45}

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{a^3\cot(c+dx)}{d}+\frac{3a^2b\log(\tan(c+dx))}{d}+\frac{3ab^2\tan(c+dx)}{d}+\frac{b^3\tan^2(c+dx)}{2d}$$

```
[In] Int[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -((a^3*Cot[c + d*x])/d) + (3*a^2*b*Log[Tan[c + d*x]])/d + (3*a*b^2*Tan[c +
d*x])/d + (b^3*Tan[c + d*x]^2)/(2*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b\text{Subst}\left(\int \frac{(a+x)^3}{x^2}\,dx, x, b\tan(c+dx)\right)}{d} \\
&= \frac{b\text{Subst}\left(\int \left(3a+\frac{a^3}{x^2}+\frac{3a^2}{x}+x\right)\,dx, x, b\tan(c+dx)\right)}{d} \\
&= -\frac{a^3\cot(c+dx)}{d}+\frac{3a^2b\log(\tan(c+dx))}{d}+\frac{3ab^2\tan(c+dx)}{d}+\frac{b^3\tan^2(c+dx)}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.52 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.97

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{\csc(c+dx)\sec^2(c+dx)\left(3a(a^2-b^2)\cos(c+dx)+(a^3+3ab^2)\cos(3(c+dx))-2b(b^2-3a^2\log(\cos(c+\right.}{4d}$$

```
[In] Integrate[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
[Out] -1/4*(Csc[c + d*x]*Sec[c + d*x]^2*(3*a*(a^2 - b^2)*Cos[c + d*x] + (a^3 + 3*
a*b^2)*Cos[3*(c + d*x)] - 2*b*(b^2 - 3*a^2*Log[Cos[c + d*x]] - 3*a^2*Cos[2*
(c + d*x)]*(Log[Cos[c + d*x]] - Log[Sin[c + d*x]]) + 3*a^2*Log[Sin[c + d*x]
])*Sin[c + d*x]))/d
```

Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{\frac{b^{3}}{2\cos(dx+c)^{2}}+3a\,b^{2}\tan(dx+c)+3a^{2}b\ln(\tan(dx+c))-a^{3}\cot(dx+c)}{d}$
default	$\frac{\frac{b^{3}}{2\cos(dx+c)^{2}}+3a\,b^{2}\tan(dx+c)+3a^{2}b\ln(\tan(dx+c))-a^{3}\cot(dx+c)}{d}$
risch	$\frac{-2ia^{3}e^{4i(dx+c)}+6ia\,b^{2}e^{4i(dx+c)}+2b^{3}e^{4i(dx+c)}-4ia^{3}e^{2i(dx+c)}-2b^{3}e^{2i(dx+c)}-2ia^{3}-6ia\,b^{2}}{d(e^{2i(dx+c)}+1)^{2}(e^{2i(dx+c)}-1)}+\frac{3a^{2}b\ln(e^{2i(dx+c)}-1)}{d}$ -

[In] int(csc(d*x+c)^2*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(1/2*b^3/cos(d*x+c)^2+3*a*b^2*tan(d*x+c)+3*a^2*b*ln(tan(d*x+c))-a^3*cot(d*x+c))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(62) = 124$.

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.98

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{3\,a^2b\cos(dx+c)^2\log\left(\cos(dx+c)^2\right)\sin(dx+c)-3\,a^2b\cos(dx+c)^2\log\left(-\frac{1}{4}\cos(dx+c)^2+\frac{1}{4}\right)\sin(c}{2\,d\cos(dx+c)^2\sin(dx+c)}$$

[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="fricas")

[Out] -1/2*(3*a^2*b*cos(d*x + c)^2*log(cos(d*x + c)^2)*sin(d*x + c) - 3*a^2*b*cos(d*x + c)^2*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) - 6*a*b^2*cos(d*x + c) + 2*(a^3 + 3*a*b^2)*cos(d*x + c)^3 - b^3*sin(d*x + c))/(d*cos(d*x + c)^2*sin(d*x + c))

Sympy [F]

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^3\,dx = \int (a+b\tan(c+dx))^3\csc^2(c+dx)\,dx$$

[In] integrate(csc(d*x+c)**2*(a+b*tan(d*x+c))**3,x)

[Out] Integral((a + b*tan(c + d*x))**3*csc(c + d*x)**2, x)

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{b^3\tan(dx+c)^2+6\,a^2b\log(\tan(dx+c))+6\,ab^2\tan(dx+c)-\frac{2\,a^3}{\tan(dx+c)}}{2\,d}$$

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b^3*tan(d*x + c)^2 + 6*a^2*b*log(tan(d*x + c)) + 6*a*b^2*tan(d*x + c)
- 2*a^3/tan(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.71 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{b^3\tan(dx+c)^2+6\,a^2b\log(|\tan(dx+c)|)+6\,ab^2\tan(dx+c)-\frac{2\,(3\,a^2b\tan(dx+c)+a^3)}{\tan(dx+c)}}{2\,d}$$

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*tan(d*x + c)^2 + 6*a^2*b*log(abs(tan(d*x + c))) + 6*a*b^2*tan(d*x
+ c) - 2*(3*a^2*b*tan(d*x + c) + a^3)/tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 4.17 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{b^3\tan(c+d\,x)^2}{2\,d}-\frac{a^3\cot(c+d\,x)}{d}+\frac{3\,a^2\,b\,\ln(\tan(c+d\,x))}{d}+\frac{3\,a\,b^2\tan(c+d\,x)}{d}$$

```
[In] int((a + b*tan(c + d*x))^3/sin(c + d*x)^2,x)
```

```
[Out] (b^3*tan(c + d*x)^2)/(2*d) - (a^3*cot(c + d*x))/d + (3*a^2*b*log(tan(c + d*
x)))/d + (3*a*b^2*tan(c + d*x))/d
```

3.37 $\int \csc^3(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 141

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{a^3\text{arctanh}(\cos(c+dx))}{2d} - \frac{3ab^2\text{arctanh}(\cos(c+dx))}{d} + \frac{3a^2b\text{arctanh}(\sin(c+dx))}{d} + \frac{b^3\text{arctanh}(\sin(c+dx))}{2d} - \frac{3a^2b\csc(c+dx)}{d} - \frac{a^3\cot(c+dx)\csc(c+dx)}{2d} + \frac{3ab^2\sec(c+dx)}{d} + \frac{b^3\sec(c+dx)\tan(c+dx)}{2d}$$

```
[Out] -1/2*a^3*arctanh(cos(d*x+c))/d-3*a*b^2*arctanh(cos(d*x+c))/d+3*a^2*b*arctan
h(sin(d*x+c))/d+1/2*b^3*arctanh(sin(d*x+c))/d-3*a^2*b*csc(d*x+c)/d-1/2*a^3*
cot(d*x+c)*csc(d*x+c)/d+3*a*b^2*sec(d*x+c)/d+1/2*b^3*sec(d*x+c)*tan(d*x+c)/
d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used

$= \{3598, 3853, 3855, 2701, 327, 213, 2702\}$

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{a^3\text{arctanh}(\cos(c+dx))}{2d} - \frac{a^3\cot(c+dx)\csc(c+dx)}{2d} + \frac{3a^2b\text{arctanh}(\sin(c+dx))}{d} - \frac{3a^2b\csc(c+dx)}{d} - \frac{3ab^2\text{arctanh}(\cos(c+dx))}{d} + \frac{3ab^2\sec(c+dx)}{d} + \frac{b^3\text{arctanh}(\sin(c+dx))}{2d} + \frac{b^3\tan(c+dx)\sec(c+dx)}{2d}$$

```
[In] Int[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -1/2*(a^3*ArcTanh[Cos[c + d*x]])/d - (3*a*b^2*ArcTanh[Cos[c + d*x]])/d + (3
*a^2*b*ArcTanh[Sin[c + d*x]])/d + (b^3*ArcTanh[Sin[c + d*x]])/(2*d) - (3*a^
2*b*Csc[c + d*x])/d - (a^3*Cot[c + d*x]*Csc[c + d*x])/(2*d) + (3*a*b^2*Sec[
c + d*x])/d + (b^3*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
```

```
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \int \left(a^3\csc^3(c+dx)+3a^2b\csc^2(c+dx)\sec(c+dx)+3ab^2\csc(c+dx)\sec^2(c+dx)+b^3\sec^3(c+dx)\right)\,dx$$

$$= a^3\int \csc^3(c+dx)\,dx+\left(3a^2b\right)\int \csc^2(c+dx)\sec(c+dx)\,dx+\left(3ab^2\right)\int \csc(c+dx)\sec^2(c+dx)\,dx+b^3\int \sec^3(c+dx)\,dx$$

$$= -\frac{a^3\cot(c+dx)\csc(c+dx)}{2d}+\frac{b^3\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}a^3\int \csc(c+dx)\,dx+\frac{1}{2}b^3\int \sec(c+dx)\,dx-\frac{\left(3a^2b\right)\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx,x,\csc(c+dx)\right)}{d}+\frac{\left(3ab^2\right)\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx,x,\sec(c+dx)\right)}{d}$$

$$\begin{aligned}
&= -\frac{a^3\operatorname{arctanh}(\cos(c+dx))}{2d} + \frac{b^3\operatorname{arctanh}(\sin(c+dx))}{2d} \\
&\quad - \frac{3a^2b\csc(c+dx)}{d} - \frac{a^3\cot(c+dx)\csc(c+dx)}{2d} + \frac{3ab^2\sec(c+dx)}{d} \\
&\quad + \frac{b^3\sec(c+dx)\tan(c+dx)}{2d} - \frac{(3a^2b)\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d} \\
&\quad + \frac{(3ab^2)\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{d} \\
&= -\frac{a^3\operatorname{arctanh}(\cos(c+dx))}{2d} - \frac{3ab^2\operatorname{arctanh}(\cos(c+dx))}{d} \\
&\quad + \frac{3a^2b\operatorname{arctanh}(\sin(c+dx))}{d} + \frac{b^3\operatorname{arctanh}(\sin(c+dx))}{2d} - \frac{3a^2b\csc(c+dx)}{d} \\
&\quad - \frac{a^3\cot(c+dx)\csc(c+dx)}{2d} + \frac{3ab^2\sec(c+dx)}{d} + \frac{b^3\sec(c+dx)\tan(c+dx)}{2d}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 897 vs. $2(141) = 282$.

Time = 7.38 (sec) , antiderivative size = 897, normalized size of antiderivative = 6.36

$$
\begin{aligned}
&\int \csc^3(c+dx)(a+b\tan(c+dx))^3\,dx \\
&= \frac{3ab^2\cos^3(c+dx)(a+b\tan(c+dx))^3}{d(a\cos(c+dx)+b\sin(c+dx))^3} \\
&\quad - \frac{3a^2b\cos^3(c+dx)\cot\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{2d(a\cos(c+dx)+b\sin(c+dx))^3} \\
&\quad - \frac{a^3\cos^3(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{8d(a\cos(c+dx)+b\sin(c+dx))^3} \\
&\quad + \frac{(-a^3-6ab^2)\cos^3(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^3}{2d(a\cos(c+dx)+b\sin(c+dx))^3} \\
&\quad + \frac{(-6a^2b-b^3)\cos^3(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^3}{2d(a\cos(c+dx)+b\sin(c+dx))^3} \\
&\quad + \frac{(a^3+6ab^2)\cos^3(c+dx)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^3}{2d(a\cos(c+dx)+b\sin(c+dx))^3} \\
&\quad + \frac{(6a^2b+b^3)\cos^3(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^3}{2d(a\cos(c+dx)+b\sin(c+dx))^3} \\
&\quad + \frac{a^3\cos^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{8d(a\cos(c+dx)+b\sin(c+dx))^3} \\
&\quad + \frac{b^3\cos^3(c+dx)(a+b\tan(c+dx))^3}{4d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a\cos(c+dx)+b\sin(c+dx))^3} \\
&\quad + \frac{3ab^2\cos^3(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^3} \\
&\quad - \frac{b^3\cos^3(c+dx)(a+b\tan(c+dx))^3}{4d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a\cos(c+dx)+b\sin(c+dx))^3} \\
&\quad - \frac{3ab^2\cos^3(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^3} \\
&\quad - \frac{3a^2b\cos^3(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{2d(a\cos(c+dx)+b\sin(c+dx))^3}
\end{aligned}
$$

```
[In] Integrate[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a*b^2*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(d*(a*Cos[c + d*x] + b*Sin[
c + d*x])^3) - (3*a^2*b*Cos[c + d*x]^3*Cot[(c + d*x)/2]*(a + b*Tan[c + d*x]
)^3)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (a^3*Cos[c + d*x]^3*Csc[(c
 + d*x)/2]^2*(a + b*Tan[c + d*x])^3)/(8*d*(a*Cos[c + d*x] + b*Sin[c + d*x])
^3) + ((-a^3 - 6*a*b^2)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2]]*(a + b*Tan[c +
 d*x])^3)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + ((-6*a^2*b - b^3)*Cos
[c + d*x]^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3
```

```
)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + ((a^3 + 6*a*b^2)*Cos[c + d*x]
^3*Log[Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3)/(2*d*(a*Cos[c + d*x] + b*S
in[c + d*x])^3) + ((6*a^2*b + b^3)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d*
x])^3) + (a^3*Cos[c + d*x]^3*Sec[(c + d*x)/2]^2*(a + b*Tan[c + d*x])^3)/(8*
d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (b^3*Cos[c + d*x]^3*(a + b*Tan[c +
 d*x])^3)/(4*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*
Sin[c + d*x])^3) + (3*a*b^2*Cos[c + d*x]^3*Sin[(c + d*x)/2]*(a + b*Tan[c +
d*x])^3)/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c
 + d*x])^3) - (b^3*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(4*d*(Cos[(c + d*
x)/2] + Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (3*a*b^2
*Cos[c + d*x]^3*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^3)/(d*(Cos[(c + d*x)/
2] + Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (3*a^2*b*Cos[
c + d*x]^3*Tan[(c + d*x)/2]*(a + b*Tan[c + d*x])^3)/(2*d*(a*Cos[c + d*x] +
b*Sin[c + d*x])^3)
```

Maple [A] (verified)

Time = 4.12 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{b^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a\,b^2\left(\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)+3a^2b\left(-\frac{1}{\sin(dx+c)}+\ln(\right.}{d}$
default	$\frac{b^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a\,b^2\left(\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)+3a^2b\left(-\frac{1}{\sin(dx+c)}+\ln(\right.}{d}$
risch	$-\frac{i e^{i(dx+c)}\left(3ia^3e^{2i(dx+c)}+3ia^3e^{4i(dx+c)}+6a^2b\,e^{6i(dx+c)}+b^3e^{6i(dx+c)}+ia^3e^{6i(dx+c)}+6ia\,b^2e^{6i(dx+c)}+6a^2b\,e^{4i(dx+c)}-3\right.}{d\left(e^{2i(dx+c)}+1\right)^2\left(e^{2i(dx+\right.}}$

```
[In] int(csc(d*x+c)^3*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^3*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+3*a*b^2*
(1/cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+3*a^2*b*(-1/sin(d*x+c)+ln(sec(d*x+
c)+tan(d*x+c)))+a^3*(-1/2*csc(d*x+c)*cot(d*x+c)+1/2*ln(csc(d*x+c)-cot(d*x+c
))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(133) = 266.

Time = 0.32 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.12

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^3\,dx =$$

$$-\frac{12\,ab^2\cos(dx+c)-2\left(a^3+6\,ab^2\right)\cos(dx+c)^3+\left(\left(a^3+6\,ab^2\right)\cos(dx+c)^4-\left(a^3+6\,ab^2\right)\cos(dx+c)^2\right.}{}$$

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(12*a*b^2*cos(d*x + c) - 2*(a^3 + 6*a*b^2)*cos(d*x + c)^3 + ((a^3 + 6*
a*b^2)*cos(d*x + c)^4 - (a^3 + 6*a*b^2)*cos(d*x + c)^2)*log(1/2*cos(d*x + c
) + 1/2) - ((a^3 + 6*a*b^2)*cos(d*x + c)^4 - (a^3 + 6*a*b^2)*cos(d*x + c)^2
)*log(-1/2*cos(d*x + c) + 1/2) - ((6*a^2*b + b^3)*cos(d*x + c)^4 - (6*a^2*b
 + b^3)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + ((6*a^2*b + b^3)*cos(d*x +
c)^4 - (6*a^2*b + b^3)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) + 2*(b^3 - (6
*a^2*b + b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^4 - d*cos(d*x +
 c)^2)
```

Sympy [F]

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^3\,dx = \int (a+b\tan{(c+dx)})^3\csc^3{(c+dx)}\,dx$$

```
[In] integrate(csc(d*x+c)**3*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*csc(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.21

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{a^3\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1} - \log(\cos(dx+c)+1) + \log(\cos(dx+c)-1)\right) - b^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1)\right.}{}$$

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - log(cos(d*x + c) + 1) + log
(cos(d*x + c) - 1)) - b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*
x + c) + 1) + log(sin(d*x + c) - 1)) + 6*a*b^2*(2/cos(d*x + c) - log(cos(d*
x + c) + 1) + log(cos(d*x + c) - 1)) - 6*a^2*b*(2/sin(d*x + c) - log(sin(d*
x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(133) = 266.

Time = 0.75 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.16

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$= \frac{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 12a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) + 4(6a^2b+b^3)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right) - 4(6a^2b+b^3)\log\cdots}{\cdots}$$

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(a^3*tan(1/2*d*x + 1/2*c)^2 - 12*a^2*b*tan(1/2*d*x + 1/2*c) + 4*(6*a^2*
b + b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 4*(6*a^2*b + b^3)*log(abs(tan
(1/2*d*x + 1/2*c) - 1)) + 4*(a^3 + 6*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))
- (2*a^3*tan(1/2*d*x + 1/2*c)^6 + 12*a*b^2*tan(1/2*d*x + 1/2*c)^6 + 12*a^2*
b*tan(1/2*d*x + 1/2*c)^5 - 8*b^3*tan(1/2*d*x + 1/2*c)^5 - 3*a^3*tan(1/2*d*x
 + 1/2*c)^4 + 24*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 24*a^2*b*tan(1/2*d*x + 1/2*
c)^3 - 8*b^3*tan(1/2*d*x + 1/2*c)^3 - 36*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 12*
a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/(tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1
/2*c))^2)/d
```

Mupad [B] (verification not implemented)

Time = 5.20 (sec) , antiderivative size = 581, normalized size of antiderivative = 4.12

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8d}$$

$$-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{a^3}{2}+24ab^2\right) - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(a^3+24ab^2) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5(6a^2b-4b^3) - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)\cdots}{d\left(4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 - 8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 + 4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}$$

$$+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(\frac{a^3}{2}+3ab^2\right)}{d}$$

$$-\frac{\mathrm{atan}\left(-\frac{\left(3a^2b+\frac{b^3}{2}\right)\left(6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(3a^2b+\frac{b^3}{2}\right)+6a^2b+b^3-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a^3+6ab^2)\right)1\mathrm{i}-\left(3a^2b+\frac{b^3}{2}\right)\left(6\mathrm{t}\cdots\right.}{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(36a^4b^2+12a^2b^4+b^6)-\left(3a^2b+\frac{b^3}{2}\right)\left(6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(3a^2b+\frac{b^3}{2}\right)+6a^2b+b^3-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a^3+6ab^2)\right)-\left(3a^2b+\cdots\right.}\right)}{d}$$

$$-\frac{3a^2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2d}$$

```
[In] int((a + b*tan(c + d*x))^3/sin(c + d*x)^3,x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^2)/(8*d) - (tan(c/2 + (d*x)/2)^4*(24*a*b^2 + a^3/2)
 - tan(c/2 + (d*x)/2)^2*(24*a*b^2 + a^3) + tan(c/2 + (d*x)/2)^5*(6*a^2*b -
```

```
4*b^3) - tan(c/2 + (d*x)/2)^3*(12*a^2*b + 4*b^3) + a^3/2 + 6*a^2*b*tan(c/2
+ (d*x)/2))/(d*(4*tan(c/2 + (d*x)/2)^2 - 8*tan(c/2 + (d*x)/2)^4 + 4*tan(c/2
 + (d*x)/2)^6)) + (log(tan(c/2 + (d*x)/2))*(3*a*b^2 + a^3/2))/d - (atan(-((
3*a^2*b + b^3/2)*(6*tan(c/2 + (d*x)/2)*(3*a^2*b + b^3/2) + 6*a^2*b + b^3 -
tan(c/2 + (d*x)/2)*(6*a*b^2 + a^3))*1i - (3*a^2*b + b^3/2)*(6*tan(c/2 + (d*
x)/2)*(3*a^2*b + b^3/2) - 6*a^2*b - b^3 + tan(c/2 + (d*x)/2)*(6*a*b^2 + a^3
))*1i)/(2*tan(c/2 + (d*x)/2)*(b^6 + 12*a^2*b^4 + 36*a^4*b^2) - (3*a^2*b + b
^3/2)*(6*tan(c/2 + (d*x)/2)*(3*a^2*b + b^3/2) + 6*a^2*b + b^3 - tan(c/2 + (
d*x)/2)*(6*a*b^2 + a^3)) - (3*a^2*b + b^3/2)*(6*tan(c/2 + (d*x)/2)*(3*a^2*b
 + b^3/2) - 6*a^2*b - b^3 + tan(c/2 + (d*x)/2)*(6*a*b^2 + a^3)) + 6*a*b^5 +
 6*a^5*b + 37*a^3*b^3))*(a^2*b*6i + b^3*1i))/d - (3*a^2*b*tan(c/2 + (d*x)/2
))/(2*d)
```

3.38 $\int \csc^4(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 113

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^3\,dx=-\frac{a(a^2+3b^2)\cot(c+dx)}{d}-\frac{3a^2b\cot^2(c+dx)}{2d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{b(3a^2+b^2)\log(\tan(c+dx))}{d}+\frac{3ab^2\tan(c+dx)}{d}+\frac{b^3\tan^2(c+dx)}{2d}$$

```
[Out] -a*(a^2+3*b^2)*cot(d*x+c)/d-3/2*a^2*b*cot(d*x+c)^2/d-1/3*a^3*cot(d*x+c)^3/d
+b*(3*a^2+b^2)*ln(tan(d*x+c))/d+3*a*b^2*tan(d*x+c)/d+1/2*b^3*tan(d*x+c)^2/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3597, 908}

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^3\,dx=-\frac{a^3\cot^3(c+dx)}{3d}-\frac{a(a^2+3b^2)\cot(c+dx)}{d}+\frac{b(3a^2+b^2)\log(\tan(c+dx))}{d}-\frac{3a^2b\cot^2(c+dx)}{2d}+\frac{3ab^2\tan(c+dx)}{d}+\frac{b^3\tan^2(c+dx)}{2d}$$

```
[In] Int[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -((a*(a^2 + 3*b^2)*Cot[c + d*x])/d) - (3*a^2*b*Cot[c + d*x]^2)/(2*d) - (a^3
*Cot[c + d*x]^3)/(3*d) + (b*(3*a^2 + b^2)*Log[Tan[c + d*x]])/d + (3*a*b^2*T
an[c + d*x])/d + (b^3*Tan[c + d*x]^2)/(2*d)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b\text{Subst}\left(\int \frac{(a+x)^3(b^2+x^2)}{x^4}\,dx, x, b\tan(c+dx)\right)}{d}\\
&= \frac{b\text{Subst}\left(\int \left(3a+\frac{a^3b^2}{x^4}+\frac{3a^2b^2}{x^3}+\frac{a^3+3ab^2}{x^2}+\frac{3a^2+b^2}{x}+x\right)\,dx, x, b\tan(c+dx)\right)}{d}\\
&= -\frac{a(a^2+3b^2)\cot(c+dx)}{d}-\frac{3a^2b\cot^2(c+dx)}{2d}-\frac{a^3\cot^3(c+dx)}{3d}\\
&\quad+\frac{b(3a^2+b^2)\log(\tan(c+dx))}{d}+\frac{3ab^2\tan(c+dx)}{d}+\frac{b^3\tan^2(c+dx)}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 4.19 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.88

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{(b+a\cot(c+dx))^3\sec^2(c+dx)\left(-16a^3\cos(c+dx)-2\sin(c+dx)\left(18a^2b-6b^3+6(3a^2b+b^3)\cos(2(c-\right.\right.}{}$$

```
[In] Integrate[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] ((b + a*Cot[c + d*x])^3*Sec[c + d*x]^2*(-16*a^3*Cos[c + d*x] - 2*Sin[c + d*
x]*(18*a^2*b - 6*b^3 + 6*(3*a^2*b + b^3)*Cos[2*(c + d*x)] + 9*a^2*b*Log[Cos
[c + d*x]] + 3*b^3*Log[Cos[c + d*x]] - 3*b*(3*a^2 + b^2)*Cos[4*(c + d*x)]*(
Log[Cos[c + d*x]] - Log[Sin[c + d*x]]) - 9*a^2*b*Log[Sin[c + d*x]] - 3*b^3*
Log[Sin[c + d*x]] + 2*a^3*Sin[4*(c + d*x)] + 18*a*b^2*Sin[4*(c + d*x)])))/(
48*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)
```

Maple [A] (verified)

Time = 6.97 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{b^3\left(\frac{1}{2\cos(dx+c)^2}+\ln(\tan(dx+c))\right)+3a\,b^2\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+3a^2b\left(-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a^3\left(\right.}{d}$
default	$\frac{b^3\left(\frac{1}{2\cos(dx+c)^2}+\ln(\tan(dx+c))\right)+3a\,b^2\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+3a^2b\left(-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a^3\left(\right.}{d}$
risch	$\frac{6a^2b\,e^{8i(dx+c)}+2b^3e^{8i(dx+c)}+\frac{4ia^3e^{2i(dx+c)}}{3}+12ia\,b^2e^{2i(dx+c)}+6a^2b\,e^{6i(dx+c)}-6b^3e^{6i(dx+c)}-12ia\,b^2e^{6i(dx+c)}+\frac{20ia^3e^{4i}}{3}}{d\left(e^{2i(dx+c)}+1\right)^2\left(e\right.}$

```
[In] int(csc(d*x+c)^4*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^3*(1/2/cos(d*x+c)^2+ln(tan(d*x+c)))+3*a*b^2*(1/sin(d*x+c)/cos(d*x+c)
-2*cot(d*x+c))+3*a^2*b*(-1/2/sin(d*x+c)^2+ln(tan(d*x+c)))+a^3*(-2/3-1/3*csc
(d*x+c)^2)*cot(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(107) = 214.

Time = 0.28 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.10

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{4\left(a^3+9\,ab^2\right)\cos\left(dx+c\right)^5+18\,ab^2\cos\left(dx+c\right)-6\left(a^3+9\,ab^2\right)\cos\left(dx+c\right)^3+3\left(\left(3\,a^2b+b^3\right)\cos\left(dx+\right.\right.}{}$$

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/6*(4*(a^3 + 9*a*b^2)*cos(d*x + c)^5 + 18*a*b^2*cos(d*x + c) - 6*(a^3 + 9
*a*b^2)*cos(d*x + c)^3 + 3*((3*a^2*b + b^3)*cos(d*x + c)^4 - (3*a^2*b + b^3
)*cos(d*x + c)^2)*log(cos(d*x + c)^2)*sin(d*x + c) - 3*((3*a^2*b + b^3)*cos
(d*x + c)^4 - (3*a^2*b + b^3)*cos(d*x + c)^2)*log(-1/4*cos(d*x + c)^2 + 1/4
)*sin(d*x + c) + 3*(b^3 - (3*a^2*b + b^3)*cos(d*x + c)^2)*sin(d*x + c))/((d
*cos(d*x + c)^4 - d*cos(d*x + c)^2)*sin(d*x + c))
```

Sympy [F]

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^3\,dx = \int (a+b\tan{(c+dx)})^3\csc^4{(c+dx)}\,dx$$

```
[In] integrate(csc(d*x+c)**4*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*csc(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.87

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{3\,b^3\tan{(dx+c)}^2 + 18\,ab^2\tan{(dx+c)} + 6\,(3\,a^2b+b^3)\log{(\tan{(dx+c)})} - \frac{9\,a^2b\tan(dx+c)+2\,a^3+6\,(a^3+3\,ab^2)\tan(\cdots}{\tan(dx+c)^3}}{6\,d}$$

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/6*(3*b^3*tan(d*x + c)^2 + 18*a*b^2*tan(d*x + c) + 6*(3*a^2*b + b^3)*log(t
an(d*x + c)) - (9*a^2*b*tan(d*x + c) + 2*a^3 + 6*(a^3 + 3*a*b^2)*tan(d*x +
c)^2)/tan(d*x + c)^3)/d
```

Giac [A] (verification not implemented)

none

Time = 0.70 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.18

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{3\,b^3\tan{(dx+c)}^2 + 18\,ab^2\tan{(dx+c)} + 6\,(3\,a^2b+b^3)\log{(|\tan{(dx+c)}|)} - \frac{33\,a^2b\tan(dx+c)^3+11\,b^3\tan(dx+c)^3+\cdots}{\cdots}}{6\,d}$$

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(3*b^3*tan(d*x + c)^2 + 18*a*b^2*tan(d*x + c) + 6*(3*a^2*b + b^3)*log(a
bs(tan(d*x + c))) - (33*a^2*b*tan(d*x + c)^3 + 11*b^3*tan(d*x + c)^3 + 6*a^
3*tan(d*x + c)^2 + 18*a*b^2*tan(d*x + c)^2 + 9*a^2*b*tan(d*x + c) + 2*a^3)/
tan(d*x + c)^3)/d
```

Mupad [B] (verification not implemented)

Time = 4.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.91

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$=\frac{\ln\left(\tan(c+d\,x)\right)\,\left(3\,a^2\,b+b^3\right)}{d}-\frac{\cot(c+d\,x)^3\,\left(\frac{a^3}{3}+\tan(c+d\,x)^2\,\left(a^3+3\,a\,b^2\right)+\frac{3\,a^2\,b\tan(c+d\,x)}{2}\right)}{d}+\frac{b^3\tan(c+d\,x)^2}{2\,d}+\frac{3\,a\,b^2\tan(c+d\,x)}{d}$$

```
[In] int((a + b*tan(c + d*x))^3/sin(c + d*x)^4,x)
```

```
[Out] (log(tan(c + d*x))*(3*a^2*b + b^3))/d - (cot(c + d*x)^3*(a^3/3 + tan(c + d*
x)^2*(3*a*b^2 + a^3) + (3*a^2*b*tan(c + d*x))/2))/d + (b^3*tan(c + d*x)^2)/
(2*d) + (3*a*b^2*tan(c + d*x))/d
```

3.39 $\int \csc^5(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 229

$$\begin{aligned}\int \csc^5(c+dx)(a+b\tan(c+dx))^3\,dx = &-\frac{3a^3\text{arctanh}(\cos(c+dx))}{8d}\\ &-\frac{9ab^2\text{arctanh}(\cos(c+dx))}{2d}\\ &+\frac{3a^2b\text{arctanh}(\sin(c+dx))}{d}\\ &+\frac{3b^3\text{arctanh}(\sin(c+dx))}{2d}-\frac{3a^2b\csc(c+dx)}{d}\\ &-\frac{3b^3\csc(c+dx)}{2d}-\frac{3a^3\cot(c+dx)\csc(c+dx)}{8d}\\ &-\frac{a^2b\csc^3(c+dx)}{d}-\frac{a^3\cot(c+dx)\csc^3(c+dx)}{4d}\\ &+\frac{9ab^2\sec(c+dx)}{2d}-\frac{3ab^2\csc^2(c+dx)\sec(c+dx)}{2d}\\ &+\frac{b^3\csc(c+dx)\sec^2(c+dx)}{2d}\end{aligned}$$

```
[Out] -3/8*a^3*arctanh(cos(d*x+c))/d-9/2*a*b^2*arctanh(cos(d*x+c))/d+3*a^2*b*arct
anh(sin(d*x+c))/d+3/2*b^3*arctanh(sin(d*x+c))/d-3*a^2*b*csc(d*x+c)/d-3/2*b^
3*csc(d*x+c)/d-3/8*a^3*cot(d*x+c)*csc(d*x+c)/d-a^2*b*csc(d*x+c)^3/d-1/4*a^3
*cot(d*x+c)*csc(d*x+c)^3/d+9/2*a*b^2*sec(d*x+c)/d-3/2*a*b^2*csc(d*x+c)^2*se
c(d*x+c)/d+1/2*b^3*csc(d*x+c)*sec(d*x+c)^2/d
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3598, 3853, 3855, 2701, 308, 213, 2702, 294, 327}

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{3a^3\text{arctanh}(\cos(c+dx))}{8d} - \frac{a^3\cot(c+dx)\csc^3(c+dx)}{4d} - \frac{3a^3\cot(c+dx)\csc(c+dx)}{8d} + \frac{3a^2b\text{arctanh}(\sin(c+dx))}{d} - \frac{a^2b\csc^3(c+dx)}{d} - \frac{3a^2b\csc(c+dx)}{d} - \frac{9ab^2\text{arctanh}(\cos(c+dx))}{2d} + \frac{9ab^2\sec(c+dx)}{2d} - \frac{3ab^2\csc^2(c+dx)\sec(c+dx)}{2d} + \frac{3b^3\text{arctanh}(\sin(c+dx))}{2d} - \frac{3b^3\csc(c+dx)}{2d} + \frac{b^3\csc(c+dx)\sec^2(c+dx)}{2d}$$

```
[In] Int[Csc[c + d*x]^5*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-3*a^3*ArcTanh[Cos[c + d*x]])/(8*d) - (9*a*b^2*ArcTanh[Cos[c + d*x]])/(2*d
) + (3*a^2*b*ArcTanh[Sin[c + d*x]])/d + (3*b^3*ArcTanh[Sin[c + d*x]])/(2*d)
 - (3*a^2*b*Csc[c + d*x])/d - (3*b^3*Csc[c + d*x])/(2*d) - (3*a^3*Cot[c + d
*x]*Csc[c + d*x])/(8*d) - (a^2*b*Csc[c + d*x]^3)/d - (a^3*Cot[c + d*x]*Csc[
c + d*x]^3)/(4*d) + (9*a*b^2*Sec[c + d*x])/(2*d) - (3*a*b^2*Csc[c + d*x]^2*
Sec[c + d*x])/(2*d) + (b^3*Csc[c + d*x]*Sec[c + d*x]^2)/(2*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a^3\csc^5(c+dx)+3a^2b\csc^4(c+dx)\sec(c+dx)+3ab^2\csc^3(c+dx)\sec^2(c+dx)\right.\\
&\qquad\left.+b^3\csc^2(c+dx)\sec^3(c+dx)\right)\,dx\\
&= a^3\int\csc^5(c+dx)\,dx+\left(3a^2b\right)\int\csc^4(c+dx)\sec(c+dx)\,dx\\
&\quad+\left(3ab^2\right)\int\csc^3(c+dx)\sec^2(c+dx)\,dx+b^3\int\csc^2(c+dx)\sec^3(c+dx)\,dx\\
&= -\frac{a^3\cot(c+dx)\csc^3(c+dx)}{4d}+\frac{1}{4}\left(3a^3\right)\int\csc^3(c+dx)\,dx\\
&\quad-\frac{\left(3a^2b\right)\text{Subst}\left(\int\frac{x^4}{-1+x^2}\,dx,x,\csc(c+dx)\right)}{d}\\
&\quad+\frac{\left(3ab^2\right)\text{Subst}\left(\int\frac{x^4}{(-1+x^2)^2}\,dx,x,\sec(c+dx)\right)}{d}\\
&\quad-\frac{b^3\text{Subst}\left(\int\frac{x^4}{(-1+x^2)^2}\,dx,x,\csc(c+dx)\right)}{d}\\
&= -\frac{3a^3\cot(c+dx)\csc(c+dx)}{8d}-\frac{a^3\cot(c+dx)\csc^3(c+dx)}{4d}\\
&\quad-\frac{3ab^2\csc^2(c+dx)\sec(c+dx)}{2d}+\frac{b^3\csc(c+dx)\sec^2(c+dx)}{2d}\\
&\quad+\frac{1}{8}\left(3a^3\right)\int\csc(c+dx)\,dx-\frac{\left(3a^2b\right)\text{Subst}\left(\int\left(1+x^2+\frac{1}{-1+x^2}\right)\,dx,x,\csc(c+dx)\right)}{d}\\
&\quad+\frac{\left(9ab^2\right)\text{Subst}\left(\int\frac{x^2}{-1+x^2}\,dx,x,\sec(c+dx)\right)}{2d}\\
&\quad-\frac{\left(3b^3\right)\text{Subst}\left(\int\frac{x^2}{-1+x^2}\,dx,x,\csc(c+dx)\right)}{2d}\\
&= -\frac{3a^3\text{arctanh}(\cos(c+dx))}{8d}-\frac{3a^2b\csc(c+dx)}{d}-\frac{3b^3\csc(c+dx)}{2d}\\
&\quad-\frac{3a^3\cot(c+dx)\csc(c+dx)}{8d}-\frac{a^2b\csc^3(c+dx)}{d}-\frac{a^3\cot(c+dx)\csc^3(c+dx)}{4d}\\
&\quad+\frac{9ab^2\sec(c+dx)}{2d}-\frac{3ab^2\csc^2(c+dx)\sec(c+dx)}{2d}\\
&\quad+\frac{b^3\csc(c+dx)\sec^2(c+dx)}{2d}-\frac{\left(3a^2b\right)\text{Subst}\left(\int\frac{1}{-1+x^2}\,dx,x,\csc(c+dx)\right)}{d}\\
&\quad+\frac{\left(9ab^2\right)\text{Subst}\left(\int\frac{1}{-1+x^2}\,dx,x,\sec(c+dx)\right)}{2d}\\
&\quad-\frac{\left(3b^3\right)\text{Subst}\left(\int\frac{1}{-1+x^2}\,dx,x,\csc(c+dx)\right)}{2d}
\end{aligned}
$$

$$= -\frac{3a^3\text{arctanh}(\cos(c+dx))}{8d} - \frac{9ab^2\text{arctanh}(\cos(c+dx))}{2d} + \frac{3a^2b\text{arctanh}(\sin(c+dx))}{d} + \frac{3b^3\text{arctanh}(\sin(c+dx))}{2d} - \frac{3a^2b\csc(c+dx)}{d} - \frac{3b^3\csc(c+dx)}{2d} - \frac{3a^3\cot(c+dx)\csc(c+dx)}{8d} - \frac{a^2b\csc^3(c+dx)}{d} - \frac{a^3\cot(c+dx)\csc^3(c+dx)}{4d} + \frac{9ab^2\sec(c+dx)}{2d} - \frac{3ab^2\csc^2(c+dx)\sec(c+dx)}{2d} + \frac{b^3\csc(c+dx)\sec^2(c+dx)}{2d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1229 vs. $2(229) = 458$.

Time = 7.64 (sec) , antiderivative size = 1229, normalized size of antiderivative = 5.37

$$\begin{aligned}
&\int \csc^5(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{3ab^2\cos^3(c+dx)(a+b\tan(c+dx))^3}{d(a\cos(c+dx)+b\sin(c+dx))^3} \\
&+ \frac{\left(-7a^2b\cos\left(\frac{1}{2}(c+dx)\right)-2b^3\cos\left(\frac{1}{2}(c+dx)\right)\right)\cos^3(c+dx)\csc\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{4d(a\cos(c+dx)+b\sin(c+dx))^3} \\
&- \frac{3(a^3+4ab^2)\cos^3(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{32d(a\cos(c+dx)+b\sin(c+dx))^3} \\
&- \frac{a^2b\cos^3(c+dx)\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{8d(a\cos(c+dx)+b\sin(c+dx))^3} \\
&- \frac{a^3\cos^3(c+dx)\csc^4\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{64d(a\cos(c+dx)+b\sin(c+dx))^3} \\
&- \frac{3(a^3+12ab^2)\cos^3(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^3}{8d(a\cos(c+dx)+b\sin(c+dx))^3} \\
&- \frac{3(2a^2b+b^3)\cos^3(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^3}{2d(a\cos(c+dx)+b\sin(c+dx))^3} \\
&+ \frac{3(a^3+12ab^2)\cos^3(c+dx)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^3}{8d(a\cos(c+dx)+b\sin(c+dx))^3} \\
&+ \frac{3(2a^2b+b^3)\cos^3(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^3}{2d(a\cos(c+dx)+b\sin(c+dx))^3} \\
&+ \frac{3(a^3+4ab^2)\cos^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{32d(a\cos(c+dx)+b\sin(c+dx))^3} \\
&+ \frac{a^3\cos^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{64d(a\cos(c+dx)+b\sin(c+dx))^3} \\
&+ \frac{b^3\cos^3(c+dx)(a+b\tan(c+dx))^3}{4d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a\cos(c+dx)+b\sin(c+dx))^3} \\
&+ \frac{3ab^2\cos^3(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^3} \\
&- \frac{b^3\cos^3(c+dx)(a+b\tan(c+dx))^3}{4d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a\cos(c+dx)+b\sin(c+dx))^3} \\
&- \frac{3ab^2\cos^3(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^3} \\
&+ \frac{\cos^3(c+dx)\sec\left(\frac{1}{2}(c+dx)\right)\left(-7a^2b\sin\left(\frac{1}{2}(c+dx)\right)-2b^3\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^3}{4d(a\cos(c+dx)+b\sin(c+dx))^3} \\
&- \frac{a^2b\cos^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{8d(a\cos(c+dx)+b\sin(c+dx))^3}
\end{aligned}$$

```
[In] Integrate[Csc[c + d*x]^5*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a*b^2*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(d*(a*Cos[c + d*x] + b*Sin[
c + d*x])^3) + ((-7*a^2*b*Cos[(c + d*x)/2] - 2*b^3*Cos[(c + d*x)/2])*Cos[c
+ d*x]^3*Csc[(c + d*x)/2]*(a + b*Tan[c + d*x])^3)/(4*d*(a*Cos[c + d*x] + b*
Sin[c + d*x])^3) - (3*(a^3 + 4*a*b^2)*Cos[c + d*x]^3*Csc[(c + d*x)/2]^2*(a
+ b*Tan[c + d*x])^3)/(32*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (a^2*b*Co
s[c + d*x]^3*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2*(a + b*Tan[c + d*x])^3)/(8
*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (a^3*Cos[c + d*x]^3*Csc[(c + d*x)
/2]^4*(a + b*Tan[c + d*x])^3)/(64*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) -
(3*(a^3 + 12*a*b^2)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2]]*(a + b*Tan[c + d*x
])^3)/(8*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (3*(2*a^2*b + b^3)*Cos[c
+ d*x]^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3)/(
2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (3*(a^3 + 12*a*b^2)*Cos[c + d*x]
^3*Log[Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3)/(8*d*(a*Cos[c + d*x] + b*S
in[c + d*x])^3) + (3*(2*a^2*b + b^3)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3)/(2*d*(a*Cos[c + d*x] + b*Sin[c +
d*x])^3) + (3*(a^3 + 4*a*b^2)*Cos[c + d*x]^3*Sec[(c + d*x)/2]^2*(a + b*Tan[
c + d*x])^3)/(32*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (a^3*Cos[c + d*x]
^3*Sec[(c + d*x)/2]^4*(a + b*Tan[c + d*x])^3)/(64*d*(a*Cos[c + d*x] + b*Sin
[c + d*x])^3) + (b^3*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(4*d*(Cos[(c +
d*x)/2] - Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (3*a*b
^2*Cos[c + d*x]^3*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^3)/(d*(Cos[(c + d*x
)/2] - Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (b^3*Cos[c
+ d*x]^3*(a + b*Tan[c + d*x])^3)/(4*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (3*a*b^2*Cos[c + d*x]^3*Sin[(c +
d*x)/2]*(a + b*Tan[c + d*x])^3)/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(a
*Cos[c + d*x] + b*Sin[c + d*x])^3) + (Cos[c + d*x]^3*Sec[(c + d*x)/2]*(-7*a
^2*b*Sin[(c + d*x)/2] - 2*b^3*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^3)/(4*
d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (a^2*b*Cos[c + d*x]^3*Sec[(c + d*x
)/2]^2*Tan[(c + d*x)/2]*(a + b*Tan[c + d*x])^3)/(8*d*(a*Cos[c + d*x] + b*Si
n[c + d*x])^3)
```

Maple [A] (verified)

Time = 10.94 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{b^3\left(\frac{1}{2\sin(dx+c)\cos(dx+c)^2}-\frac{3}{2\sin(dx+c)}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a\,b^2\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c)}+\frac{3}{2\cos(dx+c)}+\frac{3\ln(\csc}{}\right.}{}$
default	$\frac{b^3\left(\frac{1}{2\sin(dx+c)\cos(dx+c)^2}-\frac{3}{2\sin(dx+c)}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a\,b^2\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c)}+\frac{3}{2\cos(dx+c)}+\frac{3\ln(\csc}{}\right.}{}$
risch	$-\frac{i e^{i(dx+c)}\left(-5ia^3 e^{2i(dx+c)}-30ia^3 e^{6i(dx+c)}+24a^2 b\,e^{10i(dx+c)}+12b^3 e^{10i(dx+c)}+3ia^3+36ia\,b^2-56a^2 b\,e^{8i(dx+c)}-28b^3 e\right.}{}$

```
[In] int(csc(d*x+c)^5*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^3*(1/2/sin(d*x+c)/cos(d*x+c)^2-3/2/sin(d*x+c)+3/2*ln(sec(d*x+c)+tan(
d*x+c)))+3*a*b^2*(-1/2/sin(d*x+c)^2/cos(d*x+c)+3/2/cos(d*x+c)+3/2*ln(csc(d*
x+c)-cot(d*x+c)))+3*a^2*b*(-1/3/sin(d*x+c)^3-1/sin(d*x+c)+ln(sec(d*x+c)+tan
(d*x+c)))+a^3*((-1/4*csc(d*x+c)^3-3/8*csc(d*x+c))*cot(d*x+c)+3/8*ln(csc(d*x
+c)-cot(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(211) = 422$.

Time = 0.36 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.86

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{6\,(a^3+12\,ab^2)\cos{(dx+c)}^5+48\,ab^2\cos{(dx+c)}-10\,(a^3+12\,ab^2)\cos{(dx+c)}^3-3\,\left((a^3+12\,ab^2)\cos{(dx}\right.}{}$$

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/16*(6*(a^3 + 12*a*b^2)*cos(d*x + c)^5 + 48*a*b^2*cos(d*x + c) - 10*(a^3 +
 12*a*b^2)*cos(d*x + c)^3 - 3*((a^3 + 12*a*b^2)*cos(d*x + c)^6 - 2*(a^3 + 1
2*a*b^2)*cos(d*x + c)^4 + (a^3 + 12*a*b^2)*cos(d*x + c)^2)*log(1/2*cos(d*x
+ c) + 1/2) + 3*((a^3 + 12*a*b^2)*cos(d*x + c)^6 - 2*(a^3 + 12*a*b^2)*cos(d
*x + c)^4 + (a^3 + 12*a*b^2)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2) +
 12*((2*a^2*b + b^3)*cos(d*x + c)^6 - 2*(2*a^2*b + b^3)*cos(d*x + c)^4 + (2
*a^2*b + b^3)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 12*((2*a^2*b + b^3)*c
os(d*x + c)^6 - 2*(2*a^2*b + b^3)*cos(d*x + c)^4 + (2*a^2*b + b^3)*cos(d*x
+ c)^2)*log(-sin(d*x + c) + 1) + 8*(3*(2*a^2*b + b^3)*cos(d*x + c)^4 + b^3
- 4*(2*a^2*b + b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^6 - 2*d*c
os(d*x + c)^4 + d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^3\,dx = \int (a+b\tan{(c+dx)})^3\csc^5{(c+dx)}\,dx$$

```
[In] integrate(csc(d*x+c)**5*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*csc(c + d*x)**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.09

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$= \frac{a^3\left(\frac{2\left(3\cos(dx+c)^3-5\cos(dx+c)\right)}{\cos(dx+c)^4-2\cos(dx+c)^2+1}-3\log\left(\cos\left(dx+c\right)+1\right)+3\log\left(\cos\left(dx+c\right)-1\right)\right)+12\,ab^2\left(\frac{2\left(3\cos(dx+c)\right.}{\cos(dx+c)^3-\cos(\right.}}{}$$

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/16*(a^3*(2*(3*cos(d*x + c)^3 - 5*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*
x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 12*a*b
^2*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*log(cos(d*
x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 4*b^3*(2*(3*sin(d*x + c)^2 - 2)/(s
in(d*x + c)^3 - sin(d*x + c)) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c
) - 1)) - 8*a^2*b*(2*(3*sin(d*x + c)^2 + 1)/sin(d*x + c)^3 - 3*log(sin(d*x
+ c) + 1) + 3*log(sin(d*x + c) - 1)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.76 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.63

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$= \frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-8\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+8\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+24\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-120\,a^2b}{}$$

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/64*(a^3*tan(1/2*d*x + 1/2*c)^4 - 8*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 8*a^3*t
an(1/2*d*x + 1/2*c)^2 + 24*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 120*a^2*b*tan(1/2
*d*x + 1/2*c) - 32*b^3*tan(1/2*d*x + 1/2*c) + 96*(2*a^2*b + b^3)*log(abs(ta
n(1/2*d*x + 1/2*c) + 1)) - 96*(2*a^2*b + b^3)*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) + 24*(a^3 + 12*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + 64*(b^3*tan(1/
2*d*x + 1/2*c)^3 - 6*a*b^2*tan(1/2*d*x + 1/2*c)^2 + b^3*tan(1/2*d*x + 1/2*c
) + 6*a*b^2)/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 - (50*a^3*tan(1/2*d*x + 1/2*c)^
4 + 600*a*b^2*tan(1/2*d*x + 1/2*c)^4 + 120*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 3
2*b^3*tan(1/2*d*x + 1/2*c)^3 + 8*a^3*tan(1/2*d*x + 1/2*c)^2 + 24*a*b^2*tan(
1/2*d*x + 1/2*c)^2 + 8*a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*d*x + 1/2*
c)^4)/d
```

Mupad [B] (verification not implemented)

Time = 5.46 (sec) , antiderivative size = 698, normalized size of antiderivative = 3.05

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{a^3}{8}+\frac{3\,a\,b^2}{8}\right)}{d}$$
$$-\frac{\mathrm{atan}\left(\frac{\left(3\,a^2\,b+\frac{3\,b^3}{2}\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{3\,a^3}{4}+9\,a\,b^2\right)+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(3\,a^2\,b+\frac{3\,b^3}{2}\right)-6\,a^2\,b-3\,b^3\right)\,1\mathrm{i}-\left(3\,a^2\,b+3\cdots\right.}{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(36\,a^4\,b^2+36\,a^2\,b^4+9\,b^6\right)+27\,a\,b^5+\frac{9\,a^5\,b}{2}-\left(3\,a^2\,b+\frac{3\,b^3}{2}\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{3\,a^3}{4}+9\,a\,b^2\right)+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(3\,a^2\,b+\frac{3\,b^3}{2}\right)-6\cdots}\right.}{d}$$
$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{3\,a^3}{2}+6\,a\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(2\,a^3+102\,a\,b^2\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(\frac{15\,a^3}{4}+108\,a\,b^2\right)+\tan\left(\frac{c}{2}\cdots\right.}{d\,\left(16\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-32\cdots\right.}$$
$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{15\,a^2\,b}{8}+\frac{b^3}{2}\right)}{d}+\frac{3\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a^2+12\,b^2\right)}{8\,d}-\frac{a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{8\,d}$$

```
[In] int((a + b*tan(c + d*x))^3/sin(c + d*x)^5,x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^4)/(64*d) + (tan(c/2 + (d*x)/2)^2*((3*a*b^2)/8 + a^
3/8))/d - (atan(((3*a^2*b + (3*b^3)/2)*(tan(c/2 + (d*x)/2)*(9*a*b^2 + (3*a^
3)/4) + 6*tan(c/2 + (d*x)/2)*(3*a^2*b + (3*b^3)/2) - 6*a^2*b - 3*b^3)*1i -
(3*a^2*b + (3*b^3)/2)*(6*tan(c/2 + (d*x)/2)*(3*a^2*b + (3*b^3)/2) - tan(c/2
 + (d*x)/2)*(9*a*b^2 + (3*a^3)/4) + 6*a^2*b + 3*b^3)*1i)/(2*tan(c/2 + (d*x)
/2)*(9*b^6 + 36*a^2*b^4 + 36*a^4*b^2) + 27*a*b^5 + (9*a^5*b)/2 - (3*a^2*b +
 (3*b^3)/2)*(tan(c/2 + (d*x)/2)*(9*a*b^2 + (3*a^3)/4) + 6*tan(c/2 + (d*x)/2
)*(3*a^2*b + (3*b^3)/2) - 6*a^2*b - 3*b^3) - (3*a^2*b + (3*b^3)/2)*(6*tan(c
/2 + (d*x)/2)*(3*a^2*b + (3*b^3)/2) - tan(c/2 + (d*x)/2)*(9*a*b^2 + (3*a^3)
/4) + 6*a^2*b + 3*b^3) + (225*a^3*b^3)/4))*(a^2*b*6i + b^3*3i))/d - (tan(c/
2 + (d*x)/2)^2*(6*a*b^2 + (3*a^3)/2) + tan(c/2 + (d*x)/2)^6*(102*a*b^2 + 2*
a^3) - tan(c/2 + (d*x)/2)^4*(108*a*b^2 + (15*a^3)/4) + tan(c/2 + (d*x)/2)^3
*(26*a^2*b + 8*b^3) + tan(c/2 + (d*x)/2)^7*(30*a^2*b - 8*b^3) - tan(c/2 + (
d*x)/2)^5*(58*a^2*b + 32*b^3) + a^3/4 + 2*a^2*b*tan(c/2 + (d*x)/2))/(d*(16*
tan(c/2 + (d*x)/2)^4 - 32*tan(c/2 + (d*x)/2)^6 + 16*tan(c/2 + (d*x)/2)^8))
- (tan(c/2 + (d*x)/2)*((15*a^2*b)/8 + b^3/2))/d + (3*a*log(tan(c/2 + (d*x)/
2))*(a^2 + 12*b^2))/(8*d) - (a^2*b*tan(c/2 + (d*x)/2)^3)/(8*d)
```

3.40 $\int \csc^6(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 167

$$\begin{aligned}\int \csc^6(c+dx)(a+b\tan(c+dx))^3\,dx = & -\frac{a(a^2+6b^2)\cot(c+dx)}{d} \\ & -\frac{b(6a^2+b^2)\cot^2(c+dx)}{2d} \\ & -\frac{a(2a^2+3b^2)\cot^3(c+dx)}{3d}-\frac{3a^2b\cot^4(c+dx)}{4d} \\ & -\frac{a^3\cot^5(c+dx)}{5d}+\frac{b(3a^2+2b^2)\log(\tan(c+dx))}{d} \\ & +\frac{3ab^2\tan(c+dx)}{d}+\frac{b^3\tan^2(c+dx)}{2d}\end{aligned}$$

```
[Out] -a*(a^2+6*b^2)*cot(d*x+c)/d-1/2*b*(6*a^2+b^2)*cot(d*x+c)^2/d-1/3*a*(2*a^2+3
*b^2)*cot(d*x+c)^3/d-3/4*a^2*b*cot(d*x+c)^4/d-1/5*a^3*cot(d*x+c)^5/d+b*(3*a
^2+2*b^2)*ln(tan(d*x+c))/d+3*a*b^2*tan(d*x+c)/d+1/2*b^3*tan(d*x+c)^2/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used

$= \{3597, 962\}$

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{a^3\cot^5(c+dx)}{5d} - \frac{a(2a^2+3b^2)\cot^3(c+dx)}{3d} - \frac{b(6a^2+b^2)\cot^2(c+dx)}{2d} - \frac{a(a^2+6b^2)\cot(c+dx)}{d} + \frac{b(3a^2+2b^2)\log(\tan(c+dx))}{d} - \frac{3a^2b\cot^4(c+dx)}{4d} + \frac{3ab^2\tan(c+dx)}{d} + \frac{b^3\tan^2(c+dx)}{2d}$$

```
[In] Int[Csc[c + d*x]^6*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -((a*(a^2 + 6*b^2)*Cot[c + d*x])/d) - (b*(6*a^2 + b^2)*Cot[c + d*x]^2)/(2*d
) - (a*(2*a^2 + 3*b^2)*Cot[c + d*x]^3)/(3*d) - (3*a^2*b*Cot[c + d*x]^4)/(4*
d) - (a^3*Cot[c + d*x]^5)/(5*d) + (b*(3*a^2 + 2*b^2)*Log[Tan[c + d*x]])/d +
 (3*a*b^2*Tan[c + d*x])/d + (b^3*Tan[c + d*x]^2)/(2*d)
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{b\text{Subst}\left(\int \frac{(a+x)^3(b^2+x^2)^2}{x^6}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{b\text{Subst}\left(\int \left(3a + \frac{a^3b^4}{x^6} + \frac{3a^2b^4}{x^5} + \frac{2a^3b^2+3ab^4}{x^4} + \frac{6a^2b^2+b^4}{x^3} + \frac{a^3+6ab^2}{x^2} + \frac{3a^2+2b^2}{x} + x\right)dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{a(a^2+6b^2)\cot(c+dx)}{d} - \frac{b(6a^2+b^2)\cot^2(c+dx)}{2d} - \frac{a(2a^2+3b^2)\cot^3(c+dx)}{3d} - \frac{3a^2b\cot^4(c+dx)}{4d} - \frac{a^3\cot^5(c+dx)}{5d} + \frac{b(3a^2+2b^2)\log(\tan(c+dx))}{d} + \frac{3ab^2\tan(c+dx)}{d} + \frac{b^3\tan^2(c+dx)}{2d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 515 vs. 2(167) = 334.

Time = 3.55 (sec) , antiderivative size = 515, normalized size of antiderivative = 3.08

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{\csc^5(c+dx)\sec^2(c+dx)\left(40a(5a^2+3b^2)\cos(c+dx)+8(a^3+15ab^2)\cos(3(c+dx))-24a^3\cos(5(c+\right.}{}$$

```
[In] Integrate[Csc[c + d*x]^6*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -1/960*(Csc[c + d*x]^5*Sec[c + d*x]^2*(40*a*(5*a^2 + 3*b^2)*Cos[c + d*x] +
8*(a^3 + 15*a*b^2)*Cos[3*(c + d*x)] - 24*a^3*Cos[5*(c + d*x)] - 360*a*b^2*C
os[5*(c + d*x)] + 8*a^3*Cos[7*(c + d*x)] + 120*a*b^2*Cos[7*(c + d*x)] + 360
*a^2*b*Sin[c + d*x] - 240*b^3*Sin[c + d*x] + 225*a^2*b*Log[Cos[c + d*x]]*Si
n[c + d*x] + 150*b^3*Log[Cos[c + d*x]]*Sin[c + d*x] - 225*a^2*b*Log[Sin[c +
 d*x]]*Sin[c + d*x] - 150*b^3*Log[Sin[c + d*x]]*Sin[c + d*x] + 270*a^2*b*Si
n[3*(c + d*x)] + 180*b^3*Sin[3*(c + d*x)] + 45*a^2*b*Log[Cos[c + d*x]]*Sin[
3*(c + d*x)] + 30*b^3*Log[Cos[c + d*x]]*Sin[3*(c + d*x)] - 45*a^2*b*Log[Sin
[c + d*x]]*Sin[3*(c + d*x)] - 30*b^3*Log[Sin[c + d*x]]*Sin[3*(c + d*x)] - 9
0*a^2*b*Sin[5*(c + d*x)] - 60*b^3*Sin[5*(c + d*x)] - 135*a^2*b*Log[Cos[c +
d*x]]*Sin[5*(c + d*x)] - 90*b^3*Log[Cos[c + d*x]]*Sin[5*(c + d*x)] + 135*a^
2*b*Log[Sin[c + d*x]]*Sin[5*(c + d*x)] + 90*b^3*Log[Sin[c + d*x]]*Sin[5*(c
+ d*x)] + 45*a^2*b*Log[Cos[c + d*x]]*Sin[7*(c + d*x)] + 30*b^3*Log[Cos[c +
d*x]]*Sin[7*(c + d*x)] - 45*a^2*b*Log[Sin[c + d*x]]*Sin[7*(c + d*x)] - 30*b
^3*Log[Sin[c + d*x]]*Sin[7*(c + d*x)]))/d
```

Maple [A] (verified)

Time = 19.52 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{b^3\left(\frac{1}{2\sin(dx+c)^2\cos(dx+c)^2}-\frac{1}{\sin(dx+c)^2}+2\ln(\tan(dx+c))\right)+3a\,b^2\left(-\frac{1}{3\sin(dx+c)^3\cos(dx+c)}+\frac{4}{3\sin(dx+c)\cos(dx+c)}-\frac{8\cot(dx}{3}\right.}{d}$
default	$\frac{b^3\left(\frac{1}{2\sin(dx+c)^2\cos(dx+c)^2}-\frac{1}{\sin(dx+c)^2}+2\ln(\tan(dx+c))\right)+3a\,b^2\left(-\frac{1}{3\sin(dx+c)^3\cos(dx+c)}+\frac{4}{3\sin(dx+c)\cos(dx+c)}-\frac{8\cot(dx}{3}\right.}{d}$
risch	$6a^2b\,e^{12i(dx+c)}+4b^3e^{12i(dx+c)}-18a^2b\,e^{10i(dx+c)}-12b^3e^{10i(dx+c)}+\frac{16ia^3e^{2i(dx+c)}}{5}+32ia\,b^2e^{8i(dx+c)}-24a^2b\,e^{8i(dx+c)}+1$

[In] int(csc(d*x+c)^6*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(b^3*(1/2/sin(d*x+c)^2/cos(d*x+c)^2-1/sin(d*x+c)^2+2*ln(tan(d*x+c)))+3*a*b^2*(-1/3/sin(d*x+c)^3/cos(d*x+c)+4/3/sin(d*x+c)/cos(d*x+c)-8/3*cot(d*x+c))+3*a^2*b*(-1/4/sin(d*x+c)^4-1/2/sin(d*x+c)^2+ln(tan(d*x+c)))+a^3*(-8/15-1/5*csc(d*x+c)^4-4/15*csc(d*x+c)^2)*cot(d*x+c))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(157) = 314$.

Time $= 0.28$ (sec) , antiderivative size $= 343$, normalized size of antiderivative $= 2.05$

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^3\,dx =$$
$$-\frac{32\,(a^3+15\,ab^2)\cos{(dx+c)}^7-80\,(a^3+15\,ab^2)\cos{(dx+c)}^5-180\,ab^2\cos{(dx+c)}+60\,(a^3+15\,ab^2)\,\mathrm{co}}{}$$

[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^3,x, algorithm="fricas")

[Out] -1/60*(32*(a^3 + 15*a*b^2)*cos(d*x + c)^7 - 80*(a^3 + 15*a*b^2)*cos(d*x + c)^5 - 180*a*b^2*cos(d*x + c) + 60*(a^3 + 15*a*b^2)*cos(d*x + c)^3 + 30*((3*a^2*b + 2*b^3)*cos(d*x + c)^6 - 2*(3*a^2*b + 2*b^3)*cos(d*x + c)^4 + (3*a^2*b + 2*b^3)*cos(d*x + c)^2)*log(cos(d*x + c)^2)*sin(d*x + c) - 30*((3*a^2*b + 2*b^3)*cos(d*x + c)^6 - 2*(3*a^2*b + 2*b^3)*cos(d*x + c)^4 + (3*a^2*b + 2*b^3)*cos(d*x + c)^2)*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) - 15*(2*(3*a^2*b + 2*b^3)*cos(d*x + c)^4 + 2*b^3 - 3*(3*a^2*b + 2*b^3)*cos(d*x + c)^2)*sin(d*x + c))/((d*cos(d*x + c)^6 - 2*d*cos(d*x + c)^4 + d*cos(d*x + c)^2)*sin(d*x + c))

Sympy [F]

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^3\,dx = \int (a+b\tan(c+dx))^3 \csc^6(c+dx)\,dx$$

```
[In] integrate(csc(d*x+c)**6*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*csc(c + d*x)**6, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.85

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{30\,b^3\tan(dx+c)^2 + 180\,ab^2\tan(dx+c) + 60\,(3\,a^2b + 2\,b^3)\log(\tan(dx+c)) - \frac{60\,(a^3+6\,ab^2)\tan(dx+c)^4+45\,a^2b\cdots}{\cdots}}{60\,d}$$

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(30*b^3*tan(d*x + c)^2 + 180*a*b^2*tan(d*x + c) + 60*(3*a^2*b + 2*b^3)
*log(tan(d*x + c)) - (60*(a^3 + 6*a*b^2)*tan(d*x + c)^4 + 45*a^2*b*tan(d*x
+ c) + 30*(6*a^2*b + b^3)*tan(d*x + c)^3 + 12*a^3 + 20*(2*a^3 + 3*a*b^2)*ta
n(d*x + c)^2)/tan(d*x + c)^5)/d
```

Giac [A] (verification not implemented)

none

Time = 0.77 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.13

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{30\,b^3\tan(dx+c)^2 + 180\,ab^2\tan(dx+c) + 60\,(3\,a^2b + 2\,b^3)\log(|\tan(dx+c)|) - \frac{411\,a^2b\tan(dx+c)^5+274\,b^3\tan(\cdots}{\cdots}}{\cdots}$$

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(30*b^3*tan(d*x + c)^2 + 180*a*b^2*tan(d*x + c) + 60*(3*a^2*b + 2*b^3)
*log(abs(tan(d*x + c))) - (411*a^2*b*tan(d*x + c)^5 + 274*b^3*tan(d*x + c)^
5 + 60*a^3*tan(d*x + c)^4 + 360*a*b^2*tan(d*x + c)^4 + 180*a^2*b*tan(d*x +
c)^3 + 30*b^3*tan(d*x + c)^3 + 40*a^3*tan(d*x + c)^2 + 60*a*b^2*tan(d*x + c
)^2 + 45*a^2*b*tan(d*x + c) + 12*a^3)/tan(d*x + c)^5)/d
```

Mupad [B] (verification not implemented)

Time = 4.32 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.87

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{\ln\left(\tan(c+d\,x)\right)\,\left(3\,a^2\,b+2\,b^3\right)}{d}$$
$$-\frac{\cot(c+d\,x)^5\,\left(\tan(c+d\,x)^2\,\left(\frac{2\,a^3}{3}+a\,b^2\right)+\tan(c+d\,x)^3\,\left(3\,a^2\,b+\frac{b^3}{2}\right)+\frac{a^3}{5}+\tan(c+d\,x)^4\,\left(a^3+6\,a\,b^2\right)}{d}$$
$$+\frac{b^3\tan(c+d\,x)^2}{2\,d}+\frac{3\,a\,b^2\tan(c+d\,x)}{d}$$

```
[In] int((a + b*tan(c + d*x))^3/sin(c + d*x)^6,x)
```

```
[Out] (log(tan(c + d*x))*(3*a^2*b + 2*b^3))/d - (cot(c + d*x)^5*(tan(c + d*x)^2*(
a*b^2 + (2*a^3)/3) + tan(c + d*x)^3*(3*a^2*b + b^3/2) + a^3/5 + tan(c + d*x
)^4*(6*a*b^2 + a^3) + (3*a^2*b*tan(c + d*x))/4))/d + (b^3*tan(c + d*x)^2)/(
2*d) + (3*a*b^2*tan(c + d*x))/d
```

3.41 $\int \sin^3(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal result

Integrand size = 21, antiderivative size = 275

$$\begin{aligned}\int \sin^3(c+dx)(a+b\tan(c+dx))^4\,dx &= \frac{4a^3b\text{arctanh}(\sin(c+dx))}{d} \\ &- \frac{10ab^3\text{arctanh}(\sin(c+dx))}{d} - \frac{a^4\cos(c+dx)}{d} \\ &+ \frac{12a^2b^2\cos(c+dx)}{d} - \frac{3b^4\cos(c+dx)}{d} \\ &+ \frac{a^4\cos^3(c+dx)}{3d} - \frac{2a^2b^2\cos^3(c+dx)}{d} \\ &+ \frac{b^4\cos^3(c+dx)}{3d} + \frac{6a^2b^2\sec(c+dx)}{d} \\ &- \frac{3b^4\sec(c+dx)}{d} + \frac{b^4\sec^3(c+dx)}{3d} \\ &- \frac{4a^3b\sin(c+dx)}{d} + \frac{10ab^3\sin(c+dx)}{d} \\ &- \frac{4a^3b\sin^3(c+dx)}{3d} + \frac{10ab^3\sin^3(c+dx)}{3d} \\ &+ \frac{2ab^3\sin^3(c+dx)\tan^2(c+dx)}{d}\end{aligned}$$

```
[Out] 4*a^3*b*arctanh(sin(d*x+c))/d-10*a*b^3*arctanh(sin(d*x+c))/d-a^4*cos(d*x+c)
/d+12*a^2*b^2*cos(d*x+c)/d-3*b^4*cos(d*x+c)/d+1/3*a^4*cos(d*x+c)^3/d-2*a^2*
b^2*cos(d*x+c)^3/d+1/3*b^4*cos(d*x+c)^3/d+6*a^2*b^2*sec(d*x+c)/d-3*b^4*sec(
d*x+c)/d+1/3*b^4*sec(d*x+c)^3/d-4*a^3*b*sin(d*x+c)/d+10*a*b^3*sin(d*x+c)/d-
4/3*a^3*b*sin(d*x+c)^3/d+10/3*a*b^3*sin(d*x+c)^3/d+2*a*b^3*sin(d*x+c)^3*tan
(d*x+c)^2/d
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3598, 2713, 2672, 308, 212, 2670, 276, 294}

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^4\,dx = \frac{a^4\cos^3(c+dx)}{3d} - \frac{a^4\cos(c+dx)}{d} + \frac{4a^3b\text{arctanh}(\sin(c+dx))}{d} - \frac{4a^3b\sin^3(c+dx)}{3d} - \frac{4a^3b\sin(c+dx)}{d} - \frac{2a^2b^2\cos^3(c+dx)}{d} + \frac{12a^2b^2\cos(c+dx)}{d} + \frac{6a^2b^2\sec(c+dx)}{d} - \frac{10ab^3\text{arctanh}(\sin(c+dx))}{d} + \frac{10ab^3\sin^3(c+dx)}{3d} + \frac{10ab^3\sin(c+dx)}{d} + \frac{2ab^3\sin^3(c+dx)\tan^2(c+dx)}{d} + \frac{b^4\cos^3(c+dx)}{3d} - \frac{3b^4\cos(c+dx)}{d} + \frac{b^4\sec^3(c+dx)}{3d} - \frac{3b^4\sec(c+dx)}{d}$$

```
[In] Int[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (4*a^3*b*ArcTanh[Sin[c + d*x]])/d - (10*a*b^3*ArcTanh[Sin[c + d*x]])/d - (a
^4*Cos[c + d*x])/d + (12*a^2*b^2*Cos[c + d*x])/d - (3*b^4*Cos[c + d*x])/d +
 (a^4*Cos[c + d*x]^3)/(3*d) - (2*a^2*b^2*Cos[c + d*x]^3)/d + (b^4*Cos[c + d
*x]^3)/(3*d) + (6*a^2*b^2*Sec[c + d*x])/d - (3*b^4*Sec[c + d*x])/d + (b^4*S
ec[c + d*x]^3)/(3*d) - (4*a^3*b*Sin[c + d*x])/d + (10*a*b^3*Sin[c + d*x])/d
 - (4*a^3*b*Sin[c + d*x]^3)/(3*d) + (10*a*b^3*Sin[c + d*x]^3)/(3*d) + (2*a*
b^3*Sin[c + d*x]^3*Tan[c + d*x]^2)/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} = \int &\left(a^4 \sin^3(c+dx) + 4a^3 b \sin^3(c+dx)\tan(c+dx) + 6a^2b^2\sin^3(c+dx)\tan^2(c+dx)\right.\\ &\left.+ 4ab^3\sin^3(c+dx)\tan^3(c+dx) + b^4\sin^3(c+dx)\tan^4(c+dx)\right)\,dx\end{aligned}$$

$$= a^4 \int \sin^3(c+dx)\,dx + \left(4a^3b\right) \int \sin^3(c+dx)\tan(c+dx)\,dx + \left(6a^2b^2\right) \int \sin^3(c+dx)\tan^2(c+dx)\,dx + \left(4ab^3\right) \int \sin^3(c+dx)\tan^3(c+dx)\,dx + b^4 \int \sin^3(c+dx)\tan^4(c+dx)\,dx$$

$$= -\frac{a^4\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cos(c+dx)\right)}{d} + \frac{\left(4a^3b\right)\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d} - \frac{\left(6a^2b^2\right)\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^2}\,dx, x, \cos(c+dx)\right)}{d} + \frac{\left(4ab^3\right)\text{Subst}\left(\int \frac{x^6}{\left(1-x^2\right)^2}\,dx, x, \sin(c+dx)\right)}{d} - \frac{b^4\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{x^4}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{a^4\cos(c+dx)}{d} + \frac{a^4\cos^3(c+dx)}{3d} + \frac{2ab^3\sin^3(c+dx)\tan^2(c+dx)}{d} + \frac{\left(4a^3b\right)\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)\,dx, x, \sin(c+dx)\right)}{d} - \frac{\left(6a^2b^2\right)\text{Subst}\left(\int \left(-2+\frac{1}{x^2}+x^2\right)\,dx, x, \cos(c+dx)\right)}{d} - \frac{\left(10ab^3\right)\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d} - \frac{b^4\text{Subst}\left(\int \left(3+\frac{1}{x^4}-\frac{3}{x^2}-x^2\right)\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{a^4\cos(c+dx)}{d} + \frac{12a^2b^2\cos(c+dx)}{d} - \frac{3b^4\cos(c+dx)}{d} + \frac{a^4\cos^3(c+dx)}{3d} - \frac{2a^2b^2\cos^3(c+dx)}{d} + \frac{b^4\cos^3(c+dx)}{3d} + \frac{6a^2b^2\sec(c+dx)}{d} - \frac{3b^4\sec(c+dx)}{d} + \frac{b^4\sec^3(c+dx)}{3d} - \frac{4a^3b\sin(c+dx)}{d} - \frac{4a^3b\sin^3(c+dx)}{3d} + \frac{2ab^3\sin^3(c+dx)\tan^2(c+dx)}{d} + \frac{\left(4a^3b\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d} - \frac{\left(10ab^3\right)\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{4a^3b\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{a^4\cos(c+dx)}{d} + \frac{12a^2b^2\cos(c+dx)}{d} - \frac{3b^4\cos(c+dx)}{d} + \frac{a^4\cos^3(c+dx)}{3d} - \frac{2a^2b^2\cos^3(c+dx)}{d} + \frac{b^4\cos^3(c+dx)}{3d} + \frac{6a^2b^2\sec(c+dx)}{d} - \frac{3b^4\sec(c+dx)}{d} + \frac{b^4\sec^3(c+dx)}{3d} - \frac{4a^3b\sin(c+dx)}{d} + \frac{10ab^3\sin(c+dx)}{d} - \frac{4a^3b\sin^3(c+dx)}{3d} + \frac{10ab^3\sin^3(c+dx)}{3d} + \frac{2ab^3\sin^3(c+dx)\tan^2(c+dx)}{d} - \frac{(10ab^3)\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{4a^3b\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{10ab^3\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{a^4\cos(c+dx)}{d} + \frac{12a^2b^2\cos(c+dx)}{d} - \frac{3b^4\cos(c+dx)}{d} + \frac{a^4\cos^3(c+dx)}{3d} - \frac{2a^2b^2\cos^3(c+dx)}{d} + \frac{b^4\cos^3(c+dx)}{3d} + \frac{6a^2b^2\sec(c+dx)}{d} - \frac{3b^4\sec(c+dx)}{d} + \frac{b^4\sec^3(c+dx)}{3d} - \frac{4a^3b\sin(c+dx)}{d} + \frac{10ab^3\sin(c+dx)}{d} - \frac{4a^3b\sin^3(c+dx)}{3d} + \frac{10ab^3\sin^3(c+dx)}{3d} + \frac{2ab^3\sin^3(c+dx)\tan^2(c+dx)}{d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1017 vs. $2(275) = 550$.

Time = 7.87 (sec) , antiderivative size = 1017, normalized size of antiderivative = 3.70

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^4\,dx = -\frac{b^2(-36a^2+17b^2)\cos^4(c+dx)(a+b\tan(c+dx))^4}{6d(a\cos(c+dx)+b\sin(c+dx))^4}$$
$$-\frac{(3a^4-42a^2b^2+11b^4)\cos^5(c+dx)(a+b\tan(c+dx))^4}{4d(a\cos(c+dx)+b\sin(c+dx))^4}$$
$$+\frac{(a^4-6a^2b^2+b^4)\cos^4(c+dx)\cos(3(c+dx))(a+b\tan(c+dx))^4}{12d(a\cos(c+dx)+b\sin(c+dx))^4}$$
$$-\frac{2(2a^3b-5ab^3)\cos^4(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{d(a\cos(c+dx)+b\sin(c+dx))^4}$$
$$+\frac{2(2a^3b-5ab^3)\cos^4(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{d(a\cos(c+dx)+b\sin(c+dx))^4}$$
$$+\frac{(12ab^3+b^4)\cos^4(c+dx)(a+b\tan(c+dx))^4}{12d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a\cos(c+dx)+b\sin(c+dx))^4}$$
$$+\frac{b^4\cos^4(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^4}{6d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3(a\cos(c+dx)+b\sin(c+dx))^4}$$
$$-\frac{b^4\cos^4(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^4}{6d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3(a\cos(c+dx)+b\sin(c+dx))^4}$$
$$+\frac{(-12ab^3+b^4)\cos^4(c+dx)(a+b\tan(c+dx))^4}{12d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a\cos(c+dx)+b\sin(c+dx))^4}$$
$$+\frac{\cos^4(c+dx)\left(36a^2b^2\sin\left(\frac{1}{2}(c+dx)\right)-17b^4\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{6d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^4}$$
$$+\frac{\cos^4(c+dx)\left(-36a^2b^2\sin\left(\frac{1}{2}(c+dx)\right)+17b^4\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{6d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^4}$$
$$-\frac{ab(5a^2-9b^2)\cos^4(c+dx)\sin(c+dx)(a+b\tan(c+dx))^4}{d(a\cos(c+dx)+b\sin(c+dx))^4}$$
$$+\frac{ab(a^2-b^2)\cos^4(c+dx)\sin(3(c+dx))(a+b\tan(c+dx))^4}{3d(a\cos(c+dx)+b\sin(c+dx))^4}$$

```
[In] Integrate[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -1/6*(b^2*(-36*a^2 + 17*b^2)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(d*(a*C
os[c + d*x] + b*Sin[c + d*x])^4) - ((3*a^4 - 42*a^2*b^2 + 11*b^4)*Cos[c + d
*x]^5*(a + b*Tan[c + d*x])^4)/(4*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (
(a^4 - 6*a^2*b^2 + b^4)*Cos[c + d*x]^4*Cos[3*(c + d*x)]*(a + b*Tan[c + d*x]
)^4)/(12*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (2*(2*a^3*b - 5*a*b^3)*Co
s[c + d*x]^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^
4)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (2*(2*a^3*b - 5*a*b^3)*Cos[c +
 d*x]^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(d
*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((12*a*b^3 + b^4)*Cos[c + d*x]^4*(a
```

```
 + b*Tan[c + d*x])^4)/(12*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(a*Cos[
c + d*x] + b*Sin[c + d*x])^4) + (b^4*Cos[c + d*x]^4*Sin[(c + d*x)/2]*(a + b
*Tan[c + d*x])^4)/(6*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(a*Cos[c + d
*x] + b*Sin[c + d*x])^4) - (b^4*Cos[c + d*x]^4*Sin[(c + d*x)/2]*(a + b*Tan[
c + d*x])^4)/(6*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(a*Cos[c + d*x] +
 b*Sin[c + d*x])^4) + ((-12*a*b^3 + b^4)*Cos[c + d*x]^4*(a + b*Tan[c + d*x]
)^4)/(12*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[
c + d*x])^4) + (Cos[c + d*x]^4*(36*a^2*b^2*Sin[(c + d*x)/2] - 17*b^4*Sin[(c
 + d*x)/2])*(a + b*Tan[c + d*x])^4)/(6*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/
2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cos[c + d*x]^4*(-36*a^2*b^2*Sin
[(c + d*x)/2] + 17*b^4*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^4)/(6*d*(Cos[
(c + d*x)/2] + Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (a*
b*(5*a^2 - 9*b^2)*Cos[c + d*x]^4*Sin[c + d*x]*(a + b*Tan[c + d*x])^4)/(d*(a
*Cos[c + d*x] + b*Sin[c + d*x])^4) + (a*b*(a^2 - b^2)*Cos[c + d*x]^4*Sin[3*
(c + d*x)]*(a + b*Tan[c + d*x])^4)/(3*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4
)
```

Maple [A] (verified)

Time = 4.71 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{-\frac{a^4\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+4a^3b\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+6a^2b^2\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}\right.\right.}{}$
default	$\frac{-\frac{a^4\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+4a^3b\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+6a^2b^2\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}\right.\right.}{}$
risch	$-\frac{3\,e^{i(dx+c)}a^4}{8d}-\frac{11\,e^{i(dx+c)}b^4}{8d}+\frac{e^{-3i(dx+c)}a^4}{24d}+\frac{e^{-3i(dx+c)}b^4}{24d}-\frac{3\,e^{-i(dx+c)}a^4}{8d}-\frac{11\,e^{-i(dx+c)}b^4}{8d}+\frac{e^{3i(dx+c)}}{24d}$

```
[In] int(sin(d*x+c)^3*(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a^4*(2+sin(d*x+c)^2)*cos(d*x+c)+4*a^3*b*(-1/3*sin(d*x+c)^3-sin(d*
x+c)+ln(sec(d*x+c)+tan(d*x+c)))+6*a^2*b^2*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin
(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+4*a*b^3*(1/2*sin(d*x+c)^7/cos(d*x+c
)^2+1/2*sin(d*x+c)^5+5/6*sin(d*x+c)^3+5/2*sin(d*x+c)-5/2*ln(sec(d*x+c)+tan(
d*x+c)))+b^4*(1/3*sin(d*x+c)^8/cos(d*x+c)^3-5/3*sin(d*x+c)^8/cos(d*x+c)-5/3
*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.81

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^4\,dx = \frac{(a^4-6a^2b^2+b^4)\cos(dx+c)^6-3(a^4-12a^2b^2+3b^4)\cos(dx+c)^4+3(2a^3b-5ab^3)\cos(dx+c)^3\log(\mathrm{s}}{}$$

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*((a^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^6 - 3*(a^4 - 12*a^2*b^2 + 3*b^4)*
cos(d*x + c)^4 + 3*(2*a^3*b - 5*a*b^3)*cos(d*x + c)^3*log(sin(d*x + c) + 1)
 - 3*(2*a^3*b - 5*a*b^3)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + b^4 + 9*(2
*a^2*b^2 - b^4)*cos(d*x + c)^2 + 2*(2*(a^3*b - a*b^3)*cos(d*x + c)^5 + 3*a*
b^3*cos(d*x + c) - 2*(4*a^3*b - 7*a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(d*c
os(d*x + c)^3)
```

Sympy [F]

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^4\,dx = \int (a+b\tan(c+dx))^4\sin^3(c+dx)\,dx$$

```
[In] integrate(sin(d*x+c)**3*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**4*sin(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.79

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^4\,dx = \frac{(\cos(dx+c)^3-3\cos(dx+c))a^4-2\left(2\sin(dx+c)^3-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)\right.}{}$$

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*((cos(d*x + c)^3 - 3*cos(d*x + c))*a^4 - 2*(2*sin(d*x + c)^3 - 3*log(si
n(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin(d*x + c))*a^3*b - 6*(cos(
d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a^2*b^2 + (4*sin(d*x + c)^3 -
 6*sin(d*x + c)/(sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(si
n(d*x + c) - 1) + 24*sin(d*x + c))*a*b^3 + (cos(d*x + c)^3 - (9*cos(d*x + c
)^2 - 1)/cos(d*x + c)^3 - 9*cos(d*x + c))*b^4)/d
```

Giac [F(-1)]

Timed out.

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^4\,dx = \text{Timed out}$$

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 8.57 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.16

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^4\,dx =$$

$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(8\,a^4-96\,a^2\,b^2+32\,b^4\right)+4\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(20\,a\,b^3-8\,a^3\,b\right)-\frac{4\,a^4}{3}-\frac{32\,b}{3}\cdots}{\cdots}$$

$$-\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(20\,a\,b^3-8\,a^3\,b\right)}{d}$$

```
[In] int(sin(c + d*x)^3*(a + b*tan(c + d*x))^4,x)
```

```
[Out] - (tan(c/2 + (d*x)/2)^4*(8*a^4 + 32*b^4 - 96*a^2*b^2) + 4*a^4*tan(c/2 + (d*
x)/2)^8 + tan(c/2 + (d*x)/2)*(20*a*b^3 - 8*a^3*b) - (4*a^4)/3 - (32*b^4)/3
+ 32*a^2*b^2 - tan(c/2 + (d*x)/2)^6*((32*a^4)/3 - 64*a^2*b^2) + tan(c/2 + (
d*x)/2)^3*((20*a*b^3)/3 - (8*a^3*b)/3) - tan(c/2 + (d*x)/2)^11*(20*a*b^3 -
8*a^3*b) - tan(c/2 + (d*x)/2)^9*((20*a*b^3)/3 - (8*a^3*b)/3) - tan(c/2 + (d
*x)/2)^5*(56*a*b^3 - 48*a^3*b) + tan(c/2 + (d*x)/2)^7*(56*a*b^3 - 48*a^3*b)
)/(d*(3*tan(c/2 + (d*x)/2)^4 - 3*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^
12 - 1)) - (atanh(tan(c/2 + (d*x)/2))*(20*a*b^3 - 8*a^3*b))/d
```

3.42 $\int \sin^2(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal result

Integrand size = 21, antiderivative size = 139

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^4\,dx = \frac{1}{2}\left(a^4-18a^2b^2+5b^4\right)x - \frac{4ab(a^2-2b^2)\log(\cos(c+dx))}{d} + \frac{b^2(18a^2-5b^2)\tan(c+dx)}{2d} + \frac{4ab^3\tan^2(c+dx)}{d} + \frac{5b^4\tan^3(c+dx)}{6d} - \frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^4}{2d}$$

```
[Out] 1/2*(a^4-18*a^2*b^2+5*b^4)*x-4*a*b*(a^2-2*b^2)*ln(cos(d*x+c))/d+1/2*b^2*(18
*a^2-5*b^2)*tan(d*x+c)/d+4*a*b^3*tan(d*x+c)^2/d+5/6*b^4*tan(d*x+c)^3/d-1/2*
cos(d*x+c)*sin(d*x+c)*(a+b*tan(d*x+c))^4/d
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used

= {3597, 1659, 815, 649, 209, 266}

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^4\,dx = \frac{b^2(18a^2-5b^2)\tan(c+dx)}{2d} - \frac{4ab(a^2-2b^2)\log(\cos(c+dx))}{d} + \frac{1}{2}x\left(a^4-18a^2b^2+5b^4\right) + \frac{4ab^3\tan^2(c+dx)}{d} - \frac{\sin(c+dx)\cos(c+dx)(a+b\tan(c+dx))^4}{2d} + \frac{5b^4\tan^3(c+dx)}{6d}$$

```
[In] Int[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] ((a^4 - 18*a^2*b^2 + 5*b^4)*x)/2 - (4*a*b*(a^2 - 2*b^2)*Log[Cos[c + d*x]])/
d + (b^2*(18*a^2 - 5*b^2)*Tan[c + d*x])/(2*d) + (4*a*b^3*Tan[c + d*x]^2)/d
+ (5*b^4*Tan[c + d*x]^3)/(6*d) - (Cos[c + d*x]*Sin[c + d*x]*(a + b*Tan[c +
d*x])^4)/(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
```

```
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b\text{Subst}\left(\int \frac{x^2(a+x)^4}{(b^2+x^2)^2}\,dx, x, b\tan(c+dx)\right)}{d}\\
&= -\frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^4}{2d}\\
&\quad -\frac{\text{Subst}\left(\int \frac{(a+x)^3(-ab^2-5b^2x)}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{2bd}\\
&= -\frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^4}{2d}\\
&\quad -\frac{\text{Subst}\left(\int \left(-18a^2b^2+5b^4-16ab^2x-5b^2x^2-\frac{b^2(a^4-18a^2b^2+5b^4)+8ab^2(a^2-2b^2)x}{b^2+x^2}\right)dx, x, b\tan(c+dx)\right)}{2bd}\\
&= \frac{b^2(18a^2-5b^2)\tan(c+dx)}{2d}+\frac{4ab^3\tan^2(c+dx)}{d}\\
&\quad +\frac{5b^4\tan^3(c+dx)}{6d}-\frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^4}{2d}\\
&\quad +\frac{\text{Subst}\left(\int \frac{b^2(a^4-18a^2b^2+5b^4)+8ab^2(a^2-2b^2)x}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{2bd}\\
&= \frac{b^2(18a^2-5b^2)\tan(c+dx)}{2d}+\frac{4ab^3\tan^2(c+dx)}{d}\\
&\quad +\frac{5b^4\tan^3(c+dx)}{6d}-\frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^4}{2d}\\
&\quad +\frac{(4ab(a^2-2b^2))\,\text{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{d}\\
&\quad +\frac{(b(a^4-18a^2b^2+5b^4))\,\text{Subst}\left(\int \frac{1}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{2d}
\end{aligned}$$

$$= \frac{1}{2}\left(a^4-18a^2b^2+5b^4\right)x-\frac{4ab(a^2-2b^2)\log(\cos(c+dx))}{d}+\frac{b^2(18a^2-5b^2)\tan(c+dx)}{2d} + \frac{4ab^3\tan^2(c+dx)}{d}+\frac{5b^4\tan^3(c+dx)}{6d}-\frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^4}{2d}$$

Mathematica [A] (verified)

Time = 6.31 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.89

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^4\,dx$$

$$= \frac{b\left(-\frac{(a^4-6a^2b^2+b^4)\arctan(\tan(c+dx))}{2b}+2a(a-b)(a+b)\cos^2(c+dx)+\frac{1}{2}\left(4a^3-8ab^2+\frac{a^4-12a^2b^2+3b^4}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b\right.}\right.}{}$$

```
[In] Integrate[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (b*(-1/2*((a^4 - 6*a^2*b^2 + b^4)*ArcTan[Tan[c + d*x]])/b + 2*a*(a - b)*(a
+ b)*Cos[c + d*x]^2 + ((4*a^3 - 8*a*b^2 + (a^4 - 12*a^2*b^2 + 3*b^4)/Sqrt[-
b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/2 + ((4*a^3 - 8*a*b^2 - (a^4 - 12*a
^2*b^2 + 3*b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]])/2 - ((a^4 - 6
*a^2*b^2 + b^4)*Cos[c + d*x]*Sin[c + d*x])/(2*b) + 2*b*(3*a^2 - b^2)*Tan[c
+ d*x] + 2*a*b^2*Tan[c + d*x]^2 + (b^3*Tan[c + d*x]^3)/3))/d
```

Maple [A] (verified)

Time = 3.53 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.80

method	result
derivativedivides	$\frac{a^4\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+4a^3b\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+6a^2b^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx\cdots}{2}\right.\right.}{}$
default	$\frac{a^4\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+4a^3b\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+6a^2b^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx\cdots}{2}\right.\right.}{}$
risch	$\frac{i e^{2i(dx+c)}b^4}{8d}-\frac{i e^{-2i(dx+c)}b^4}{8d}+\frac{x\,a^4}{2}-9x\,a^2b^2+\frac{5x\,b^4}{2}+\frac{e^{2i(dx+c)}a^3b}{2d}-\frac{e^{2i(dx+c)}a\,b^3}{2d}-8ixa\,b^3+4i$

```
[In] int(sin(d*x+c)^2*(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(-1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+4*a^3*b*(-1/2*sin(d*x+c
)^2-ln(cos(d*x+c)))+6*a^2*b^2*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*si
n(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c)+4*a*b^3*(1/2*sin(d*x+c)^6/cos(d*x+c)^2+
1/2*sin(d*x+c)^4+sin(d*x+c)^2+2*ln(cos(d*x+c)))+b^4*(1/3*sin(d*x+c)^7/cos(d
```

```
*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8
*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.34

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^4\,dx = \frac{12\left(a^3b-ab^3\right)\cos\left(dx+c\right)^5+12\,ab^3\cos\left(dx+c\right)-24\left(a^3b-2\,ab^3\right)\cos\left(dx+c\right)^3\log\left(-\cos\left(dx+c\right)\right)-3\,(2$$

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
[Out] 1/6*(12*(a^3*b - a*b^3)*cos(d*x + c)^5 + 12*a*b^3*cos(d*x + c) - 24*(a^3*b
- 2*a*b^3)*cos(d*x + c)^3*log(-cos(d*x + c)) - 3*(2*a^3*b - 2*a*b^3 - (a^4
- 18*a^2*b^2 + 5*b^4)*d*x)*cos(d*x + c)^3 - (3*(a^4 - 6*a^2*b^2 + b^4)*cos(
d*x + c)^4 - 2*b^4 - 2*(18*a^2*b^2 - 7*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(
d*cos(d*x + c)^3)
```

Sympy [F]

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^4\,dx = \int \left(a+b\tan\left(c+dx\right)\right)^4\sin^2\left(c+dx\right)\,dx$$

```
[In] integrate(sin(d*x+c)**2*(a+b*tan(d*x+c))**4,x)
[Out] Integral((a + b*tan(c + d*x))**4*sin(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.70 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.11

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^4\,dx = \frac{2\,b^4\tan\left(dx+c\right)^3+12\,ab^3\tan\left(dx+c\right)^2+3\left(a^4-18\,a^2b^2+5\,b^4\right)\left(dx+c\right)+12\left(a^3b-2\,ab^3\right)\log\left(\tan\left(dx+c\right.\right.}{6\,d}$$

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
[Out] 1/6*(2*b^4*tan(d*x + c)^3 + 12*a*b^3*tan(d*x + c)^2 + 3*(a^4 - 18*a^2*b^2 +
 5*b^4)*(d*x + c) + 12*(a^3*b - 2*a*b^3)*log(tan(d*x + c)^2 + 1) + 12*(3*a^
2*b^2 - b^4)*tan(d*x + c) + 3*(4*a^3*b - 4*a*b^3 - (a^4 - 6*a^2*b^2 + b^4)*
tan(d*x + c))/(tan(d*x + c)^2 + 1))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3651 vs. 2(131) = 262.

Time = 2.57 (sec) , antiderivative size = 3651, normalized size of antiderivative = 26.27

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^4\,dx = \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/6*(3*a^4*d*x*tan(d*x)^5*tan(c)^5 - 54*a^2*b^2*d*x*tan(d*x)^5*tan(c)^5 + 1
5*b^4*d*x*tan(d*x)^5*tan(c)^5 - 12*a^3*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan
(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*
x)^5*tan(c)^5 + 24*a*b^3*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1
)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^5*tan(c)^5 +
3*a^4*d*x*tan(d*x)^5*tan(c)^3 - 54*a^2*b^2*d*x*tan(d*x)^5*tan(c)^3 + 15*b^4
*d*x*tan(d*x)^5*tan(c)^3 - 9*a^4*d*x*tan(d*x)^4*tan(c)^4 + 162*a^2*b^2*d*x*
tan(d*x)^4*tan(c)^4 - 45*b^4*d*x*tan(d*x)^4*tan(c)^4 + 3*a^4*d*x*tan(d*x)^3
*tan(c)^5 - 54*a^2*b^2*d*x*tan(d*x)^3*tan(c)^5 + 15*b^4*d*x*tan(d*x)^3*tan(
c)^5 + 6*a^3*b*tan(d*x)^5*tan(c)^5 + 6*a*b^3*tan(d*x)^5*tan(c)^5 - 12*a^3*b
*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 +
 tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^5*tan(c)^3 + 24*a*b^3*log(4*(tan(d*x)
^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + ta
n(c)^2 + 1))*tan(d*x)^5*tan(c)^3 + 36*a^3*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*
tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan
(d*x)^4*tan(c)^4 - 72*a*b^3*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c)
+ 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4
 + 3*a^4*tan(d*x)^5*tan(c)^4 - 54*a^2*b^2*tan(d*x)^5*tan(c)^4 + 15*b^4*tan(
d*x)^5*tan(c)^4 - 12*a^3*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) +
 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^3*tan(c)^5
+ 24*a*b^3*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*
tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^3*tan(c)^5 + 3*a^4*tan(d*x)
^4*tan(c)^5 - 54*a^2*b^2*tan(d*x)^4*tan(c)^5 + 15*b^4*tan(d*x)^4*tan(c)^5 -
 9*a^4*d*x*tan(d*x)^4*tan(c)^2 + 162*a^2*b^2*d*x*tan(d*x)^4*tan(c)^2 - 45*b
^4*d*x*tan(d*x)^4*tan(c)^2 + 12*a^4*d*x*tan(d*x)^3*tan(c)^3 - 216*a^2*b^2*d
*x*tan(d*x)^3*tan(c)^3 + 60*b^4*d*x*tan(d*x)^3*tan(c)^3 - 6*a^3*b*tan(d*x)^
5*tan(c)^3 + 30*a*b^3*tan(d*x)^5*tan(c)^3 - 9*a^4*d*x*tan(d*x)^2*tan(c)^4 +
 162*a^2*b^2*d*x*tan(d*x)^2*tan(c)^4 - 45*b^4*d*x*tan(d*x)^2*tan(c)^4 - 42*
a^3*b*tan(d*x)^4*tan(c)^4 + 30*a*b^3*tan(d*x)^4*tan(c)^4 - 6*a^3*b*tan(d*x)
^3*tan(c)^5 + 30*a*b^3*tan(d*x)^3*tan(c)^5 + 36*a^3*b*log(4*(tan(d*x)^2*tan
(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2
 + 1))*tan(d*x)^4*tan(c)^2 - 72*a*b^3*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*
x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^
4*tan(c)^2 - 36*a^2*b^2*tan(d*x)^5*tan(c)^2 + 10*b^4*tan(d*x)^5*tan(c)^2 -
48*a^3*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*ta
```

```
n(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 + 96*a*b^3*log(4*(
tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x
)^2 + tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 - 12*a^4*tan(d*x)^4*tan(c)^3 + 108
*a^2*b^2*tan(d*x)^4*tan(c)^3 - 30*b^4*tan(d*x)^4*tan(c)^3 + 36*a^3*b*log(4*
(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*
x)^2 + tan(c)^2 + 1))*tan(d*x)^2*tan(c)^4 - 72*a*b^3*log(4*(tan(d*x)^2*tan(
c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2
+ 1))*tan(d*x)^2*tan(c)^4 - 12*a^4*tan(d*x)^3*tan(c)^4 + 108*a^2*b^2*tan(d*
x)^3*tan(c)^4 - 30*b^4*tan(d*x)^3*tan(c)^4 - 36*a^2*b^2*tan(d*x)^2*tan(c)^5
 + 10*b^4*tan(d*x)^2*tan(c)^5 + 9*a^4*d*x*tan(d*x)^3*tan(c) - 162*a^2*b^2*d
*x*tan(d*x)^3*tan(c) + 45*b^4*d*x*tan(d*x)^3*tan(c) + 12*a*b^3*tan(d*x)^5*t
an(c) - 12*a^4*d*x*tan(d*x)^2*tan(c)^2 + 216*a^2*b^2*d*x*tan(d*x)^2*tan(c)^
2 - 60*b^4*d*x*tan(d*x)^2*tan(c)^2 + 18*a^3*b*tan(d*x)^4*tan(c)^2 - 42*a*b^
3*tan(d*x)^4*tan(c)^2 + 9*a^4*d*x*tan(d*x)*tan(c)^3 - 162*a^2*b^2*d*x*tan(d
*x)*tan(c)^3 + 45*b^4*d*x*tan(d*x)*tan(c)^3 + 96*a^3*b*tan(d*x)^3*tan(c)^3
- 48*a*b^3*tan(d*x)^3*tan(c)^3 + 18*a^3*b*tan(d*x)^2*tan(c)^4 - 42*a*b^3*ta
n(d*x)^2*tan(c)^4 + 12*a*b^3*tan(d*x)*tan(c)^5 - 2*b^4*tan(d*x)^5 - 36*a^3*
b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2
+ tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^3*tan(c) + 72*a*b^3*log(4*(tan(d*x)^
2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan
(c)^2 + 1))*tan(d*x)^3*tan(c) + 72*a^2*b^2*tan(d*x)^4*tan(c) - 30*b^4*tan(d
*x)^4*tan(c) + 48*a^3*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)
/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 - 9
6*a*b^3*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan
(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 + 18*a^4*tan(d*x)^3
*tan(c)^2 - 108*a^2*b^2*tan(d*x)^3*tan(c)^2 + 10*b^4*tan(d*x)^3*tan(c)^2 -
36*a^3*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*ta
n(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)*tan(c)^3 + 72*a*b^3*log(4*(ta
n(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^
2 + tan(c)^2 + 1))*tan(d*x)*tan(c)^3 + 18*a^4*tan(d*x)^2*tan(c)^3 - 108*a^2
*b^2*tan(d*x)^2*tan(c)^3 + 10*b^4*tan(d*x)^2*tan(c)^3 + 72*a^2*b^2*tan(d*x)
*tan(c)^4 - 30*b^4*tan(d*x)*tan(c)^4 - 2*b^4*tan(c)^5 - 3*a^4*d*x*tan(d*x)^
2 + 54*a^2*b^2*d*x*tan(d*x)^2 - 15*b^4*d*x*tan(d*x)^2 - 12*a*b^3*tan(d*x)^4
 + 9*a^4*d*x*tan(d*x)*tan(c) - 162*a^2*b^2*d*x*tan(d*x)*tan(c) + 45*b^4*d*x
*tan(d*x)*tan(c) - 18*a^3*b*tan(d*x)^3*tan(c) + 42*a*b^3*tan(d*x)^3*tan(c)
- 3*a^4*d*x*tan(c)^2 + 54*a^2*b^2*d*x*tan(c)^2 - 15*b^4*d*x*tan(c)^2 - 96*a
^3*b*tan(d*x)^2*tan(c)^2 + 48*a*b^3*tan(d*x)^2*tan(c)^2 - 18*a^3*b*tan(d*x)
*tan(c)^3 + 42*a*b^3*tan(d*x)*tan(c)^3 - 12*a*b^3*tan(c)^4 + 12*a^3*b*log(4
*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d
*x)^2 + tan(c)^2 + 1))*tan(d*x)^2 - 24*a*b^3*log(4*(tan(d*x)^2*tan(c)^2 - 2
*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*ta
n(d*x)^2 - 36*a^2*b^2*tan(d*x)^3 + 10*b^4*tan(d*x)^3 - 36*a^3*b*log(4*(tan(
d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2
+ tan(c)^2 + 1))*tan(d*x)*tan(c) + 72*a*b^3*log(4*(tan(d*x)^2*tan(c)^2 - 2*
tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan
```

```
(d*x)*tan(c) - 12*a^4*tan(d*x)^2*tan(c) + 108*a^2*b^2*tan(d*x)^2*tan(c) - 3
0*b^4*tan(d*x)^2*tan(c) + 12*a^3*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*
tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(c)^2 - 2
4*a*b^3*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan
(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(c)^2 - 12*a^4*tan(d*x)*tan(c)^2 + 1
08*a^2*b^2*tan(d*x)*tan(c)^2 - 30*b^4*tan(d*x)*tan(c)^2 - 36*a^2*b^2*tan(c)
^3 + 10*b^4*tan(c)^3 - 3*a^4*d*x + 54*a^2*b^2*d*x - 15*b^4*d*x + 6*a^3*b*ta
n(d*x)^2 - 30*a*b^3*tan(d*x)^2 + 42*a^3*b*tan(d*x)*tan(c) - 30*a*b^3*tan(d*
x)*tan(c) + 6*a^3*b*tan(c)^2 - 30*a*b^3*tan(c)^2 + 12*a^3*b*log(4*(tan(d*x)
^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + ta
n(c)^2 + 1)) - 24*a*b^3*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)
/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1)) + 3*a^4*tan(d*x) - 54*a
^2*b^2*tan(d*x) + 15*b^4*tan(d*x) + 3*a^4*tan(c) - 54*a^2*b^2*tan(c) + 15*b
^4*tan(c) - 6*a^3*b - 6*a*b^3)/(d*tan(d*x)^5*tan(c)^5 + d*tan(d*x)^5*tan(c)
^3 - 3*d*tan(d*x)^4*tan(c)^4 + d*tan(d*x)^3*tan(c)^5 - 3*d*tan(d*x)^4*tan(c
)^2 + 4*d*tan(d*x)^3*tan(c)^3 - 3*d*tan(d*x)^2*tan(c)^4 + 3*d*tan(d*x)^3*ta
n(c) - 4*d*tan(d*x)^2*tan(c)^2 + 3*d*tan(d*x)*tan(c)^3 - d*tan(d*x)^2 + 3*d
*tan(d*x)*tan(c) - d*tan(c)^2 - d)
```

Mupad [B] (verification not implemented)

Time = 4.98 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.16

$$\begin{aligned}\int \sin^2(c+dx)(a+b\tan(c+dx))^4\,dx &= x\left(\frac{a^4}{2}-9\,a^2\,b^2+\frac{5\,b^4}{2}\right)-\frac{\ln\left(\tan(c+d\,x)^2+1\right)\,\left(4\,a\,b^3-2\,a^3\,b\right)}{d}\\&\quad-\frac{\cos\left(c+d\,x\right)^2\left(\tan(c+d\,x)\,\left(\frac{a^4}{2}-3\,a^2\,b^2+\frac{b^4}{2}\right)+2\,a\,b^3-2\,a^3\,b\right)}{d}\\&\quad+\frac{b^4\tan(c+d\,x)^3}{3\,d}-\frac{\tan(c+d\,x)\,\left(2\,b^4-6\,a^2\,b^2\right)}{d}+\frac{2\,a\,b^3\tan(c+d\,x)^2}{d}\end{aligned}$$

```
[In] int(sin(c + d*x)^2*(a + b*tan(c + d*x))^4,x)
```

```
[Out] x*(a^4/2 + (5*b^4)/2 - 9*a^2*b^2) - (log(tan(c + d*x)^2 + 1)*(4*a*b^3 - 2*a
^3*b))/d - (cos(c + d*x)^2*(tan(c + d*x)*(a^4/2 + b^4/2 - 3*a^2*b^2) + 2*a*
b^3 - 2*a^3*b))/d + (b^4*tan(c + d*x)^3)/(3*d) - (tan(c + d*x)*(2*b^4 - 6*a
^2*b^2))/d + (2*a*b^3*tan(c + d*x)^2)/d
```

3.43 $\int \sin(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal result

Integrand size = 19, antiderivative size = 180

$$\begin{aligned}\int \sin(c+dx)(a+b\tan(c+dx))^4\,dx = {} & \frac{4a^3b\operatorname{arctanh}(\sin(c+dx))}{d} \\ & - \frac{6ab^3\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{a^4\cos(c+dx)}{d} \\ & + \frac{6a^2b^2\cos(c+dx)}{d} - \frac{b^4\cos(c+dx)}{d} \\ & + \frac{6a^2b^2\sec(c+dx)}{d} - \frac{2b^4\sec(c+dx)}{d} \\ & + \frac{b^4\sec^3(c+dx)}{3d} - \frac{4a^3b\sin(c+dx)}{d} \\ & + \frac{6ab^3\sin(c+dx)}{d} + \frac{2ab^3\sin(c+dx)\tan^2(c+dx)}{d}\end{aligned}$$

```
[Out] 4*a^3*b*arctanh(sin(d*x+c))/d-6*a*b^3*arctanh(sin(d*x+c))/d-a^4*cos(d*x+c)/
d+6*a^2*b^2*cos(d*x+c)/d-b^4*cos(d*x+c)/d+6*a^2*b^2*sec(d*x+c)/d-2*b^4*sec(
d*x+c)/d+1/3*b^4*sec(d*x+c)^3/d-4*a^3*b*sin(d*x+c)/d+6*a*b^3*sin(d*x+c)/d+2
*a*b^3*sin(d*x+c)*tan(d*x+c)^2/d
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used

$= \{3598, 2718, 2672, 327, 212, 2670, 14, 294, 276\}$

$$\int \sin(c+dx)(a+b\tan(c+dx))^4\,dx = -\frac{a^4\cos(c+dx)}{d}+\frac{4a^3b\text{arctanh}(\sin(c+dx))}{d} - \frac{4a^3b\sin(c+dx)}{d}+\frac{6a^2b^2\cos(c+dx)}{d} + \frac{6a^2b^2\sec(c+dx)}{d}-\frac{6ab^3\text{arctanh}(\sin(c+dx))}{d} + \frac{6ab^3\sin(c+dx)}{d}+\frac{2ab^3\sin(c+dx)\tan^2(c+dx)}{d} - \frac{b^4\cos(c+dx)}{d}+\frac{b^4\sec^3(c+dx)}{3d}-\frac{2b^4\sec(c+dx)}{d}$$

```
[In] Int[Sin[c + d*x]*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (4*a^3*b*ArcTanh[Sin[c + d*x]])/d - (6*a*b^3*ArcTanh[Sin[c + d*x]])/d - (a^
4*Cos[c + d*x])/d + (6*a^2*b^2*Cos[c + d*x])/d - (b^4*Cos[c + d*x])/d + (6*
a^2*b^2*Sec[c + d*x])/d - (2*b^4*Sec[c + d*x])/d + (b^4*Sec[c + d*x]^3)/(3*
d) - (4*a^3*b*Sin[c + d*x])/d + (6*a*b^3*Sin[c + d*x])/d + (2*a*b^3*Sin[c +
 d*x]*Tan[c + d*x]^2)/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^4\sin(c+dx)+4a^3b\sin(c+dx)\tan(c+dx)+6a^2b^2\sin(c+dx)\tan^2(c+dx)\right.\\
&\qquad \left.+4ab^3\sin(c+dx)\tan^3(c+dx)+b^4\sin(c+dx)\tan^4(c+dx)\right)\,dx\\
&= a^4\int\sin(c+dx)\,dx+\left(4a^3b\right)\int\sin(c+dx)\tan(c+dx)\,dx\\
&\quad+\left(6a^2b^2\right)\int\sin(c+dx)\tan^2(c+dx)\,dx\\
&\quad+\left(4ab^3\right)\int\sin(c+dx)\tan^3(c+dx)\,dx+b^4\int\sin(c+dx)\tan^4(c+dx)\,dx
\end{aligned}$$

$$= -\frac{a^4\cos(c+dx)}{d} + \frac{(4a^3b)\,\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d} - \frac{(6a^2b^2)\,\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, \cos(c+dx)\right)}{d} + \frac{(4ab^3)\,\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx, x, \sin(c+dx)\right)}{d} - \frac{b^4\text{Subst}\left(\int \frac{(1-x^2)^2}{x^4}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{a^4\cos(c+dx)}{d} - \frac{4a^3b\sin(c+dx)}{d} + \frac{2ab^3\sin(c+dx)\tan^2(c+dx)}{d} + \frac{(4a^3b)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d} - \frac{(6a^2b^2)\,\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)\,dx, x, \cos(c+dx)\right)}{d} - \frac{(6ab^3)\,\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d} - \frac{b^4\text{Subst}\left(\int \left(1+\frac{1}{x^4}-\frac{2}{x^2}\right)\,dx, x, \cos(c+dx)\right)}{d}$$

$$= \frac{4a^3b\text{arctanh}(\sin(c+dx))}{d} - \frac{a^4\cos(c+dx)}{d} + \frac{6a^2b^2\cos(c+dx)}{d} - \frac{b^4\cos(c+dx)}{d} + \frac{6a^2b^2\sec(c+dx)}{d} - \frac{2b^4\sec(c+dx)}{d} + \frac{b^4\sec^3(c+dx)}{3d} - \frac{4a^3b\sin(c+dx)}{d} + \frac{6ab^3\sin(c+dx)}{d} + \frac{2ab^3\sin(c+dx)\tan^2(c+dx)}{d} - \frac{(6ab^3)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{4a^3b\text{arctanh}(\sin(c+dx))}{d} - \frac{6ab^3\text{arctanh}(\sin(c+dx))}{d} - \frac{a^4\cos(c+dx)}{d} + \frac{6a^2b^2\cos(c+dx)}{d} - \frac{b^4\cos(c+dx)}{d} + \frac{6a^2b^2\sec(c+dx)}{d} - \frac{2b^4\sec(c+dx)}{d} + \frac{b^4\sec^3(c+dx)}{3d} - \frac{4a^3b\sin(c+dx)}{d} + \frac{6ab^3\sin(c+dx)}{d} + \frac{2ab^3\sin(c+dx)\tan^2(c+dx)}{d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 383 vs. $2(180) = 360$.

Time = 6.41 (sec) , antiderivative size = 383, normalized size of antiderivative = 2.13

$$\int \sin(c+dx)(a+b\tan(c+dx))^4\,dx$$
$$= \frac{72a^2b^2 - 22b^4 - 12(a^4 - 6a^2b^2 + b^4)\cos(c+dx) - 24ab(2a^2 - 3b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right)}{}$$

```
[In] Integrate[Sin[c + d*x]*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (72*a^2*b^2 - 22*b^4 - 12*(a^4 - 6*a^2*b^2 + b^4)*Cos[c + d*x] - 24*a*b*(2*
a^2 - 3*b^2)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 24*a*b*(2*a^2 - 3*b
^2)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (b^3*(12*a + b))/(Cos[(c + d
*x)/2] - Sin[(c + d*x)/2])^2 + (2*b^4*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2])^3 + (2*b^2*(36*a^2 - 11*b^2)*Sin[(c + d*x)/2])/(Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2]) - (2*b^4*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2])^3 + (b^3*(-12*a + b))/(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])^2 + (2*b^2*(-36*a^2 + 11*b^2)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]) - 48*a*b*(a^2 - b^2)*Sin[c + d*x])/(12*d)
```

Maple [A] (verified)

Time = 3.04 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{-a^4\cos(dx+c)+4a^3b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+6a^2b^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+4a\,b^3}{}$
default	$\frac{-a^4\cos(dx+c)+4a^3b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+6a^2b^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+4a\,b^3}{}$
risch	$\frac{2ie^{i(dx+c)}a^3b}{d} - \frac{2ie^{i(dx+c)}a\,b^3}{d} - \frac{e^{i(dx+c)}a^4}{2d} + \frac{3\,e^{i(dx+c)}a^2b^2}{d} - \frac{e^{i(dx+c)}b^4}{2d} - \frac{2ie^{-i(dx+c)}a^3b}{d} + \frac{2ie^{-i(dx+c)}a\,b}{d}$

```
[In] int(sin(d*x+c)*(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^4*cos(d*x+c)+4*a^3*b*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+6*a^2*
b^2*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+4*a*b^3*(1/2*sin(
d*x+c)^5/cos(d*x+c)^2+1/2*sin(d*x+c)^3+3/2*sin(d*x+c)-3/2*ln(sec(d*x+c)+tan
(d*x+c)))+b^4*(1/3*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+s
in(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.98

$$\int \sin(c+dx)(a+b\tan(c+dx))^4\,dx =$$
$$-\frac{3\,(a^4-6\,a^2b^2+b^4)\cos{(dx+c)}^4-3\,(2\,a^3b-3\,ab^3)\cos{(dx+c)}^3\log{(\sin{(dx+c)}+1)}+3\,(2\,a^3b-3\,ab}{}$$

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*(a^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^4 - 3*(2*a^3*b - 3*a*b^3)*cos(
d*x + c)^3*log(sin(d*x + c) + 1) + 3*(2*a^3*b - 3*a*b^3)*cos(d*x + c)^3*log
(-sin(d*x + c) + 1) - b^4 - 6*(3*a^2*b^2 - b^4)*cos(d*x + c)^2 - 6*(a*b^3*c
os(d*x + c) - 2*(a^3*b - a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x +
c)^3)
```

Sympy [F]

$$\int \sin(c+dx)(a+b\tan(c+dx))^4\,dx = \int (a+b\tan{(c+dx)})^4\sin{(c+dx)}\,dx$$

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**4*sin(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.92

$$\int \sin(c+dx)(a+b\tan(c+dx))^4\,dx =$$
$$-\frac{3\,ab^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+3\,\log{(\sin{(dx+c)}+1)}-3\,\log{(\sin{(dx+c)}-1)}-4\,\sin{(dx+c)}\right)-18\,a^2b^2\left(\frac{1}{\cos(d}\right.}{}$$

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*a*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(d*x + c) + 1
) - 3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)) - 18*a^2*b^2*(1/cos(d*x + c)
+ cos(d*x + c)) + b^4*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x +
c)) - 6*a^3*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x +
c)) + 3*a^4*cos(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19074 vs. 2(178) = 356.

Time = 11.22 (sec) , antiderivative size = 19074, normalized size of antiderivative = 105.97

$$\int \sin(c+dx)(a+b\tan(c+dx))^4\,dx = \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(6*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2
*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1
/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 +
 tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 - 9*a*b^3*log(2*(tan(1/2*d*
x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)
^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(ta
n(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x
)^8*tan(1/2*c)^8 - 6*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d
*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c
)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan
(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 + 9*a*b^3*log(
2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*
x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/
2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1)
)*tan(1/2*d*x)^8*tan(1/2*c)^8 + 3*a^4*tan(1/2*d*x)^8*tan(1/2*c)^8 - 36*a^2*
b^2*tan(1/2*d*x)^8*tan(1/2*c)^8 + 8*b^4*tan(1/2*d*x)^8*tan(1/2*c)^8 - 12*a^
3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*ta
n(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) -
2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c
)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^6 + 18*a*b^3*log(2*(tan(1/2*d*x)^2*tan(
1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(
1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x
)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8*tan(1
/2*c)^6 + 12*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*ta
n(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*
tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x
)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^6 - 18*a*b^3*log(2*(tan(
1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(
1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) +
1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1
/2*d*x)^8*tan(1/2*c)^6 - 72*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + ta
n(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^
2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^7*tan(1/2*c)^7 + 108*a
*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*t
```

an(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^7*tan(1/2*c)^7 + 72*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^7*tan(1/2*c)^7 - 108*a*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^7*tan(1/2*c)^7 - 24*a^3*b*tan(1/2*d*x)^8*tan(1/2*c)^7 + 36*a*b^3*tan(1/2*d*x)^8*tan(1/2*c)^7 - 12*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^8 + 18*a*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^8 + 12*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^8 - 18*a*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^8 - 24*a^3*b*tan(1/2*d*x)^7*tan(1/2*c)^8 + 36*a*b^3*tan(1/2*d*x)^7*tan(1/2*c)^8 - 12*a^4*tan(1/2*d*x)^8*tan(1/2*c)^6 + 72*a^2*b^2*tan(1/2*d*x)^8*tan(1/2*c)^6 - 16*b^4*tan(1/2*d*x)^8*tan(1/2*c)^6 - 48*a^4*tan(1/2*d*x)^7*tan(1/2*c)^7 + 432*a^2*b^2*tan(1/2*d*x)^7*tan(1/2*c)^7 - 96*b^4*tan(1/2*d*x)^7*tan(1/2*c)^7 - 12*a^4*tan(1/2*d*x)^6*tan(1/2*c)^8 + 72*a^2*b^2*tan(1/2*d*x)^6*tan(1/2*c)^8 - 16*b^4*tan(1/2*d*x)^6*tan(1/2*c)^8 + 72*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^7*tan(1/2*c)^5 - 108*a*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^7*tan(1/2*c)^5 - 72*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^7*tan(1/2*c)^5 + 108*a*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d

*x)^7*tan(1/2*c)^5 + 72*a^3*b*tan(1/2*d*x)^8*tan(1/2*c)^5 - 60*a*b^3*tan(1/
2*d*x)^8*tan(1/2*c)^5 + 312*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + ta
n(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^
2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 - 468*a
*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*t
an(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) -
 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*
c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 - 312*a^3*b*log(2*(tan(1/2*d*x)^2*ta
n(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + ta
n(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d
*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan
(1/2*c)^6 + 468*a*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2
*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 +
 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*
d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 + 384*a^3*b*tan(1/2
*d*x)^7*tan(1/2*c)^6 - 432*a*b^3*tan(1/2*d*x)^7*tan(1/2*c)^6 + 72*a^3*b*log
(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d
*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1
/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1
))*tan(1/2*d*x)^5*tan(1/2*c)^7 - 108*a*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)
^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*
x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*ta
n(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^5*tan(1/2*c)^
7 - 72*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*
c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/
2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 1))*tan(1/2*d*x)^5*tan(1/2*c)^7 + 108*a*b^3*log(2*(tan(1/2*d
*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c
)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(t
an(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*
x)^5*tan(1/2*c)^7 + 384*a^3*b*tan(1/2*d*x)^6*tan(1/2*c)^7 - 432*a*b^3*tan(1
/2*d*x)^6*tan(1/2*c)^7 + 72*a^3*b*tan(1/2*d*x)^5*tan(1/2*c)^8 - 60*a*b^3*ta
n(1/2*d*x)^5*tan(1/2*c)^8 + 18*a^4*tan(1/2*d*x)^8*tan(1/2*c)^4 - 72*a^2*b^2
*tan(1/2*d*x)^8*tan(1/2*c)^4 + 144*a^4*tan(1/2*d*x)^7*tan(1/2*c)^5 - 720*a^
2*b^2*tan(1/2*d*x)^7*tan(1/2*c)^5 + 96*b^4*tan(1/2*d*x)^7*tan(1/2*c)^5 + 33
6*a^4*tan(1/2*d*x)^6*tan(1/2*c)^6 - 2304*a^2*b^2*tan(1/2*d*x)^6*tan(1/2*c)^
6 + 416*b^4*tan(1/2*d*x)^6*tan(1/2*c)^6 + 144*a^4*tan(1/2*d*x)^5*tan(1/2*c)
^7 - 720*a^2*b^2*tan(1/2*d*x)^5*tan(1/2*c)^7 + 96*b^4*tan(1/2*d*x)^5*tan(1/
2*c)^7 + 18*a^4*tan(1/2*d*x)^4*tan(1/2*c)^8 - 72*a^2*b^2*tan(1/2*d*x)^4*tan
(1/2*c)^8 + 12*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*
tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 -
2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d
*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^2 - 18*a*b^3*log(2*(ta
n(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*ta

```
n(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c)
+ 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan
(1/2*d*x)^8*tan(1/2*c)^2 - 12*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*
tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c
)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^2 + 18*
a*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*
tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x)
+ 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2
*c)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^2 + 72*a^3*b*log(2*(tan(1/2*d*x)^2*ta
n(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + ta
n(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d
*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^7*tan
(1/2*c)^3 - 108*a*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2
*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 -
 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*
d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^7*tan(1/2*c)^3 - 72*a^3*b*log(2*(t
an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*t
an(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c)
 + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*ta
n(1/2*d*x)^7*tan(1/2*c)^3 + 108*a*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 -
2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2
+ tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2
*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^7*tan(1/2*c)^3 - 7
2*a^3*b*tan(1/2*d*x)^8*tan(1/2*c)^3 + 60*a*b^3*tan(1/2*d*x)^8*tan(1/2*c)^3
- 720*a^3*b*tan(1/2*d*x)^7*tan(1/2*c)^4 + 600*a*b^3*tan(1/2*d*x)^7*tan(1/2*
c)^4 - 456*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(
1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*ta
n(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^
2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^5*tan(1/2*c)^5 + 684*a*b^3*log(2*(tan(1
/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1
/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1
)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/
2*d*x)^5*tan(1/2*c)^5 + 456*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*ta
n(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + ta
n(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^
2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^5*tan(1/2*c)^5 - 684*a
*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*t
an(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) +
 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*
c)^2 + 1))*tan(1/2*d*x)^5*tan(1/2*c)^5 - 2304*a^3*b*tan(1/2*d*x)^6*tan(1/2*
c)^5 + 2256*a*b^3*tan(1/2*d*x)^6*tan(1/2*c)^5 - 2304*a^3*b*tan(1/2*d*x)^5*t
an(1/2*c)^6 + 2256*a*b^3*tan(1/2*d*x)^5*tan(1/2*c)^6 + 72*a^3*b*log(2*(tan(
1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(
1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) +
```

1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1
/2*d*x)^3*tan(1/2*c)^7 - 108*a*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*t
an(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + t
an(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)
^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c)^7 - 72*a
^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*t
an(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) +
 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*
c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c)^7 + 108*a*b^3*log(2*(tan(1/2*d*x)^2*ta
n(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + ta
n(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d
*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan
(1/2*c)^7 - 720*a^3*b*tan(1/2*d*x)^4*tan(1/2*c)^7 + 600*a*b^3*tan(1/2*d*x)^
4*tan(1/2*c)^7 + 12*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*
x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)
^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(
1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^8 - 18*a*b^3*log(
2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*
x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/
2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1)
)*tan(1/2*d*x)^2*tan(1/2*c)^8 - 12*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2
 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)
^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(
1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^8
+ 18*a*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c)
 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*
d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + ta
n(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^8 - 72*a^3*b*tan(1/2*d*x)^3*tan(
1/2*c)^8 + 60*a*b^3*tan(1/2*d*x)^3*tan(1/2*c)^8 - 12*a^4*tan(1/2*d*x)^8*tan
(1/2*c)^2 + 72*a^2*b^2*tan(1/2*d*x)^8*tan(1/2*c)^2 - 16*b^4*tan(1/2*d*x)^8*
tan(1/2*c)^2 - 144*a^4*tan(1/2*d*x)^7*tan(1/2*c)^3 + 720*a^2*b^2*tan(1/2*d*
x)^7*tan(1/2*c)^3 - 96*b^4*tan(1/2*d*x)^7*tan(1/2*c)^3 - 648*a^4*tan(1/2*d*
x)^6*tan(1/2*c)^4 + 3312*a^2*b^2*tan(1/2*d*x)^6*tan(1/2*c)^4 - 480*b^4*tan(
1/2*d*x)^6*tan(1/2*c)^4 - 1200*a^4*tan(1/2*d*x)^5*tan(1/2*c)^5 + 7344*a^2*b
^2*tan(1/2*d*x)^5*tan(1/2*c)^5 - 1248*b^4*tan(1/2*d*x)^5*tan(1/2*c)^5 - 648
*a^4*tan(1/2*d*x)^4*tan(1/2*c)^6 + 3312*a^2*b^2*tan(1/2*d*x)^4*tan(1/2*c)^6
 - 480*b^4*tan(1/2*d*x)^4*tan(1/2*c)^6 - 144*a^4*tan(1/2*d*x)^3*tan(1/2*c)^
7 + 720*a^2*b^2*tan(1/2*d*x)^3*tan(1/2*c)^7 - 96*b^4*tan(1/2*d*x)^3*tan(1/2
*c)^7 - 12*a^4*tan(1/2*d*x)^2*tan(1/2*c)^8 + 72*a^2*b^2*tan(1/2*d*x)^2*tan(
1/2*c)^8 - 16*b^4*tan(1/2*d*x)^2*tan(1/2*c)^8 - 6*a^3*b*log(2*(tan(1/2*d*x)
^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2
 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(
1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^
8 + 9*a*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c
) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2

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*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + t
an(1/2*c)^2 + 1))*tan(1/2*d*x)^8 + 6*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)
^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*
x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*ta
n(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8 - 9*a*b^3*l
og(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2
*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan
(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 +
 1))*tan(1/2*d*x)^8 - 72*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1
/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1
/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 +
 tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^7*tan(1/2*c) + 108*a*b^3*
log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/
2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*ta
n(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2
+ 1))*tan(1/2*d*x)^7*tan(1/2*c) + 72*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)
^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*
x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*ta
n(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^7*tan(1/2*c)
- 108*a*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c
) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2
*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + t
an(1/2*c)^2 + 1))*tan(1/2*d*x)^7*tan(1/2*c) + 24*a^3*b*tan(1/2*d*x)^8*tan(1
/2*c) - 36*a*b^3*tan(1/2*d*x)^8*tan(1/2*c) - 312*a^3*b*log(2*(tan(1/2*d*x)^
2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2
+ tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1
/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6
*tan(1/2*c)^2 + 468*a*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*
x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)
^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(
1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 + 312*a^3*b*log
(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d
*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1
/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1
))*tan(1/2*d*x)^6*tan(1/2*c)^2 - 468*a*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)
^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*
x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*ta
n(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^
2 + 384*a^3*b*tan(1/2*d*x)^7*tan(1/2*c)^2 - 432*a*b^3*tan(1/2*d*x)^7*tan(1/
2*c)^2 - 456*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*ta
n(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*
tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x
)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^5*tan(1/2*c)^3 + 684*a*b^3*log(2*(tan
(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan
(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) +
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1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(
1/2*d*x)^5*tan(1/2*c)^3 + 456*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*
tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c
)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^5*tan(1/2*c)^3 - 684
*a*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2
*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x)
 + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/
2*c)^2 + 1))*tan(1/2*d*x)^5*tan(1/2*c)^3 + 2304*a^3*b*tan(1/2*d*x)^6*tan(1/
2*c)^3 - 2256*a*b^3*tan(1/2*d*x)^6*tan(1/2*c)^3 + 6000*a^3*b*tan(1/2*d*x)^5
*tan(1/2*c)^4 - 6120*a*b^3*tan(1/2*d*x)^5*tan(1/2*c)^4 - 456*a^3*b*log(2*(t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*t
an(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c)
 + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*ta
n(1/2*d*x)^3*tan(1/2*c)^5 + 684*a*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 +
2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2
+ tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2
*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c)^5 + 4
56*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) -
 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*
x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(
1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c)^5 - 684*a*b^3*log(2*(tan(1/2*d*x)^
2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2
+ tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1
/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3
*tan(1/2*c)^5 + 6000*a^3*b*tan(1/2*d*x)^4*tan(1/2*c)^5 - 6120*a*b^3*tan(1/2
*d*x)^4*tan(1/2*c)^5 - 312*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan
(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan
(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2
 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 + 468*a*
b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*ta
n(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) -
2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c
)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 + 312*a^3*b*log(2*(tan(1/2*d*x)^2*tan
(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan
(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*
x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(
1/2*c)^6 - 468*a*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*
tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 +
2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d
*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 + 2304*a^3*b*tan(1/2
*d*x)^3*tan(1/2*c)^6 - 2256*a*b^3*tan(1/2*d*x)^3*tan(1/2*c)^6 - 72*a^3*b*lo
g(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*
d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(
1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 +

1))*tan(1/2*d*x)*tan(1/2*c)^7 + 108*a*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^
2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x
)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan
(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)*tan(1/2*c)^7 +
72*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c)
- 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d
*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan
(1/2*c)^2 + 1))*tan(1/2*d*x)*tan(1/2*c)^7 - 108*a*b^3*log(2*(tan(1/2*d*x)^2
*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 +
tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/
2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)*ta
n(1/2*c)^7 + 384*a^3*b*tan(1/2*d*x)^2*tan(1/2*c)^7 - 432*a*b^3*tan(1/2*d*x)
^2*tan(1/2*c)^7 - 6*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*
x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)
^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(
1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^8 + 9*a*b^3*log(2*(tan(1/2*d*x)^
2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2
+ tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1
/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^8 +
6*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) -
2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*
x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(
1/2*c)^2 + 1))*tan(1/2*c)^8 - 9*a*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 -
2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2
+ tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2
*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^8 + 24*a^3*b*tan(1/2
*d*x)*tan(1/2*c)^8 - 36*a*b^3*tan(1/2*d*x)*tan(1/2*c)^8 + 3*a^4*tan(1/2*d*x
)^8 - 36*a^2*b^2*tan(1/2*d*x)^8 + 8*b^4*tan(1/2*d*x)^8 + 48*a^4*tan(1/2*d*x
)^7*tan(1/2*c) - 432*a^2*b^2*tan(1/2*d*x)^7*tan(1/2*c) + 96*b^4*tan(1/2*d*x
)^7*tan(1/2*c) + 336*a^4*tan(1/2*d*x)^6*tan(1/2*c)^2 - 2304*a^2*b^2*tan(1/2
*d*x)^6*tan(1/2*c)^2 + 416*b^4*tan(1/2*d*x)^6*tan(1/2*c)^2 + 1200*a^4*tan(1
/2*d*x)^5*tan(1/2*c)^3 - 7344*a^2*b^2*tan(1/2*d*x)^5*tan(1/2*c)^3 + 1248*b^
4*tan(1/2*d*x)^5*tan(1/2*c)^3 + 2028*a^4*tan(1/2*d*x)^4*tan(1/2*c)^4 - 1339
2*a^2*b^2*tan(1/2*d*x)^4*tan(1/2*c)^4 + 2400*b^4*tan(1/2*d*x)^4*tan(1/2*c)^
4 + 1200*a^4*tan(1/2*d*x)^3*tan(1/2*c)^5 - 7344*a^2*b^2*tan(1/2*d*x)^3*tan(
1/2*c)^5 + 1248*b^4*tan(1/2*d*x)^3*tan(1/2*c)^5 + 336*a^4*tan(1/2*d*x)^2*ta
n(1/2*c)^6 - 2304*a^2*b^2*tan(1/2*d*x)^2*tan(1/2*c)^6 + 416*b^4*tan(1/2*d*x
)^2*tan(1/2*c)^6 + 48*a^4*tan(1/2*d*x)*tan(1/2*c)^7 - 432*a^2*b^2*tan(1/2*d
*x)*tan(1/2*c)^7 + 96*b^4*tan(1/2*d*x)*tan(1/2*c)^7 + 3*a^4*tan(1/2*c)^8 -
36*a^2*b^2*tan(1/2*c)^8 + 8*b^4*tan(1/2*c)^8 + 12*a^3*b*log(2*(tan(1/2*d*x)
^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2
+ tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(
1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^
6 - 18*a*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*
c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/

2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6 - 12*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6 + 18*a*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6 - 24*a^3*b*tan(1/2*d*x)^7 + 36*a*b^3*tan(1/2*d*x)^7 + 72*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^5*tan(1/2*c) - 108*a*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^5*tan(1/2*c) - 72*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^5*tan(1/2*c) + 108*a*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^5*tan(1/2*c) - 384*a^3*b*tan(1/2*d*x)^6*tan(1/2*c) + 432*a*b^3*tan(1/2*d*x)^6*tan(1/2*c) - 2304*a^3*b*tan(1/2*d*x)^5*tan(1/2*c)^2 + 2256*a*b^3*tan(1/2*d*x)^5*tan(1/2*c)^2 - 456*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c)^3 + 684*a*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c)^3 + 456*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c)^3 - 684*a*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c)^3 - 6000*a^3*b*tan(1/2*d*x)^4*tan(1/2*c)^3 + 6120*a*b^3*tan(1/2*d*x)^4*tan(1/2*c)^3 - 6000*a^3*b*tan(1/2*d*x)^3*tan(1/2*c)^4 + 6120*a*b^3*tan(1/2*d*x)^3*tan(1/2*c)^4 + 72*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2

+ tan(1/2*c)^2 + 1))*tan(1/2*d*x)*tan(1/2*c)^5 - 108*a*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)*tan(1/2*c)^5 - 72*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)*tan(1/2*c)^5 + 108*a*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)*tan(1/2*c)^5 - 2304*a^3*b*tan(1/2*d*x)^2*tan(1/2*c)^5 + 2256*a*b^3*tan(1/2*d*x)^2*tan(1/2*c)^5 + 12*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^6 - 18*a*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^6 - 12*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^6 + 18*a*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^6 - 384*a^3*b*tan(1/2*d*x)*tan(1/2*c)^6 + 432*a*b^3*tan(1/2*d*x)*tan(1/2*c)^6 - 24*a^3*b*tan(1/2*c)^7 + 36*a*b^3*tan(1/2*c)^7 - 12*a^4*tan(1/2*d*x)^6 + 72*a^2*b^2*tan(1/2*d*x)^6 - 16*b^4*tan(1/2*d*x)^6 - 144*a^4*tan(1/2*d*x)^5*tan(1/2*c) + 720*a^2*b^2*tan(1/2*d*x)^5*tan(1/2*c) - 96*b^4*tan(1/2*d*x)^5*tan(1/2*c) - 648*a^4*tan(1/2*d*x)^4*tan(1/2*c)^2 + 3312*a^2*b^2*tan(1/2*d*x)^4*tan(1/2*c)^2 - 480*b^4*tan(1/2*d*x)^4*tan(1/2*c)^2 - 1200*a^4*tan(1/2*d*x)^3*tan(1/2*c)^3 + 7344*a^2*b^2*tan(1/2*d*x)^3*tan(1/2*c)^3 - 1248*b^4*tan(1/2*d*x)^3*tan(1/2*c)^3 - 648*a^4*tan(1/2*d*x)^2*tan(1/2*c)^4 + 3312*a^2*b^2*tan(1/2*d*x)^2*tan(1/2*c)^4 - 480*b^4*tan(1/2*d*x)^2*tan(1/2*c)^4 - 144*a^4*tan(1/2*d*x)*tan(1/2*c)^5 + 720*a^2*b^2*tan(1/2*d*x)*tan(1/2*c)^5 - 96*b^4*tan(1/2*d*x)*tan(1/2*c)^5 - 12*a^4*tan(1/2*c)^6 + 72*a^2*b^2*tan(1/2*c)^6 - 16*b^4*tan(1/2*c)^6 + 72*a^3*b*tan(1/2*d*x)^5 - 60*a*b^3*tan(1/2*d*x)^5 + 72*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c) - 108*a*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1

/2*d*x)^3*tan(1/2*c) - 72*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(
1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(
1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2
+ tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c) + 108*a*b^3
log(2(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1
/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*t
an(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2
 + 1))*tan(1/2*d*x)^3*tan(1/2*c) + 720*a^3*b*tan(1/2*d*x)^4*tan(1/2*c) - 60
0*a*b^3*tan(1/2*d*x)^4*tan(1/2*c) + 312*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2
*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2
*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2
*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*
c)^2 - 468*a*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(
1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*ta
n(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^
2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 312*a^3*b*log(2*(tan(1
/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1
/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1
)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/
2*d*x)^2*tan(1/2*c)^2 + 468*a*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*ta
n(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + ta
n(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^
2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2304*
a^3*b*tan(1/2*d*x)^3*tan(1/2*c)^2 - 2256*a*b^3*tan(1/2*d*x)^3*tan(1/2*c)^2
+ 72*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c)
 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*
d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + ta
n(1/2*c)^2 + 1))*tan(1/2*d*x)*tan(1/2*c)^3 - 108*a*b^3*log(2*(tan(1/2*d*x)^
2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2
+ tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1
/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)*t
an(1/2*c)^3 - 72*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^
2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2
+ 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2
*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)*tan(1/2*c)^3 + 108*a*b^3*log(2*(t
an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*t
an(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c)
 + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*ta
n(1/2*d*x)*tan(1/2*c)^3 + 2304*a^3*b*tan(1/2*d*x)^2*tan(1/2*c)^3 - 2256*a*b
^3*tan(1/2*d*x)^2*tan(1/2*c)^3 + 720*a^3*b*tan(1/2*d*x)*tan(1/2*c)^4 - 600*
a*b^3*tan(1/2*d*x)*tan(1/2*c)^4 + 72*a^3*b*tan(1/2*c)^5 - 60*a*b^3*tan(1/2*
c)^5 + 18*a^4*tan(1/2*d*x)^4 - 72*a^2*b^2*tan(1/2*d*x)^4 + 144*a^4*tan(1/2*
d*x)^3*tan(1/2*c) - 720*a^2*b^2*tan(1/2*d*x)^3*tan(1/2*c) + 96*b^4*tan(1/2*
d*x)^3*tan(1/2*c) + 336*a^4*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2304*a^2*b^2*tan(
1/2*d*x)^2*tan(1/2*c)^2 + 416*b^4*tan(1/2*d*x)^2*tan(1/2*c)^2 + 144*a^4*tan

(1/2*d*x)*tan(1/2*c)^3 - 720*a^2*b^2*tan(1/2*d*x)*tan(1/2*c)^3 + 96*b^4*tan
(1/2*d*x)*tan(1/2*c)^3 + 18*a^4*tan(1/2*c)^4 - 72*a^2*b^2*tan(1/2*c)^4 - 12
*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2
*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x)
 - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/
2*c)^2 + 1))*tan(1/2*d*x)^2 + 18*a*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 +
 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2
 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/
2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2 + 12*a^3*b*log(
2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*
x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/
2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1)
)*tan(1/2*d*x)^2 - 18*a*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*
d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*
c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + ta
n(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2 - 72*a^3*b*tan(1/2*d*x)^3
+ 60*a*b^3*tan(1/2*d*x)^3 - 72*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2
*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 +
 tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*
c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)*tan(1/2*c) + 108*a*
b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*ta
n(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) -
2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c
)^2 + 1))*tan(1/2*d*x)*tan(1/2*c) + 72*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*
c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*
d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*
tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)*tan(1/2*c)
- 108*a*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c
) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2
*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + t
an(1/2*c)^2 + 1))*tan(1/2*d*x)*tan(1/2*c) - 384*a^3*b*tan(1/2*d*x)^2*tan(1/
2*c) + 432*a*b^3*tan(1/2*d*x)^2*tan(1/2*c) - 12*a^3*b*log(2*(tan(1/2*d*x)^2
*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 +
 tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/
2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^2 +
18*a*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) +
 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*
x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(
1/2*c)^2 + 1))*tan(1/2*c)^2 + 12*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 -
 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2
 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/
2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^2 - 18*a*b^3*log(2*
(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)
*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*
c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*

```
tan(1/2*c)^2 - 384*a^3*b*tan(1/2*d*x)*tan(1/2*c)^2 + 432*a*b^3*tan(1/2*d*x)
*tan(1/2*c)^2 - 72*a^3*b*tan(1/2*c)^3 + 60*a*b^3*tan(1/2*c)^3 - 12*a^4*tan(
1/2*d*x)^2 + 72*a^2*b^2*tan(1/2*d*x)^2 - 16*b^4*tan(1/2*d*x)^2 - 48*a^4*tan
(1/2*d*x)*tan(1/2*c) + 432*a^2*b^2*tan(1/2*d*x)*tan(1/2*c) - 96*b^4*tan(1/2
*d*x)*tan(1/2*c) - 12*a^4*tan(1/2*c)^2 + 72*a^2*b^2*tan(1/2*c)^2 - 16*b^4*t
an(1/2*c)^2 + 6*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2
*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 -
 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*
d*x)^2 + tan(1/2*c)^2 + 1)) - 9*a*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 +
2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2
+ tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2
*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1)) - 6*a^3*b*log(2*(tan(1/2*d*x)^2
*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 +
 tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/
2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1)) + 9*a*b^3*log(2
*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x
)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2
*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))
 + 24*a^3*b*tan(1/2*d*x) - 36*a*b^3*tan(1/2*d*x) + 24*a^3*b*tan(1/2*c) - 36
*a*b^3*tan(1/2*c) + 3*a^4 - 36*a^2*b^2 + 8*b^4)/(d*tan(1/2*d*x)^8*tan(1/2*c
)^8 - 2*d*tan(1/2*d*x)^8*tan(1/2*c)^6 - 12*d*tan(1/2*d*x)^7*tan(1/2*c)^7 -
2*d*tan(1/2*d*x)^6*tan(1/2*c)^8 + 12*d*tan(1/2*d*x)^7*tan(1/2*c)^5 + 52*d*t
an(1/2*d*x)^6*tan(1/2*c)^6 + 12*d*tan(1/2*d*x)^5*tan(1/2*c)^7 + 2*d*tan(1/2
*d*x)^8*tan(1/2*c)^2 + 12*d*tan(1/2*d*x)^7*tan(1/2*c)^3 - 76*d*tan(1/2*d*x)
^5*tan(1/2*c)^5 + 12*d*tan(1/2*d*x)^3*tan(1/2*c)^7 + 2*d*tan(1/2*d*x)^2*tan
(1/2*c)^8 - d*tan(1/2*d*x)^8 - 12*d*tan(1/2*d*x)^7*tan(1/2*c) - 52*d*tan(1/
2*d*x)^6*tan(1/2*c)^2 - 76*d*tan(1/2*d*x)^5*tan(1/2*c)^3 - 76*d*tan(1/2*d*x
)^3*tan(1/2*c)^5 - 52*d*tan(1/2*d*x)^2*tan(1/2*c)^6 - 12*d*tan(1/2*d*x)*tan
(1/2*c)^7 - d*tan(1/2*c)^8 + 2*d*tan(1/2*d*x)^6 + 12*d*tan(1/2*d*x)^5*tan(1
/2*c) - 76*d*tan(1/2*d*x)^3*tan(1/2*c)^3 + 12*d*tan(1/2*d*x)*tan(1/2*c)^5 +
 2*d*tan(1/2*c)^6 + 12*d*tan(1/2*d*x)^3*tan(1/2*c) + 52*d*tan(1/2*d*x)^2*ta
n(1/2*c)^2 + 12*d*tan(1/2*d*x)*tan(1/2*c)^3 - 2*d*tan(1/2*d*x)^2 - 12*d*tan
(1/2*d*x)*tan(1/2*c) - 2*d*tan(1/2*c)^2 + d)
```

Mupad [B] (verification not implemented)

Time = 8.15 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.49

$$\int \sin(c+dx)(a+b\tan(c+dx))^4\,dx =$$

$$-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(6\,a^4-48\,a^2\,b^2+\frac{32\,b^4}{3}\right)+2\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(12\,a\,b^3-8\,a^3\,b\right)-2\,a^4-\frac{16\,b^4}{3}}{d\,\left(\tan\left(\frac{c}{2}\right.\right.}$$

$$-\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,\left(12\,a\,b^3-8\,a^3\,b\right)}{d}$$

```
[In] int(sin(c + d*x)*(a + b*tan(c + d*x))^4,x)

[Out] - (tan(c/2 + (d*x)/2)^2*(6*a^4 + (32*b^4)/3 - 48*a^2*b^2) + 2*a^4*tan(c/2 +
 (d*x)/2)^6 + tan(c/2 + (d*x)/2)*(12*a*b^3 - 8*a^3*b) - 2*a^4 - (16*b^4)/3
+ 24*a^2*b^2 - tan(c/2 + (d*x)/2)^4*(6*a^4 - 24*a^2*b^2) - tan(c/2 + (d*x)/
2)^7*(12*a*b^3 - 8*a^3*b) - tan(c/2 + (d*x)/2)^3*(20*a*b^3 - 24*a^3*b) + ta
n(c/2 + (d*x)/2)^5*(20*a*b^3 - 24*a^3*b))/(d*(2*tan(c/2 + (d*x)/2)^2 - 2*ta
n(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 - 1)) - (atanh(tan(c/2 + (d*x)/2)
)*(12*a*b^3 - 8*a^3*b))/d
```

3.44 $\int \csc(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal result

Integrand size = 19, antiderivative size = 118

$$\int \csc(c+dx)(a+b\tan(c+dx))^4\,dx = -\frac{a^4\operatorname{arctanh}(\cos(c+dx))}{d} + \frac{4a^3b\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{2ab^3\operatorname{arctanh}(\sin(c+dx))}{d} + \frac{6a^2b^2\sec(c+dx)}{d} - \frac{b^4\sec(c+dx)}{d} + \frac{b^4\sec^3(c+dx)}{3d} + \frac{2ab^3\sec(c+dx)\tan(c+dx)}{d}$$

```
[Out] -a^4*arctanh(cos(d*x+c))/d+4*a^3*b*arctanh(sin(d*x+c))/d-2*a*b^3*arctanh(si
n(d*x+c))/d+6*a^2*b^2*sec(d*x+c)/d-b^4*sec(d*x+c)/d+1/3*b^4*sec(d*x+c)^3/d+
2*a*b^3*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used

$= \{3598, 3855, 2686, 8, 2691\}$

$$\int \csc(c+dx)(a+b\tan(c+dx))^4\,dx = -\frac{a^4\operatorname{arctanh}(\cos(c+dx))}{d} + \frac{4a^3b\operatorname{arctanh}(\sin(c+dx))}{d} + \frac{6a^2b^2\sec(c+dx)}{d} - \frac{2ab^3\operatorname{arctanh}(\sin(c+dx))}{d} + \frac{2ab^3\tan(c+dx)\sec(c+dx)}{d} + \frac{b^4\sec^3(c+dx)}{3d} - \frac{b^4\sec(c+dx)}{d}$$

```
[In] Int[Csc[c + d*x]*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -((a^4*ArcTanh[Cos[c + d*x]])/d) + (4*a^3*b*ArcTanh[Sin[c + d*x]])/d - (2*a
*b^3*ArcTanh[Sin[c + d*x]])/d + (6*a^2*b^2*Sec[c + d*x])/d - (b^4*Sec[c + d
*x])/d + (b^4*Sec[c + d*x]^3)/(3*d) + (2*a*b^3*Sec[c + d*x]*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a^4\csc(c+dx) + 4a^3b\sec(c+dx) + 6a^2b^2\sec(c+dx)\tan(c+dx) \right. \\
&\qquad \left. + 4ab^3\sec(c+dx)\tan^2(c+dx) + b^4\sec(c+dx)\tan^3(c+dx)\right)\,dx \\
&= a^4\int \csc(c+dx)\,dx + \left(4a^3b\right)\int \sec(c+dx)\,dx + \left(6a^2b^2\right)\int \sec(c+dx)\tan(c+dx)\,dx \\
&\quad + \left(4ab^3\right)\int \sec(c+dx)\tan^2(c+dx)\,dx + b^4\int \sec(c+dx)\tan^3(c+dx)\,dx \\
&= -\frac{a^4\text{arctanh}(\cos(c+dx))}{d} + \frac{4a^3b\text{arctanh}(\sin(c+dx))}{d} \\
&\quad + \frac{2ab^3\sec(c+dx)\tan(c+dx)}{d} - \left(2ab^3\right)\int \sec(c+dx)\,dx \\
&\quad + \frac{\left(6a^2b^2\right)\text{Subst}\left(\int 1\,dx, x, \sec(c+dx)\right)}{d} + \frac{b^4\text{Subst}\left(\int \left(-1+x^2\right)\,dx, x, \sec(c+dx)\right)}{d} \\
&= -\frac{a^4\text{arctanh}(\cos(c+dx))}{d} + \frac{4a^3b\text{arctanh}(\sin(c+dx))}{d} - \frac{2ab^3\text{arctanh}(\sin(c+dx))}{d} \\
&\quad + \frac{6a^2b^2\sec(c+dx)}{d} - \frac{b^4\sec(c+dx)}{d} + \frac{b^4\sec^3(c+dx)}{3d} + \frac{2ab^3\sec(c+dx)\tan(c+dx)}{d}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 352 vs. 2(118) = 236.

Time = 6.85 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.98

$$
\int \csc(c+dx)(a+b\tan(c+dx))^4\,dx
= \frac{72a^2b^2 - 10b^4 - 12a^4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - 48a^3b\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) + 24ab^3\log\left(\text{co}\right.}{}
$$

```
[In] Integrate[Csc[c + d*x]*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (72*a^2*b^2 - 10*b^4 - 12*a^4*Log[Cos[(c + d*x)/2]] - 48*a^3*b*Log[Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2]] + 24*a*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]] + 12*a^4*Log[Sin[(c + d*x)/2]] + 48*a^3*b*Log[Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]] - 24*a*b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (12*a*b
^3)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + b^4/(Cos[(c + d*x)/2] - Sin[(
c + d*x)/2])^2 + 2*b^2*(36*a^2 - b^2 + 2*b^2*Cos[c + d*x] + (36*a^2 - 5*b^2
)*Cos[2*(c + d*x)])*Sec[c + d*x]^3*Sin[(c + d*x)/2]^2 - (12*a*b^3)/(Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2])^2 + b^4/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
^2)/(12*d)
```

Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.44

method	result
derivativedivides	$\frac{b^4\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{(2+\sin^2(dx+c))\cos(dx+c)}{3}\right)+4a\,b^3\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+}{d}$
default	$\frac{b^4\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{(2+\sin^2(dx+c))\cos(dx+c)}{3}\right)+4a\,b^3\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+}{d}$
risch	$-\frac{2b^2e^{i(dx+c)}(-18a^2e^{4i(dx+c)}+3b^2e^{4i(dx+c)}+6iab\,e^{4i(dx+c)}-36a^2e^{2i(dx+c)}+2b^2e^{2i(dx+c)}-18a^2+3b^2-6iab)}{3d(e^{2i(dx+c)}+1)^3}-\frac{4a^3}{}$

```
[In] int(csc(d*x+c)*(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^4*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+
sin(d*x+c)^2)*cos(d*x+c))+4*a*b^3*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*
x+c)-1/2*ln(sec(d*x+c)+tan(d*x+c)))+6*a^2*b^2/cos(d*x+c)+4*a^3*b*ln(sec(d*x
+c)+tan(d*x+c))+a^4*ln(csc(d*x+c)-cot(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.48

$$\int \csc(c+dx)(a+b\tan(c+dx))^4\,dx = -\frac{3\,a^4\cos(dx+c)^3\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-3\,a^4\cos(dx+c)^3\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-12\,ab^3\cos(d}{}$$

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/6*(3*a^4*cos(d*x + c)^3*log(1/2*cos(d*x + c) + 1/2) - 3*a^4*cos(d*x + c)
^3*log(-1/2*cos(d*x + c) + 1/2) - 12*a*b^3*cos(d*x + c)*sin(d*x + c) - 6*(2
*a^3*b - a*b^3)*cos(d*x + c)^3*log(sin(d*x + c) + 1) + 6*(2*a^3*b - a*b^3)*
cos(d*x + c)^3*log(-sin(d*x + c) + 1) - 2*b^4 - 6*(6*a^2*b^2 - b^4)*cos(d*x
 + c)^2)/(d*cos(d*x + c)^3)
```

Sympy [F]

$$\int \csc(c+dx)(a+b\tan(c+dx))^4\,dx = \int (a+b\tan(c+dx))^4 \csc(c+dx)\,dx$$

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))**4,x)
[Out] Integral((a + b*tan(c + d*x))**4*csc(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.18

$$\int \csc(c+dx)(a+b\tan(c+dx))^4\,dx =$$

$$-\frac{3\,ab^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)-6\,a^3b(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(s\right.}{3\,d}$$

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
[Out] -1/3*(3*a*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1)
- log(sin(d*x + c) - 1)) - 6*a^3*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c
) - 1)) + 3*a^4*log(cot(d*x + c) + csc(d*x + c)) - 18*a^2*b^2/cos(d*x + c)
+ (3*cos(d*x + c)^2 - 1)*b^4/cos(d*x + c)^3)/d
```

Giac [A] (verification not implemented)

none

Time = 1.04 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.64

$$\int \csc(c+dx)(a+b\tan(c+dx))^4\,dx$$

$$= \frac{3\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+6\left(2\,a^3b-ab^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-6\left(2\,a^3b-ab^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\right.\right.\right.}{}$$

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^4,x, algorithm="giac")
[Out] 1/3*(3*a^4*log(abs(tan(1/2*d*x + 1/2*c))) + 6*(2*a^3*b - a*b^3)*log(abs(tan
(1/2*d*x + 1/2*c) + 1)) - 6*(2*a^3*b - a*b^3)*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) + 4*(3*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 9*a^2*b^2*tan(1/2*d*x + 1/2*c)^
4 + 18*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 3*b^4*tan(1/2*d*x + 1/2*c)^2 - 3*a*
b^3*tan(1/2*d*x + 1/2*c) - 9*a^2*b^2 + b^4)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)
/d
```

Mupad [B] (verification not implemented)

Time = 6.07 (sec) , antiderivative size = 496, normalized size of antiderivative = 4.20

$$\int \csc(c+dx)(a+b\tan(c+dx))^4\,dx = \frac{a^4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{12\,a^2\,b^2-\frac{4\,b^4}{3}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(4\,b^4-24\,a^2\,b^2\right)+12\,a^2\,b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\,a\,b^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-4\,a\,b^3\,\tan}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)} - \frac{a\,b\,\mathrm{atan}\left(\frac{a\,b\,\left(2\,a^2-b^2\right)\,\left(4\,a\,b^3-8\,a^3\,b+2\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-12\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^2-b^2\right)\right)\,2\mathrm{i}+a\,b\,\left(2\,a^2-b^2\right)\,\left(\right.}{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(64\,a^6\,b^2-64\,a^4\,b^4+16\,a^2\,b^6\right)+16\,a^7\,b-8\,a^5\,b^3+2\,a\,b\,\left(2\,a^2-b^2\right)\,\left(4\,a\,b^3-8\,a^3\,b+2\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-12\,a\,b\,\tan\left(\frac{c}{2}+\frac{d}{2}\right.\right.}\right.}{d}$$

```
[In] int((a + b*tan(c + d*x))^4/sin(c + d*x),x)
```

```
[Out] (a^4*log(tan(c/2 + (d*x)/2)))/d - (12*a^2*b^2 - (4*b^4)/3 + tan(c/2 + (d*x)
/2)^2*(4*b^4 - 24*a^2*b^2) + 12*a^2*b^2*tan(c/2 + (d*x)/2)^4 + 4*a*b^3*tan(
c/2 + (d*x)/2) - 4*a*b^3*tan(c/2 + (d*x)/2)^5)/(d*(3*tan(c/2 + (d*x)/2)^2 -
 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1)) - (a*b*atan((a*b*(2*a^
2 - b^2)*(4*a*b^3 - 8*a^3*b + 2*a^4*tan(c/2 + (d*x)/2) - 12*a*b*tan(c/2 + (
d*x)/2)*(2*a^2 - b^2))*2i + a*b*(2*a^2 - b^2)*(4*a*b^3 - 8*a^3*b + 2*a^4*ta
n(c/2 + (d*x)/2) + 12*a*b*tan(c/2 + (d*x)/2)*(2*a^2 - b^2))*2i)/(2*tan(c/2
+ (d*x)/2)*(16*a^2*b^6 - 64*a^4*b^4 + 64*a^6*b^2) + 16*a^7*b - 8*a^5*b^3 +
2*a*b*(2*a^2 - b^2)*(4*a*b^3 - 8*a^3*b + 2*a^4*tan(c/2 + (d*x)/2) - 12*a*b*
tan(c/2 + (d*x)/2)*(2*a^2 - b^2)) - 2*a*b*(2*a^2 - b^2)*(4*a*b^3 - 8*a^3*b
+ 2*a^4*tan(c/2 + (d*x)/2) + 12*a*b*tan(c/2 + (d*x)/2)*(2*a^2 - b^2))))*(2*
a^2 - b^2)*4i)/d
```

3.45 $\int \csc^2(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal result

Integrand size = 21, antiderivative size = 83

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^4\,dx = -\frac{a^4\cot(c+dx)}{d}+\frac{4a^3b\log(\tan(c+dx))}{d}+\frac{6a^2b^2\tan(c+dx)}{d}+\frac{2ab^3\tan^2(c+dx)}{d}+\frac{b^4\tan^3(c+dx)}{3d}$$

```
[Out] -a^4*cot(d*x+c)/d+4*a^3*b*ln(tan(d*x+c))/d+6*a^2*b^2*tan(d*x+c)/d+2*a*b^3*t
an(d*x+c)^2/d+1/3*b^4*tan(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3597, 45}

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^4\,dx = -\frac{a^4\cot(c+dx)}{d}+\frac{4a^3b\log(\tan(c+dx))}{d}+\frac{6a^2b^2\tan(c+dx)}{d}+\frac{2ab^3\tan^2(c+dx)}{d}+\frac{b^4\tan^3(c+dx)}{3d}$$

```
[In] Int[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -((a^4*Cot[c + d*x])/d) + (4*a^3*b*Log[Tan[c + d*x]])/d + (6*a^2*b^2*Tan[c
+ d*x])/d + (2*a*b^3*Tan[c + d*x]^2)/d + (b^4*Tan[c + d*x]^3)/(3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{b\text{Subst}\left(\int \frac{(a+x)^4}{x^2}\,dx, x, b\tan(c+dx)\right)}{d}\\ &= \frac{b\text{Subst}\left(\int \left(6a^2+\frac{a^4}{x^2}+\frac{4a^3}{x}+4ax+x^2\right)\,dx, x, b\tan(c+dx)\right)}{d}\\ &= -\frac{a^4\cot(c+dx)}{d}+\frac{4a^3b\log(\tan(c+dx))}{d}\\ &\quad+\frac{6a^2b^2\tan(c+dx)}{d}+\frac{2ab^3\tan^2(c+dx)}{d}+\frac{b^4\tan^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] (verified)

Time = 3.30 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.95

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^4\,dx = -\frac{\csc(c+dx)\sec^3(c+dx)\left(4(3a^4+b^4)\cos(2(c+dx))+(3a^4+18a^2b^2-b^4)\cos(4(c+dx))+3(3a^4-6a\right.}{}$$

```
[In] Integrate[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -1/24*(Csc[c + d*x]*Sec[c + d*x]^3*(4*(3*a^4 + b^4)*Cos[2*(c + d*x)] + (3*a
^4 + 18*a^2*b^2 - b^4)*Cos[4*(c + d*x)] + 3*(3*a^4 - 6*a^2*b^2 - b^4 + 8*a*
b*(-b^2 + a^2*Log[Cos[c + d*x]] - a^2*Log[Sin[c + d*x]])*Sin[2*(c + d*x)] +
 4*a^3*b*(Log[Cos[c + d*x]] - Log[Sin[c + d*x]])*Sin[4*(c + d*x)])))/d
```

Maple [A] (verified)

Time = 3.40 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{\frac{b^4\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+\frac{2ab^3}{\cos(dx+c)^2}+6a^2b^2\tan(dx+c)+4a^3b\ln(\tan(dx+c))-a^4\cot(dx+c)}{d}$
default	$\frac{\frac{b^4\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+\frac{2ab^3}{\cos(dx+c)^2}+6a^2b^2\tan(dx+c)+4a^3b\ln(\tan(dx+c))-a^4\cot(dx+c)}{d}$
risch	$-\frac{2i(12iab^3e^{6i(dx+c)}+3a^4e^{6i(dx+c)}-18a^2b^2e^{6i(dx+c)}+3b^4e^{6i(dx+c)}+9a^4e^{4i(dx+c)}-18a^2b^2e^{4i(dx+c)}-3b^4e^{4i(dx+c)}-12i}{3d\left(e^{2i(dx+c)}+1\right)^3\left(e^{2i(dx+c)}-1\right)}$

```
[In] int(csc(d*x+c)^2*(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*b^4*sin(d*x+c)^3/cos(d*x+c)^3+2*a*b^3/cos(d*x+c)^2+6*a^2*b^2*tan(d
*x+c)+4*a^3*b*ln(tan(d*x+c))-a^4*cot(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.92

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^4\,dx = -\frac{6\,a^3b\cos\left(dx+c\right)^3\log\left(\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)-6\,a^3b\cos\left(dx+c\right)^3\log\left(-\frac{1}{4}\,\cos\left(dx+c\right)^2+\frac{1}{4}\right)\sin\left(dx\right.}{3\,d\cos\left(dx\right.}$$

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(6*a^3*b*cos(d*x + c)^3*log(cos(d*x + c)^2)*sin(d*x + c) - 6*a^3*b*cos
(d*x + c)^3*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) - 6*a*b^3*cos(d*x +
 c)*sin(d*x + c) + (3*a^4 + 18*a^2*b^2 - b^4)*cos(d*x + c)^4 - b^4 - 2*(9*a
^2*b^2 - b^4)*cos(d*x + c)^2)/(d*cos(d*x + c)^3*sin(d*x + c))
```

Sympy [F]

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^4\,dx = \int \left(a+b\tan\left(c+dx\right)\right)^4\csc^2\left(c+dx\right)\,dx$$

```
[In] integrate(csc(d*x+c)**2*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**4*csc(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.87

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^4\,dx$$
$$=\frac{b^4\tan\left(dx+c\right)^3+6\,ab^3\tan\left(dx+c\right)^2+12\,a^3b\log\left(\tan\left(dx+c\right)\right)+18\,a^2b^2\tan\left(dx+c\right)-\frac{3\,a^4}{\tan(dx+c)}}{3\,d}$$

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
[Out] 1/3*(b^4*tan(d*x + c)^3 + 6*a*b^3*tan(d*x + c)^2 + 12*a^3*b*log(tan(d*x + c
)) + 18*a^2*b^2*tan(d*x + c) - 3*a^4/tan(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 1.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.04

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^4\,dx$$
$$=\frac{b^4\tan\left(dx+c\right)^3+6\,ab^3\tan\left(dx+c\right)^2+12\,a^3b\log\left(\left|\tan\left(dx+c\right)\right|\right)+18\,a^2b^2\tan\left(dx+c\right)-\frac{3\,(4\,a^3b\tan(dx+c)+}{\tan(dx+c)}}{3\,d}$$

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^4,x, algorithm="giac")
[Out] 1/3*(b^4*tan(d*x + c)^3 + 6*a*b^3*tan(d*x + c)^2 + 12*a^3*b*log(abs(tan(d*x
 + c))) + 18*a^2*b^2*tan(d*x + c) - 3*(4*a^3*b*tan(d*x + c) + a^4)/tan(d*x
+ c))/d
```

Mupad [B] (verification not implemented)

Time = 4.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.98

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^4\,dx=\frac{b^4\tan(c+d\,x)^3}{3\,d}-\frac{a^4\cot(c+d\,x)}{d}+\frac{6\,a^2\,b^2\tan(c+d\,x)}{d}+\frac{2\,a\,b^3\tan(c+d\,x)^2}{d}+\frac{4\,a^3\,b\,\ln\left(\tan(c+d\,x)\right)}{d}$$

```
[In] int((a + b*tan(c + d*x))^4/sin(c + d*x)^2,x)
[Out] (b^4*tan(c + d*x)^3)/(3*d) - (a^4*cot(c + d*x))/d + (6*a^2*b^2*tan(c + d*x)
)/d + (2*a*b^3*tan(c + d*x)^2)/d + (4*a^3*b*log(tan(c + d*x)))/d
```

3.46 $\int \csc^3(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal result

Integrand size = 21, antiderivative size = 161

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^4\,dx = -\frac{a^4\text{arctanh}(\cos(c+dx))}{2d} - \frac{6a^2b^2\text{arctanh}(\cos(c+dx))}{d} + \frac{4a^3b\text{arctanh}(\sin(c+dx))}{d} + \frac{2ab^3\text{arctanh}(\sin(c+dx))}{d} - \frac{4a^3b\csc(c+dx)}{d} - \frac{a^4\cot(c+dx)\csc(c+dx)}{2d} + \frac{6a^2b^2\sec(c+dx)}{d} + \frac{b^4\sec^3(c+dx)}{3d} + \frac{2ab^3\sec(c+dx)\tan(c+dx)}{d}$$

```
[Out] -1/2*a^4*arctanh(cos(d*x+c))/d-6*a^2*b^2*arctanh(cos(d*x+c))/d+4*a^3*b*arct
anh(sin(d*x+c))/d+2*a*b^3*arctanh(sin(d*x+c))/d-4*a^3*b*csc(d*x+c)/d-1/2*a^
4*cot(d*x+c)*csc(d*x+c)/d+6*a^2*b^2*sec(d*x+c)/d+1/3*b^4*sec(d*x+c)^3/d+2*a
*b^3*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used

$= \{3598, 3853, 3855, 2701, 327, 213, 2702, 2686, 30\}$

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^4\,dx = -\frac{a^4\text{arctanh}(\cos(c+dx))}{2d} - \frac{a^4\cot(c+dx)\csc(c+dx)}{2d} + \frac{4a^3b\text{arctanh}(\sin(c+dx))}{d} - \frac{4a^3b\csc(c+dx)}{d} - \frac{6a^2b^2\text{arctanh}(\cos(c+dx))}{d} + \frac{6a^2b^2\sec(c+dx)}{d} + \frac{2ab^3\text{arctanh}(\sin(c+dx))}{d} + \frac{2ab^3\tan(c+dx)\sec(c+dx)}{d} + \frac{b^4\sec^3(c+dx)}{3d}$$

```
[In] Int[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -1/2*(a^4*ArcTanh[Cos[c + d*x]])/d - (6*a^2*b^2*ArcTanh[Cos[c + d*x]])/d +
(4*a^3*b*ArcTanh[Sin[c + d*x]])/d + (2*a*b^3*ArcTanh[Sin[c + d*x]])/d - (4*
a^3*b*Csc[c + d*x])/d - (a^4*Cot[c + d*x]*Csc[c + d*x])/(2*d) + (6*a^2*b^2*
Sec[c + d*x])/d + (b^4*Sec[c + d*x]^3)/(3*d) + (2*a*b^3*Sec[c + d*x]*Tan[c
+ d*x])/d
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^4 \csc^3(c+dx) + 4a^3 b \csc^2(c+dx)\sec(c+dx) + 6a^2b^2 \csc(c+dx)\sec^2(c+dx) \right. \\
&\qquad \left. + 4ab^3 \sec^3(c+dx) + b^4 \sec^3(c+dx)\tan(c+dx)\right)\, dx \\
&= a^4 \int \csc^3(c+dx)\, dx + \left(4a^3b\right) \int \csc^2(c+dx)\sec(c+dx)\, dx \\
&\quad + \left(6a^2b^2\right) \int \csc(c+dx)\sec^2(c+dx)\, dx \\
&\quad + \left(4ab^3\right) \int \sec^3(c+dx)\, dx + b^4 \int \sec^3(c+dx)\tan(c+dx)\, dx
\end{aligned}$$

$$= -\frac{a^4\cot(c+dx)\csc(c+dx)}{2d} + \frac{2ab^3\sec(c+dx)\tan(c+dx)}{d} + \frac{1}{2}a^4\int\csc(c+dx)\,dx$$
$$+ (2ab^3)\int\sec(c+dx)\,dx - \frac{(4a^3b)\,\text{Subst}\left(\int\frac{x^2}{-1+x^2}\,dx,x,\csc(c+dx)\right)}{d}$$
$$+ \frac{(6a^2b^2)\,\text{Subst}\left(\int\frac{x^2}{-1+x^2}\,dx,x,\sec(c+dx)\right)}{d} + \frac{b^4\text{Subst}\left(\int x^2\,dx,x,\sec(c+dx)\right)}{d}$$
$$= -\frac{a^4\text{arctanh}(\cos(c+dx))}{2d} + \frac{2ab^3\text{arctanh}(\sin(c+dx))}{d} - \frac{4a^3b\csc(c+dx)}{d}$$
$$- \frac{a^4\cot(c+dx)\csc(c+dx)}{2d} + \frac{6a^2b^2\sec(c+dx)}{d} + \frac{b^4\sec^3(c+dx)}{3d}$$
$$+ \frac{2ab^3\sec(c+dx)\tan(c+dx)}{d} - \frac{(4a^3b)\,\text{Subst}\left(\int\frac{1}{-1+x^2}\,dx,x,\csc(c+dx)\right)}{d}$$
$$+ \frac{(6a^2b^2)\,\text{Subst}\left(\int\frac{1}{-1+x^2}\,dx,x,\sec(c+dx)\right)}{d}$$
$$= -\frac{a^4\text{arctanh}(\cos(c+dx))}{2d} - \frac{6a^2b^2\text{arctanh}(\cos(c+dx))}{d} + \frac{4a^3b\text{arctanh}(\sin(c+dx))}{d}$$
$$+ \frac{2ab^3\text{arctanh}(\sin(c+dx))}{d} - \frac{4a^3b\csc(c+dx)}{d} - \frac{a^4\cot(c+dx)\csc(c+dx)}{2d}$$
$$+ \frac{6a^2b^2\sec(c+dx)}{d} + \frac{b^4\sec^3(c+dx)}{3d} + \frac{2ab^3\sec(c+dx)\tan(c+dx)}{d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1128 vs. $2(161) = 322$.

Time = 8.04 (sec) , antiderivative size = 1128, normalized size of antiderivative = 7.01

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^4\,dx$$
$$= \frac{b^2(36a^2+b^2)\cos^4(c+dx)(a+b\tan(c+dx))^4}{6d(a\cos(c+dx)+b\sin(c+dx))^4} - \frac{2a^3b\cos^4(c+dx)\cot\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^4}{d(a\cos(c+dx)+b\sin(c+dx))^4} - \frac{a^4\cos^4(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^4}{8d(a\cos(c+dx)+b\sin(c+dx))^4} + \frac{(-a^4-12a^2b^2)\cos^4(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{2d(a\cos(c+dx)+b\sin(c+dx))^4} - \frac{2(2a^3b+ab^3)\cos^4(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{d(a\cos(c+dx)+b\sin(c+dx))^4} + \frac{(a^4+12a^2b^2)\cos^4(c+dx)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{2d(a\cos(c+dx)+b\sin(c+dx))^4} + \frac{2(2a^3b+ab^3)\cos^4(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{d(a\cos(c+dx)+b\sin(c+dx))^4} + \frac{a^4\cos^4(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^4}{8d(a\cos(c+dx)+b\sin(c+dx))^4} + \frac{(12ab^3+b^4)\cos^4(c+dx)(a+b\tan(c+dx))^4}{12d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a\cos(c+dx)+b\sin(c+dx))^4} + \frac{b^4\cos^4(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^4}{6d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3(a\cos(c+dx)+b\sin(c+dx))^4} - \frac{b^4\cos^4(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^4}{6d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3(a\cos(c+dx)+b\sin(c+dx))^4} + \frac{(-12ab^3+b^4)\cos^4(c+dx)(a+b\tan(c+dx))^4}{12d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a\cos(c+dx)+b\sin(c+dx))^4} + \frac{\cos^4(c+dx)\left(-36a^2b^2\sin\left(\frac{1}{2}(c+dx)\right)-b^4\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{6d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^4} + \frac{\cos^4(c+dx)\left(36a^2b^2\sin\left(\frac{1}{2}(c+dx)\right)+b^4\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{6d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^4} - \frac{2a^3b\cos^4(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^4}{d(a\cos(c+dx)+b\sin(c+dx))^4}$$

```
[In] Integrate[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (b^2*(36*a^2 + b^2)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(6*d*(a*Cos[c +
d*x] + b*Sin[c + d*x])^4) - (2*a^3*b*Cos[c + d*x]^4*Cot[(c + d*x)/2]*(a + b
```

```
*Tan[c + d*x])^4)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (a^4*Cos[c + d*
x]^4*Csc[(c + d*x)/2]^2*(a + b*Tan[c + d*x])^4)/(8*d*(a*Cos[c + d*x] + b*Si
n[c + d*x])^4) + ((-a^4 - 12*a^2*b^2)*Cos[c + d*x]^4*Log[Cos[(c + d*x)/2]]*
(a + b*Tan[c + d*x])^4)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (2*(2*a
^3*b + a*b^3)*Cos[c + d*x]^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a +
b*Tan[c + d*x])^4)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((a^4 + 12*a^2
*b^2)*Cos[c + d*x]^4*Log[Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(2*d*(a*
Cos[c + d*x] + b*Sin[c + d*x])^4) + (2*(2*a^3*b + a*b^3)*Cos[c + d*x]^4*Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(d*(a*Cos[c +
 d*x] + b*Sin[c + d*x])^4) + (a^4*Cos[c + d*x]^4*Sec[(c + d*x)/2]^2*(a + b*
Tan[c + d*x])^4)/(8*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((12*a*b^3 + b
^4)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(12*d*(Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (b^4*Cos[c + d*x]^4*Si
n[(c + d*x)/2]*(a + b*Tan[c + d*x])^4)/(6*d*(Cos[(c + d*x)/2] - Sin[(c + d*
x)/2])^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (b^4*Cos[c + d*x]^4*Sin[(c
+ d*x)/2]*(a + b*Tan[c + d*x])^4)/(6*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
)^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((-12*a*b^3 + b^4)*Cos[c + d*x]^
4*(a + b*Tan[c + d*x])^4)/(12*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(a*
Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cos[c + d*x]^4*(-36*a^2*b^2*Sin[(c + d
*x)/2] - b^4*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^4)/(6*d*(Cos[(c + d*x)/
2] + Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cos[c + d*x]
^4*(36*a^2*b^2*Sin[(c + d*x)/2] + b^4*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x]
)^4)/(6*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c +
 d*x])^4) - (2*a^3*b*Cos[c + d*x]^4*Tan[(c + d*x)/2]*(a + b*Tan[c + d*x])^4
)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4)
```

Maple [A] (verified)

Time = 6.39 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{\frac{b^4}{3\cos(dx+c)^3}+4a\,b^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+6a^2b^2\left(\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)+4a}{d}$
default	$\frac{\frac{b^4}{3\cos(dx+c)^3}+4a\,b^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+6a^2b^2\left(\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)+4a}{d}$
risch	$-\frac{i\mathrm{e}^{i(dx+c)}\left(8ib^4\mathrm{e}^{6i(dx+c)}+8ib^4\mathrm{e}^{2i(dx+c)}+24a^3b\,\mathrm{e}^{8i(dx+c)}+12a\,b^3\mathrm{e}^{8i(dx+c)}+36ia^2b^2-16ib^4\mathrm{e}^{4i(dx+c)}+48a^3b\,\mathrm{e}^{6i(dx+c)}\right.}{}$

```
[In] int(csc(d*x+c)^3*(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*b^4/cos(d*x+c)^3+4*a*b^3*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x
+c)+tan(d*x+c)))+6*a^2*b^2*(1/cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+4*a^3*b
*(-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a^4*(-1/2*csc(d*x+c)*cot(d*x+c)+
1/2*ln(csc(d*x+c)-cot(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 346 vs. 2(155) = 310.

Time = 0.33 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.15

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^4\,dx = \frac{6\,(a^4+12\,a^2b^2)\cos(dx+c)^4-4\,b^4-4\,(18\,a^2b^2-b^4)\cos(dx+c)^2-3\,((a^4+12\,a^2b^2)\cos(dx+c)^5-(a^4}{}$$

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/12*(6*(a^4 + 12*a^2*b^2)*cos(d*x + c)^4 - 4*b^4 - 4*(18*a^2*b^2 - b^4)*co
s(d*x + c)^2 - 3*((a^4 + 12*a^2*b^2)*cos(d*x + c)^5 - (a^4 + 12*a^2*b^2)*co
s(d*x + c)^3)*log(1/2*cos(d*x + c) + 1/2) + 3*((a^4 + 12*a^2*b^2)*cos(d*x +
 c)^5 - (a^4 + 12*a^2*b^2)*cos(d*x + c)^3)*log(-1/2*cos(d*x + c) + 1/2) + 1
2*((2*a^3*b + a*b^3)*cos(d*x + c)^5 - (2*a^3*b + a*b^3)*cos(d*x + c)^3)*log
(sin(d*x + c) + 1) - 12*((2*a^3*b + a*b^3)*cos(d*x + c)^5 - (2*a^3*b + a*b^
3)*cos(d*x + c)^3)*log(-sin(d*x + c) + 1) - 24*(a*b^3*cos(d*x + c) - (2*a^3
*b + a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^5 - d*cos(d*x + c
)^3)
```

Sympy [F]

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^4\,dx = \int (a+b\tan(c+dx))^4\csc^3(c+dx)\,dx$$

```
[In] integrate(csc(d*x+c)**3*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**4*csc(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.17

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^4\,dx = \frac{3\,a^4\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1)\right)-12\,ab^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)\right.}{}$$

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/12*(3*a^4*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - log(cos(d*x + c) + 1) +
log(cos(d*x + c) - 1)) - 12*a*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - lo
g(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 36*a^2*b^2*(2/cos(d*x + c) -
 log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 24*a^3*b*(2/sin(d*x + c)
- log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 4*b^4/cos(d*x + c)^3)/d
```

Giac [A] (verification not implemented)

none

Time = 1.07 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.86

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^4\,dx$$
$$= \frac{3\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 48\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + 48\,(2\,a^3b+ab^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 48\,(2\,a^3b\cdots}{\cdots}$$

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(3*a^4*tan(1/2*d*x + 1/2*c)^2 - 48*a^3*b*tan(1/2*d*x + 1/2*c) + 48*(2*
a^3*b + a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 48*(2*a^3*b + a*b^3)*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) + 12*(a^4 + 12*a^2*b^2)*log(abs(tan(1/2*d*
x + 1/2*c))) - 3*(6*a^4*tan(1/2*d*x + 1/2*c)^2 + 72*a^2*b^2*tan(1/2*d*x + 1
/2*c)^2 + 16*a^3*b*tan(1/2*d*x + 1/2*c) + a^4)/tan(1/2*d*x + 1/2*c)^2 + 16*
(6*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 18*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 - 3*b^4
*tan(1/2*d*x + 1/2*c)^4 + 36*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 6*a*b^3*tan(1
/2*d*x + 1/2*c) - 18*a^2*b^2 - b^4)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 5.10 (sec) , antiderivative size = 670, normalized size of antiderivative = 4.16

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^4\,dx = \frac{a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}$$
$$-\frac{\frac{a^4}{2}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(\frac{3\,a^4}{2}+48\,a^2\,b^2+\frac{8\,b^4}{3}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(\frac{a^4}{2}+48\,a^2\,b^2+8\,b^4\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(\frac{3\,a^4}{2}\cdots\right.}{d\,\left(-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+12\tan\cdots\right.}$$
$$+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{a^4}{2}+6\,a^2\,b^2\right)}{d}-\frac{2\,a^3\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d}$$
$$-\frac{a\,b\,\mathrm{atan}\left(\frac{a\,b\,(2\,a^2+b^2)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a^4+12\,a^2\,b^2)-4\,a\,b^3-8\,a^3\,b+12\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(2\,a^2+b^2)\right)\,2\mathrm{i}-\cdots}{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(64\,a^6\,b^2+64\,a^4\,b^4+16\,a^2\,b^6)+8\,a^7\,b+48\,a^3\,b^5+100\,a^5\,b^3-2\,a\,b\,(2\,a^2+b^2)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a^4+12\,a^2\,b^2)-4\,a\,b^3-8\,a\cdots\right.}\right.}{\cdots}$$

```
[In] int((a + b*tan(c + d*x))^4/sin(c + d*x)^3,x)
```

```
[Out] (a^4*tan(c/2 + (d*x)/2)^2)/(8*d) - (a^4/2 - tan(c/2 + (d*x)/2)^2*((3*a^4)/2
 + (8*b^4)/3 + 48*a^2*b^2) - tan(c/2 + (d*x)/2)^6*(a^4/2 + 8*b^4 + 48*a^2*b
^2) + tan(c/2 + (d*x)/2)^4*((3*a^4)/2 + 96*a^2*b^2) + tan(c/2 + (d*x)/2)^7*
(16*a*b^3 - 8*a^3*b) - tan(c/2 + (d*x)/2)^3*(16*a*b^3 + 24*a^3*b) + 8*a^3*b
*tan(c/2 + (d*x)/2) + 24*a^3*b*tan(c/2 + (d*x)/2)^5)/(d*(4*tan(c/2 + (d*x)/
2)^2 - 12*tan(c/2 + (d*x)/2)^4 + 12*tan(c/2 + (d*x)/2)^6 - 4*tan(c/2 + (d*x
)/2)^8)) + (log(tan(c/2 + (d*x)/2))*(a^4/2 + 6*a^2*b^2))/d - (2*a^3*b*tan(c
/2 + (d*x)/2))/d - (a*b*atan((a*b*(2*a^2 + b^2)*(tan(c/2 + (d*x)/2)*(a^4 +
12*a^2*b^2) - 4*a*b^3 - 8*a^3*b + 12*a*b*tan(c/2 + (d*x)/2)*(2*a^2 + b^2))*
2i - a*b*(2*a^2 + b^2)*(4*a*b^3 - tan(c/2 + (d*x)/2)*(a^4 + 12*a^2*b^2) + 8
*a^3*b + 12*a*b*tan(c/2 + (d*x)/2)*(2*a^2 + b^2))*2i)/(2*tan(c/2 + (d*x)/2)
*(16*a^2*b^6 + 64*a^4*b^4 + 64*a^6*b^2) + 8*a^7*b + 48*a^3*b^5 + 100*a^5*b^
3 - 2*a*b*(2*a^2 + b^2)*(tan(c/2 + (d*x)/2)*(a^4 + 12*a^2*b^2) - 4*a*b^3 -
8*a^3*b + 12*a*b*tan(c/2 + (d*x)/2)*(2*a^2 + b^2)) - 2*a*b*(2*a^2 + b^2)*(4
*a*b^3 - tan(c/2 + (d*x)/2)*(a^4 + 12*a^2*b^2) + 8*a^3*b + 12*a*b*tan(c/2 +
 (d*x)/2)*(2*a^2 + b^2))))*(2*a^2 + b^2)*4i)/d
```

3.47 $\int \csc^4(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal result

Integrand size = 21, antiderivative size = 137

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^4\,dx = -\frac{a^2(a^2+6b^2)\cot(c+dx)}{d}-\frac{2a^3b\cot^2(c+dx)}{d}-\frac{a^4\cot^3(c+dx)}{3d}+\frac{4ab(a^2+b^2)\log(\tan(c+dx))}{d}+\frac{b^2(6a^2+b^2)\tan(c+dx)}{d}+\frac{2ab^3\tan^2(c+dx)}{d}+\frac{b^4\tan^3(c+dx)}{3d}$$

```
[Out] -a^2*(a^2+6*b^2)*cot(d*x+c)/d-2*a^3*b*cot(d*x+c)^2/d-1/3*a^4*cot(d*x+c)^3/d
+4*a*b*(a^2+b^2)*ln(tan(d*x+c))/d+b^2*(6*a^2+b^2)*tan(d*x+c)/d+2*a*b^3*tan(
d*x+c)^2/d+1/3*b^4*tan(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3597, 908}

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^4\,dx = -\frac{a^4\cot^3(c+dx)}{3d}-\frac{2a^3b\cot^2(c+dx)}{d}+\frac{b^2(6a^2+b^2)\tan(c+dx)}{d}-\frac{a^2(a^2+6b^2)\cot(c+dx)}{d}+\frac{4ab(a^2+b^2)\log(\tan(c+dx))}{d}+\frac{2ab^3\tan^2(c+dx)}{d}+\frac{b^4\tan^3(c+dx)}{3d}$$

```
[In] Int[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -((a^2*(a^2 + 6*b^2)*Cot[c + d*x])/d) - (2*a^3*b*Cot[c + d*x]^2)/d - (a^4*C
ot[c + d*x]^3)/(3*d) + (4*a*b*(a^2 + b^2)*Log[Tan[c + d*x]])/d + (b^2*(6*a^
2 + b^2)*Tan[c + d*x])/d + (2*a*b^3*Tan[c + d*x]^2)/d + (b^4*Tan[c + d*x]^3
)/(3*d)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b\text{Subst}\left(\int \frac{(a+x)^4(b^2+x^2)}{x^4}\,dx, x, b\tan(c+dx)\right)}{d}\\
&= \frac{b\text{Subst}\left(\int \left(6a^2\left(1+\frac{b^2}{6a^2}\right)+\frac{a^4b^2}{x^4}+\frac{4a^3b^2}{x^3}+\frac{a^4+6a^2b^2}{x^2}+\frac{4a(a^2+b^2)}{x}+4ax+x^2\right)\,dx, x, b\tan(c+dx)\right)}{d}\\
&= -\frac{a^2(a^2+6b^2)\cot(c+dx)}{d}-\frac{2a^3b\cot^2(c+dx)}{d}\\
&\quad -\frac{a^4\cot^3(c+dx)}{3d}+\frac{4ab(a^2+b^2)\log(\tan(c+dx))}{d}\\
&\quad +\frac{b^2(6a^2+b^2)\tan(c+dx)}{d}+\frac{2ab^3\tan^2(c+dx)}{d}+\frac{b^4\tan^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 5.98 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.37

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^4\,dx =$$

$$-\frac{(b+a\cot(c+dx))^4\sin(c+dx)\left(\cos(c+dx)\left(-6ab^3+6a^3b\cot^2(c+dx)+a^4\cot^3(c+dx)\right)+2a\cos^3(c-\right.}{}$$

```
[In] Integrate[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -1/3*((b + a*Cot[c + d*x])^4*Sin[c + d*x]*(Cos[c + d*x]*(-6*a*b^3 + 6*a^3*b
*Cot[c + d*x]^2 + a^4*Cot[c + d*x]^3) + 2*a*Cos[c + d*x]^3*(a*(a^2 + 9*b^2)
*Cot[c + d*x] + 6*b*(a^2 + b^2)*(Log[Cos[c + d*x]] - Log[Sin[c + d*x]])) -
b^4*Sin[c + d*x] - 2*b^2*(9*a^2 + b^2)*Cos[c + d*x]^2*Sin[c + d*x])*Tan[c +
 d*x]^3)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4)
```

Maple [A] (verified)

Time = 11.77 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{-b^4\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+4a\,b^3\left(\frac{1}{2\cos(dx+c)^2}+\ln(\tan(dx+c))\right)+6a^2b^2\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)}{d}$
default	$\frac{-b^4\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+4a\,b^3\left(\frac{1}{2\cos(dx+c)^2}+\ln(\tan(dx+c))\right)+6a^2b^2\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)}{d}$
risch	$8a^3b\,e^{10i(dx+c)}+8a\,b^3e^{10i(dx+c)}-\frac{32ib^4e^{6i(dx+c)}}{3}-\frac{4ib^4}{3}-24ia^2b^2+16a^3b\,e^{8i(dx+c)}-16a\,b^3e^{8i(dx+c)}+8ib^4e^{4i(dx+c)}+8ia$

```
[In] int(csc(d*x+c)^4*(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-b^4*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+4*a*b^3*(1/2/cos(d*x+c)^2+ln(t
an(d*x+c)))+6*a^2*b^2*(1/sin(d*x+c)/cos(d*x+c)-2*cot(d*x+c))+4*a^3*b*(-1/2/
sin(d*x+c)^2+ln(tan(d*x+c)))+a^4*(-2/3-1/3*csc(d*x+c)^2)*cot(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(133) = 266.

Time = 0.28 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.95

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^4\,dx = -\frac{2\left(a^4+18\,a^2b^2+b^4\right)\cos\left(dx+c\right)^6+18\,a^2b^2\cos\left(dx+c\right)^2-3\left(a^4+18\,a^2b^2+b^4\right)\cos\left(dx+c\right)^4+b^4+6}{}$$

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(2*(a^4 + 18*a^2*b^2 + b^4)*cos(d*x + c)^6 + 18*a^2*b^2*cos(d*x + c)^2
 - 3*(a^4 + 18*a^2*b^2 + b^4)*cos(d*x + c)^4 + b^4 + 6*((a^3*b + a*b^3)*cos
(d*x + c)^5 - (a^3*b + a*b^3)*cos(d*x + c)^3)*log(cos(d*x + c)^2)*sin(d*x +
 c) - 6*((a^3*b + a*b^3)*cos(d*x + c)^5 - (a^3*b + a*b^3)*cos(d*x + c)^3)*l
og(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) + 6*(a*b^3*cos(d*x + c) - (a^3*b
 + a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/((d*cos(d*x + c)^5 - d*cos(d*x + c)
^3)*sin(d*x + c))
```

Sympy [F]

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^4\,dx = \int (a+b\tan(c+dx))^4\csc^4(c+dx)\,dx$$

```
[In] integrate(csc(d*x+c)**4*(a+b*tan(d*x+c))**4,x)
[Out] Integral((a + b*tan(c + d*x))**4*csc(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.88

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^4\,dx$$
$$= \frac{b^4\tan(dx+c)^3 + 6\,ab^3\tan(dx+c)^2 + 12\,(a^3b+ab^3)\log(\tan(dx+c)) + 3\,(6\,a^2b^2+b^4)\tan(dx+c) - \frac{6\,a^3 \cdots}{\cdots}}{3\,d}$$

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
[Out] 1/3*(b^4*tan(d*x + c)^3 + 6*a*b^3*tan(d*x + c)^2 + 12*(a^3*b + a*b^3)*log(t
an(d*x + c)) + 3*(6*a^2*b^2 + b^4)*tan(d*x + c) - (6*a^3*b*tan(d*x + c) + a
^4 + 3*(a^4 + 6*a^2*b^2)*tan(d*x + c)^2)/tan(d*x + c)^3)/d
```

Giac [A] (verification not implemented)

none

Time = 1.16 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.18

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^4\,dx$$
$$= \frac{b^4\tan(dx+c)^3 + 6\,ab^3\tan(dx+c)^2 + 18\,a^2b^2\tan(dx+c) + 3\,b^4\tan(dx+c) + 12\,(a^3b+ab^3)\log(|\tan(d \cdots}{3\,d}$$

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^4,x, algorithm="giac")
[Out] 1/3*(b^4*tan(d*x + c)^3 + 6*a*b^3*tan(d*x + c)^2 + 18*a^2*b^2*tan(d*x + c)
+ 3*b^4*tan(d*x + c) + 12*(a^3*b + a*b^3)*log(abs(tan(d*x + c))) - (22*a^3*
b*tan(d*x + c)^3 + 22*a*b^3*tan(d*x + c)^3 + 3*a^4*tan(d*x + c)^2 + 18*a^2*
b^2*tan(d*x + c)^2 + 6*a^3*b*tan(d*x + c) + a^4)/tan(d*x + c)^3)/d
```

Mupad [B] (verification not implemented)

Time = 4.78 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.96

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^4\,dx$$
$$=\frac{\ln\left(\tan(c+d\,x)\right)\,\left(4\,a^3\,b+4\,a\,b^3\right)}{d}-\frac{\cot(c+d\,x)^3\,\left(\tan(c+d\,x)^2\,\left(a^4+6\,a^2\,b^2\right)+\frac{a^4}{3}+2\,a^3\,b\tan(c+d\,x)\right)}{d}+\frac{\tan(c+d\,x)\,\left(6\,a^2\,b^2+b^4\right)}{d}+\frac{b^4\tan(c+d\,x)^3}{3\,d}+\frac{2\,a\,b^3\tan(c+d\,x)^2}{d}$$

```
[In] int((a + b*tan(c + d*x))^4/sin(c + d*x)^4,x)
```

```
[Out] (log(tan(c + d*x))*(4*a*b^3 + 4*a^3*b))/d - (cot(c + d*x)^3*(tan(c + d*x)^2
*(a^4 + 6*a^2*b^2) + a^4/3 + 2*a^3*b*tan(c + d*x)))/d + (tan(c + d*x)*(b^4
+ 6*a^2*b^2))/d + (b^4*tan(c + d*x)^3)/(3*d) + (2*a*b^3*tan(c + d*x)^2)/d
```

3.48 $\int \csc^5(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal result

Integrand size = 21, antiderivative size = 274

$$\begin{aligned}\int \csc^5(c+dx)(a+b\tan(c+dx))^4\,dx =& -\frac{3a^4\text{arctanh}(\cos(c+dx))}{8d} \\ &-\frac{9a^2b^2\text{arctanh}(\cos(c+dx))}{d} \\ &-\frac{b^4\text{arctanh}(\cos(c+dx))}{d} \\ &+\frac{4a^3b\text{arctanh}(\sin(c+dx))}{d} \\ &+\frac{6ab^3\text{arctanh}(\sin(c+dx))}{d}-\frac{4a^3b\csc(c+dx)}{d} \\ &-\frac{6ab^3\csc(c+dx)}{d}-\frac{3a^4\cot(c+dx)\csc(c+dx)}{8d} \\ &-\frac{4a^3b\csc^3(c+dx)}{3d}-\frac{a^4\cot(c+dx)\csc^3(c+dx)}{4d} \\ &+\frac{9a^2b^2\sec(c+dx)}{d}+\frac{b^4\sec(c+dx)}{d} \\ &-\frac{3a^2b^2\csc^2(c+dx)\sec(c+dx)}{d} \\ &+\frac{2ab^3\csc(c+dx)\sec^2(c+dx)}{d}+\frac{b^4\sec^3(c+dx)}{3d}\end{aligned}$$

```
[Out] -3/8*a^4*arctanh(cos(d*x+c))/d-9*a^2*b^2*arctanh(cos(d*x+c))/d-b^4*arctanh(
cos(d*x+c))/d+4*a^3*b*arctanh(sin(d*x+c))/d+6*a*b^3*arctanh(sin(d*x+c))/d-4
*a^3*b*csc(d*x+c)/d-6*a*b^3*csc(d*x+c)/d-3/8*a^4*cot(d*x+c)*csc(d*x+c)/d-4/
3*a^3*b*csc(d*x+c)^3/d-1/4*a^4*cot(d*x+c)*csc(d*x+c)^3/d+9*a^2*b^2*sec(d*x+
c)/d+b^4*sec(d*x+c)/d-3*a^2*b^2*csc(d*x+c)^2*sec(d*x+c)/d+2*a*b^3*csc(d*x+c
)*sec(d*x+c)^2/d+1/3*b^4*sec(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3598, 3853, 3855, 2701, 308, 213, 2702, 294, 327}

$$\begin{aligned}\int \csc^5(c+dx)(a+b\tan(c+dx))^4\,dx = &-\frac{3a^4\text{arctanh}(\cos(c+dx))}{8d} \\ &-\frac{a^4\cot(c+dx)\csc^3(c+dx)}{4d} \\ &-\frac{3a^4\cot(c+dx)\csc(c+dx)}{8d} \\ &+\frac{4a^3b\text{arctanh}(\sin(c+dx))}{d} \\ &-\frac{4a^3b\csc^3(c+dx)}{3d}-\frac{4a^3b\csc(c+dx)}{d} \\ &-\frac{9a^2b^2\text{arctanh}(\cos(c+dx))}{d}+\frac{9a^2b^2\sec(c+dx)}{d} \\ &-\frac{3a^2b^2\csc^2(c+dx)\sec(c+dx)}{d} \\ &+\frac{6ab^3\text{arctanh}(\sin(c+dx))}{d}-\frac{6ab^3\csc(c+dx)}{d} \\ &+\frac{2ab^3\csc(c+dx)\sec^2(c+dx)}{d} \\ &-\frac{b^4\text{arctanh}(\cos(c+dx))}{d} \\ &+\frac{b^4\sec^3(c+dx)}{3d}+\frac{b^4\sec(c+dx)}{d}\end{aligned}$$

```
[In] Int[Csc[c + d*x]^5*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (-3*a^4*ArcTanh[Cos[c + d*x]])/(8*d) - (9*a^2*b^2*ArcTanh[Cos[c + d*x]])/d
- (b^4*ArcTanh[Cos[c + d*x]])/d + (4*a^3*b*ArcTanh[Sin[c + d*x]])/d + (6*a*
b^3*ArcTanh[Sin[c + d*x]])/d - (4*a^3*b*Csc[c + d*x])/d - (6*a*b^3*Csc[c +
d*x])/d - (3*a^4*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (4*a^3*b*Csc[c + d*x]^3
)/(3*d) - (a^4*Cot[c + d*x]*Csc[c + d*x]^3)/(4*d) + (9*a^2*b^2*Sec[c + d*x]
)/d + (b^4*Sec[c + d*x])/d - (3*a^2*b^2*Csc[c + d*x]^2*Sec[c + d*x])/d + (2
*a*b^3*Csc[c + d*x]*Sec[c + d*x]^2)/d + (b^4*Sec[c + d*x]^3)/(3*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^4\csc^5(c+dx)+4a^3b\csc^4(c+dx)\sec(c+dx)+6a^2b^2\csc^3(c+dx)\sec^2(c+dx)\right.\\
&\qquad\left.+4ab^3\csc^2(c+dx)\sec^3(c+dx)+b^4\csc(c+dx)\sec^4(c+dx)\right)\,dx\\
&= a^4\int\csc^5(c+dx)\,dx+\left(4a^3b\right)\int\csc^4(c+dx)\sec(c+dx)\,dx\\
&\quad+\left(6a^2b^2\right)\int\csc^3(c+dx)\sec^2(c+dx)\,dx\\
&\quad+\left(4ab^3\right)\int\csc^2(c+dx)\sec^3(c+dx)\,dx+b^4\int\csc(c+dx)\sec^4(c+dx)\,dx\\
&= -\frac{a^4\cot(c+dx)\csc^3(c+dx)}{4d}+\frac{1}{4}\left(3a^4\right)\int\csc^3(c+dx)\,dx\\
&\quad-\frac{\left(4a^3b\right)\text{Subst}\left(\int\frac{x^4}{-1+x^2}\,dx,x,\csc(c+dx)\right)}{d}\\
&\quad+\frac{\left(6a^2b^2\right)\text{Subst}\left(\int\frac{x^4}{(-1+x^2)^2}\,dx,x,\sec(c+dx)\right)}{d}\\
&\quad-\frac{\left(4ab^3\right)\text{Subst}\left(\int\frac{x^4}{(-1+x^2)^2}\,dx,x,\csc(c+dx)\right)}{d}\\
&\quad+\frac{b^4\text{Subst}\left(\int\frac{x^4}{-1+x^2}\,dx,x,\sec(c+dx)\right)}{d}\\
&= -\frac{3a^4\cot(c+dx)\csc(c+dx)}{8d}-\frac{a^4\cot(c+dx)\csc^3(c+dx)}{4d}\\
&\quad-\frac{3a^2b^2\csc^2(c+dx)\sec(c+dx)}{d}+\frac{2ab^3\csc(c+dx)\sec^2(c+dx)}{d}\\
&\quad+\frac{1}{8}\left(3a^4\right)\int\csc(c+dx)\,dx-\frac{\left(4a^3b\right)\text{Subst}\left(\int\left(1+x^2+\frac{1}{-1+x^2}\right)\,dx,x,\csc(c+dx)\right)}{d}\\
&\quad+\frac{\left(9a^2b^2\right)\text{Subst}\left(\int\frac{x^2}{-1+x^2}\,dx,x,\sec(c+dx)\right)}{d}\\
&\quad-\frac{\left(6ab^3\right)\text{Subst}\left(\int\frac{x^2}{-1+x^2}\,dx,x,\csc(c+dx)\right)}{d}\\
&\quad+\frac{b^4\text{Subst}\left(\int\left(1+x^2+\frac{1}{-1+x^2}\right)\,dx,x,\sec(c+dx)\right)}{d}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{3a^4\text{arctanh}(\cos(c+dx))}{8d} - \frac{4a^3b\csc(c+dx)}{d} - \frac{6ab^3\csc(c+dx)}{d} \\
&\quad - \frac{3a^4\cot(c+dx)\csc(c+dx)}{8d} - \frac{4a^3b\csc^3(c+dx)}{3d} \\
&\quad - \frac{a^4\cot(c+dx)\csc^3(c+dx)}{4d} + \frac{9a^2b^2\sec(c+dx)}{d} + \frac{b^4\sec(c+dx)}{d} \\
&\quad - \frac{3a^2b^2\csc^2(c+dx)\sec(c+dx)}{d} + \frac{2ab^3\csc(c+dx)\sec^2(c+dx)}{d} \\
&\quad + \frac{b^4\sec^3(c+dx)}{3d} - \frac{(4a^3b)\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d} \\
&\quad + \frac{(9a^2b^2)\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{d} \\
&\quad - \frac{(6ab^3)\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d} + \frac{b^4\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{d} \\
&= -\frac{3a^4\text{arctanh}(\cos(c+dx))}{8d} - \frac{9a^2b^2\text{arctanh}(\cos(c+dx))}{d} - \frac{b^4\text{arctanh}(\cos(c+dx))}{d} \\
&\quad + \frac{4a^3b\text{arctanh}(\sin(c+dx))}{d} + \frac{6ab^3\text{arctanh}(\sin(c+dx))}{d} - \frac{4a^3b\csc(c+dx)}{d} \\
&\quad - \frac{6ab^3\csc(c+dx)}{d} - \frac{3a^4\cot(c+dx)\csc(c+dx)}{8d} - \frac{4a^3b\csc^3(c+dx)}{3d} \\
&\quad - \frac{a^4\cot(c+dx)\csc^3(c+dx)}{4d} + \frac{9a^2b^2\sec(c+dx)}{d} + \frac{b^4\sec(c+dx)}{d} \\
&\quad - \frac{3a^2b^2\csc^2(c+dx)\sec(c+dx)}{d} + \frac{2ab^3\csc(c+dx)\sec^2(c+dx)}{d} + \frac{b^4\sec^3(c+dx)}{3d}
\end{aligned}
$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1491 vs. $2(274) = 548$.

Time = 8.07 (sec) , antiderivative size = 1491, normalized size of antiderivative = 5.44

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^4\,dx = \frac{b^2(36a^2+7b^2)\cos^4(c+dx)(a+b\tan(c+dx))^4}{6d(a\cos(c+dx)+b\sin(c+dx))^4}$$
$$+\frac{\left(-7a^3b\cos\left(\frac{1}{2}(c+dx)\right)-6ab^3\cos\left(\frac{1}{2}(c+dx)\right)\right)\cos^4(c+dx)\csc\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^4}{3d(a\cos(c+dx)+b\sin(c+dx))^4}$$
$$-\frac{3(a^4+8a^2b^2)\cos^4(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^4}{32d(a\cos(c+dx)+b\sin(c+dx))^4}$$
$$-\frac{a^3b\cos^4(c+dx)\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^4}{6d(a\cos(c+dx)+b\sin(c+dx))^4}$$
$$-\frac{a^4\cos^4(c+dx)\csc^4\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^4}{64d(a\cos(c+dx)+b\sin(c+dx))^4}$$
$$+\frac{(-3a^4-72a^2b^2-8b^4)\cos^4(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{8d(a\cos(c+dx)+b\sin(c+dx))^4}$$
$$-\frac{2(2a^3b+3ab^3)\cos^4(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{d(a\cos(c+dx)+b\sin(c+dx))^4}$$
$$+\frac{(3a^4+72a^2b^2+8b^4)\cos^4(c+dx)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{8d(a\cos(c+dx)+b\sin(c+dx))^4}$$
$$+\frac{2(2a^3b+3ab^3)\cos^4(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{d(a\cos(c+dx)+b\sin(c+dx))^4}$$
$$+\frac{3(a^4+8a^2b^2)\cos^4(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^4}{32d(a\cos(c+dx)+b\sin(c+dx))^4}$$
$$+\frac{a^4\cos^4(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^4}{64d(a\cos(c+dx)+b\sin(c+dx))^4}$$
$$+\frac{(12ab^3+b^4)\cos^4(c+dx)(a+b\tan(c+dx))^4}{12d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a\cos(c+dx)+b\sin(c+dx))^4}$$
$$+\frac{b^4\cos^4(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^4}{6d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3(a\cos(c+dx)+b\sin(c+dx))^4}$$
$$-\frac{b^4\cos^4(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^4}{6d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3(a\cos(c+dx)+b\sin(c+dx))^4}$$
$$+\frac{(-12ab^3+b^4)\cos^4(c+dx)(a+b\tan(c+dx))^4}{12d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a\cos(c+dx)+b\sin(c+dx))^4}$$
$$+\frac{\cos^4(c+dx)\sec\left(\frac{1}{2}(c+dx)\right)\left(-7a^3b\sin\left(\frac{1}{2}(c+dx)\right)-6ab^3\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{3d(a\cos(c+dx)+b\sin(c+dx))^4}$$
$$+\frac{\cos^4(c+dx)\left(-36a^2b^2\sin\left(\frac{1}{2}(c+dx)\right)-7b^4\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{6d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^4}$$
$$+\frac{\cos^4(c+dx)\left(36a^2b^2\sin\left(\frac{1}{2}(c+dx)\right)+7b^4\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{6d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^4}$$
$$-\frac{a^3b\cos^4(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^4}{6d(a\cos(c+dx)+b\sin(c+dx))^4}$$

```
[In] Integrate[Csc[c + d*x]^5*(a + b*Tan[c + d*x])^4,x]

[Out] (b^2*(36*a^2 + 7*b^2)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(6*d*(a*Cos[c
+ d*x] + b*Sin[c + d*x])^4) + ((-7*a^3*b*Cos[(c + d*x)/2] - 6*a*b^3*Cos[(c
+ d*x)/2])*Cos[c + d*x]^4*Csc[(c + d*x)/2]*(a + b*Tan[c + d*x])^4)/(3*d*(a*
Cos[c + d*x] + b*Sin[c + d*x])^4) - (3*(a^4 + 8*a^2*b^2)*Cos[c + d*x]^4*Csc
[(c + d*x)/2]^2*(a + b*Tan[c + d*x])^4)/(32*d*(a*Cos[c + d*x] + b*Sin[c + d
*x])^4) - (a^3*b*Cos[c + d*x]^4*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2*(a + b*
Tan[c + d*x])^4)/(6*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (a^4*Cos[c + d
*x]^4*Csc[(c + d*x)/2]^4*(a + b*Tan[c + d*x])^4)/(64*d*(a*Cos[c + d*x] + b*
Sin[c + d*x])^4) + ((-3*a^4 - 72*a^2*b^2 - 8*b^4)*Cos[c + d*x]^4*Log[Cos[(c
 + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(8*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^
4) - (2*(2*a^3*b + 3*a*b^3)*Cos[c + d*x]^4*Log[Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]]*(a + b*Tan[c + d*x])^4)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) +
((3*a^4 + 72*a^2*b^2 + 8*b^4)*Cos[c + d*x]^4*Log[Sin[(c + d*x)/2]]*(a + b*T
an[c + d*x])^4)/(8*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (2*(2*a^3*b + 3
*a*b^3)*Cos[c + d*x]^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Tan[
c + d*x])^4)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (3*(a^4 + 8*a^2*b^2)
*Cos[c + d*x]^4*Sec[(c + d*x)/2]^2*(a + b*Tan[c + d*x])^4)/(32*d*(a*Cos[c +
 d*x] + b*Sin[c + d*x])^4) + (a^4*Cos[c + d*x]^4*Sec[(c + d*x)/2]^4*(a + b*
Tan[c + d*x])^4)/(64*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((12*a*b^3 +
b^4)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(12*d*(Cos[(c + d*x)/2] - Sin[(
c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (b^4*Cos[c + d*x]^4*S
in[(c + d*x)/2]*(a + b*Tan[c + d*x])^4)/(6*d*(Cos[(c + d*x)/2] - Sin[(c + d
*x)/2])^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (b^4*Cos[c + d*x]^4*Sin[(c
 + d*x)/2]*(a + b*Tan[c + d*x])^4)/(6*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
])^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((-12*a*b^3 + b^4)*Cos[c + d*x]
^4*(a + b*Tan[c + d*x])^4)/(12*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(a
*Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cos[c + d*x]^4*Sec[(c + d*x)/2]*(-7*a
^3*b*Sin[(c + d*x)/2] - 6*a*b^3*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^4)/(
3*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cos[c + d*x]^4*(-36*a^2*b^2*Sin
[(c + d*x)/2] - 7*b^4*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^4)/(6*d*(Cos[(
c + d*x)/2] + Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cos
[c + d*x]^4*(36*a^2*b^2*Sin[(c + d*x)/2] + 7*b^4*Sin[(c + d*x)/2])*(a + b*T
an[c + d*x])^4)/(6*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(a*Cos[c + d*x]
+ b*Sin[c + d*x])^4) - (a^3*b*Cos[c + d*x]^4*Sec[(c + d*x)/2]^2*Tan[(c + d*
x)/2]*(a + b*Tan[c + d*x])^4)/(6*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4)
```

Maple [A] (verified)

Time = 17.35 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{b^4\left(\frac{1}{3\cos(dx+c)^3}+\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)+4a\,b^3\left(\frac{1}{2\sin(dx+c)\cos(dx+c)^2}-\frac{3}{2\sin(dx+c)}+\frac{3\ln(\sec(dx+c)+\tan(}{2}\right.}{}$
default	$\frac{b^4\left(\frac{1}{3\cos(dx+c)^3}+\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)+4a\,b^3\left(\frac{1}{2\sin(dx+c)\cos(dx+c)^2}-\frac{3}{2\sin(dx+c)}+\frac{3\ln(\sec(dx+c)+\tan(}{2}\right.}{}$
risch	$\frac{e^{i(dx+c)}\left(216a^2b^2+9a^4+24b^4+216a^2b^2e^{12i(dx+c)}-144a^2b^2e^{10i(dx+c)}-216a^2b^2e^{8i(dx+c)}+144ia\,b^3+192ia\,b^3e^{10i(dx+c)}\right.}{}$

```
[In] int(csc(d*x+c)^5*(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^4*(1/3/cos(d*x+c)^3+1/cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+4*a*b^3*
(1/2/sin(d*x+c)/cos(d*x+c)^2-3/2/sin(d*x+c)+3/2*ln(sec(d*x+c)+tan(d*x+c)))+
6*a^2*b^2*(-1/2/sin(d*x+c)^2/cos(d*x+c)+3/2/cos(d*x+c)+3/2*ln(csc(d*x+c)-co
t(d*x+c)))+4*a^3*b*(-1/3/sin(d*x+c)^3-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)
))+a^4*((-1/4*csc(d*x+c)^3-3/8*csc(d*x+c))*cot(d*x+c)+3/8*ln(csc(d*x+c)-cot
(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 547 vs. 2(264) = 528.

Time = 0.40 (sec) , antiderivative size = 547, normalized size of antiderivative = 2.00

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^4\,dx$$
$$= \frac{6\,(3\,a^4+72\,a^2b^2+8\,b^4)\cos{(dx+c)}^6-10\,(3\,a^4+72\,a^2b^2+8\,b^4)\cos{(dx+c)}^4+16\,b^4+16\,(18\,a^2b^2+b^4)\,}{}$$

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/48*(6*(3*a^4 + 72*a^2*b^2 + 8*b^4)*cos(d*x + c)^6 - 10*(3*a^4 + 72*a^2*b^
2 + 8*b^4)*cos(d*x + c)^4 + 16*b^4 + 16*(18*a^2*b^2 + b^4)*cos(d*x + c)^2 -
 3*((3*a^4 + 72*a^2*b^2 + 8*b^4)*cos(d*x + c)^7 - 2*(3*a^4 + 72*a^2*b^2 + 8
*b^4)*cos(d*x + c)^5 + (3*a^4 + 72*a^2*b^2 + 8*b^4)*cos(d*x + c)^3)*log(1/2
*cos(d*x + c) + 1/2) + 3*((3*a^4 + 72*a^2*b^2 + 8*b^4)*cos(d*x + c)^7 - 2*(
3*a^4 + 72*a^2*b^2 + 8*b^4)*cos(d*x + c)^5 + (3*a^4 + 72*a^2*b^2 + 8*b^4)*c
os(d*x + c)^3)*log(-1/2*cos(d*x + c) + 1/2) + 48*((2*a^3*b + 3*a*b^3)*cos(d
*x + c)^7 - 2*(2*a^3*b + 3*a*b^3)*cos(d*x + c)^5 + (2*a^3*b + 3*a*b^3)*cos(
d*x + c)^3)*log(sin(d*x + c) + 1) - 48*((2*a^3*b + 3*a*b^3)*cos(d*x + c)^7
- 2*(2*a^3*b + 3*a*b^3)*cos(d*x + c)^5 + (2*a^3*b + 3*a*b^3)*cos(d*x + c)^3
)*log(-sin(d*x + c) + 1) + 32*(3*(2*a^3*b + 3*a*b^3)*cos(d*x + c)^5 + 3*a*b
```

```
^3*cos(d*x + c) - 4*(2*a^3*b + 3*a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(d*co
s(d*x + c)^7 - 2*d*cos(d*x + c)^5 + d*cos(d*x + c)^3)
```

Sympy [F]

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^4\,dx = \int (a+b\tan(c+dx))^4\csc^5(c+dx)\,dx$$

```
[In] integrate(csc(d*x+c)**5*(a+b*tan(d*x+c))**4,x)
[Out] Integral((a + b*tan(c + d*x))**4*csc(c + d*x)**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.11

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^4\,dx$$

$$= \frac{3\,a^4\left(\frac{2\left(3\,\cos(dx+c)^3-5\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)+72\,a^2b^2\left(\frac{2\left(3\,\cos(dx+c\ldots}{\cos(dx+c)^3-\mathrm{co}\ldots}\right.}{\ldots}$$

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
[Out] 1/48*(3*a^4*(2*(3*cos(d*x + c)^3 - 5*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(
d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 72*a
^2*b^2*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*log(co
s(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 48*a*b^3*(2*(3*sin(d*x + c)^2
- 2)/(sin(d*x + c)^3 - sin(d*x + c)) - 3*log(sin(d*x + c) + 1) + 3*log(sin(
d*x + c) - 1)) + 8*b^4*(2*(3*cos(d*x + c)^2 + 1)/cos(d*x + c)^3 - 3*log(cos
(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 32*a^3*b*(2*(3*sin(d*x + c)^2 +
 1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)))/d
```

Giac [A] (verification not implemented)

none

Time = 1.09 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.75

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^4\,dx$$

$$= \frac{3\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-32\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+144\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-48\ldots}{\ldots}$$

```
[In] integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^4*tan(1/2*d*x + 1/2*c)^4 - 32*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 24*
a^4*tan(1/2*d*x + 1/2*c)^2 + 144*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 480*a^3*b
*tan(1/2*d*x + 1/2*c) - 384*a*b^3*tan(1/2*d*x + 1/2*c) + 384*(2*a^3*b + 3*a
*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 384*(2*a^3*b + 3*a*b^3)*log(abs(
tan(1/2*d*x + 1/2*c) - 1)) + 24*(3*a^4 + 72*a^2*b^2 + 8*b^4)*log(abs(tan(1/
2*d*x + 1/2*c))) + 256*(3*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 9*a^2*b^2*tan(1/2*
d*x + 1/2*c)^4 - 3*b^4*tan(1/2*d*x + 1/2*c)^4 + 18*a^2*b^2*tan(1/2*d*x + 1/
2*c)^2 + 3*b^4*tan(1/2*d*x + 1/2*c)^2 - 3*a*b^3*tan(1/2*d*x + 1/2*c) - 9*a^
2*b^2 - 2*b^4)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3 - (150*a^4*tan(1/2*d*x + 1/2*
c)^4 + 3600*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 + 400*b^4*tan(1/2*d*x + 1/2*c)^4
 + 480*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 384*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 24
*a^4*tan(1/2*d*x + 1/2*c)^2 + 144*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 32*a^3*b
*tan(1/2*d*x + 1/2*c) + 3*a^4)/tan(1/2*d*x + 1/2*c)^4)/d
```

Mupad [B] (verification not implemented)

Time = 4.90 (sec) , antiderivative size = 857, normalized size of antiderivative = 3.13

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^4\,dx = \text{Too large to display}$$

```
[In] int((a + b*tan(c + d*x))^4/sin(c + d*x)^5,x)
```

```
[Out] (a^4*tan(c/2 + (d*x)/2)^4)/(64*d) - (atan(-((6*a*b^3 + 4*a^3*b)*(12*a*b^3 +
 8*a^3*b - 6*tan(c/2 + (d*x)/2)*(6*a*b^3 + 4*a^3*b) - tan(c/2 + (d*x)/2)*((
3*a^4)/4 + 2*b^4 + 18*a^2*b^2))*1i + (6*a*b^3 + 4*a^3*b)*(12*a*b^3 + 8*a^3*
b + 6*tan(c/2 + (d*x)/2)*(6*a*b^3 + 4*a^3*b) - tan(c/2 + (d*x)/2)*((3*a^4)/
4 + 2*b^4 + 18*a^2*b^2))*1i)/(2*tan(c/2 + (d*x)/2)*(144*a^2*b^6 + 192*a^4*b
^4 + 64*a^6*b^2) + (6*a*b^3 + 4*a^3*b)*(12*a*b^3 + 8*a^3*b - 6*tan(c/2 + (d
*x)/2)*(6*a*b^3 + 4*a^3*b) - tan(c/2 + (d*x)/2)*((3*a^4)/4 + 2*b^4 + 18*a^2
*b^2)) - (6*a*b^3 + 4*a^3*b)*(12*a*b^3 + 8*a^3*b + 6*tan(c/2 + (d*x)/2)*(6*
a*b^3 + 4*a^3*b) - tan(c/2 + (d*x)/2)*((3*a^4)/4 + 2*b^4 + 18*a^2*b^2)) + 2
4*a*b^7 + 6*a^7*b + 232*a^3*b^5 + 153*a^5*b^3))*(a*b^3*12i + a^3*b*8i))/d -
 (tan(c/2 + (d*x)/2)*(2*a^3*b + (a*b*(a^2 + 4*b^2))/2))/d + (log(tan(c/2 +
(d*x)/2))*((3*a^4)/8 + b^4 + 9*a^2*b^2))/d + (tan(c/2 + (d*x)/2)^2*(a^4/8 +
 (3*a^2*b^2)/4))/d - (tan(c/2 + (d*x)/2)^6*((23*a^4)/4 + 64*b^4 + 420*a^2*b
^2) - tan(c/2 + (d*x)/2)^4*((21*a^4)/4 + (128*b^4)/3 + 228*a^2*b^2) - tan(c
/2 + (d*x)/2)^8*(2*a^4 + 64*b^4 + 204*a^2*b^2) + a^4/4 + tan(c/2 + (d*x)/2)
^2*((5*a^4)/4 + 12*a^2*b^2) + tan(c/2 + (d*x)/2)^3*(32*a*b^3 + 32*a^3*b) +
tan(c/2 + (d*x)/2)^9*(32*a*b^3 - 40*a^3*b) - tan(c/2 + (d*x)/2)^5*(160*a*b^
3 + 112*a^3*b) + tan(c/2 + (d*x)/2)^7*(96*a*b^3 + (352*a^3*b)/3) + (8*a^3*b
*tan(c/2 + (d*x)/2))/3)/(d*(16*tan(c/2 + (d*x)/2)^4 - 48*tan(c/2 + (d*x)/2)
^6 + 48*tan(c/2 + (d*x)/2)^8 - 16*tan(c/2 + (d*x)/2)^10)) - (a^3*b*tan(c/2
+ (d*x)/2)^3)/(6*d)
```

3.49 $\int \csc^6(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal result

Integrand size = 21, antiderivative size = 194

$$\begin{aligned}\int \csc^6(c+dx)(a+b\tan(c+dx))^4\,dx =& -\frac{(a^4+12a^2b^2+b^4)\cot(c+dx)}{d} \\ &-\frac{2ab(2a^2+b^2)\cot^2(c+dx)}{d} \\ &-\frac{2a^2(a^2+3b^2)\cot^3(c+dx)}{3d}-\frac{a^3b\cot^4(c+dx)}{d} \\ &-\frac{a^4\cot^5(c+dx)}{5d}+\frac{4ab(a^2+2b^2)\log(\tan(c+dx))}{d} \\ &+\frac{2b^2(3a^2+b^2)\tan(c+dx)}{d} \\ &+\frac{2ab^3\tan^2(c+dx)}{d}+\frac{b^4\tan^3(c+dx)}{3d}\end{aligned}$$

```
[Out] -(a^4+12*a^2*b^2+b^4)*cot(d*x+c)/d-2*a*b*(2*a^2+b^2)*cot(d*x+c)^2/d-2/3*a^2
*(a^2+3*b^2)*cot(d*x+c)^3/d-a^3*b*cot(d*x+c)^4/d-1/5*a^4*cot(d*x+c)^5/d+4*a
*b*(a^2+2*b^2)*ln(tan(d*x+c))/d+2*b^2*(3*a^2+b^2)*tan(d*x+c)/d+2*a*b^3*tan(
d*x+c)^2/d+1/3*b^4*tan(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used

$= \{3597, 962\}$

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^4\,dx = -\frac{a^4\cot^5(c+dx)}{5d} - \frac{a^3b\cot^4(c+dx)}{d} + \frac{2b^2(3a^2+b^2)\tan(c+dx)}{d} - \frac{2a^2(a^2+3b^2)\cot^3(c+dx)}{3d} - \frac{2ab(2a^2+b^2)\cot^2(c+dx)}{d} + \frac{4ab(a^2+2b^2)\log(\tan(c+dx))}{d} - \frac{(a^4+12a^2b^2+b^4)\cot(c+dx)}{d} + \frac{2ab^3\tan^2(c+dx)}{d} + \frac{b^4\tan^3(c+dx)}{3d}$$

```
[In] Int[Csc[c + d*x]^6*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -(((a^4 + 12*a^2*b^2 + b^4)*Cot[c + d*x])/d) - (2*a*b*(2*a^2 + b^2)*Cot[c +
 d*x]^2)/d - (2*a^2*(a^2 + 3*b^2)*Cot[c + d*x]^3)/(3*d) - (a^3*b*Cot[c + d*
x]^4)/d - (a^4*Cot[c + d*x]^5)/(5*d) + (4*a*b*(a^2 + 2*b^2)*Log[Tan[c + d*x
]])/d + (2*b^2*(3*a^2 + b^2)*Tan[c + d*x])/d + (2*a*b^3*Tan[c + d*x]^2)/d +
 (b^4*Tan[c + d*x]^3)/(3*d)
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{b\text{Subst}\left(\int \frac{(a+x)^4(b^2+x^2)^2}{x^6}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{b\text{Subst}\left(\int \left(2(3a^2+b^2) + \frac{a^4b^4}{x^6} + \frac{4a^3b^4}{x^5} + \frac{2a^2b^2(a^2+3b^2)}{x^4} + \frac{4ab^2(2a^2+b^2)}{x^3} + \frac{a^4+12a^2b^2+b^4}{x^2} + \frac{4(a^3+2ab^2)}{x} + 4a\right.}{d}$$

$$= -\frac{(a^4+12a^2b^2+b^4)\cot(c+dx)}{d} - \frac{2ab(2a^2+b^2)\cot^2(c+dx)}{d} - \frac{2a^2(a^2+3b^2)\cot^3(c+dx)}{3d} - \frac{a^3b\cot^4(c+dx)}{d} - \frac{a^4\cot^5(c+dx)}{5d} + \frac{4ab(a^2+2b^2)\log(\tan(c+dx))}{d} + \frac{2b^2(3a^2+b^2)\tan(c+dx)}{d} + \frac{2ab^3\tan^2(c+dx)}{d} + \frac{b^4\tan^3(c+dx)}{3d}$$

Mathematica [A] (verified)

Time = 6.10 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.20

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^4\,dx = -\frac{(15a^3b\cot^4(c+dx)+3a^4\cot^5(c+dx)+2a\cos^2(c+dx)\left(-15b^3+15b(a^2+b^2)\cot^2(c+dx)+a(2a^2+\right.}{}$$

```
[In] Integrate[Csc[c + d*x]^6*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -1/15*((15*a^3*b*Cot[c + d*x]^4 + 3*a^4*Cot[c + d*x]^5 + 2*a*Cos[c + d*x]^2
*(-15*b^3 + 15*b*(a^2 + b^2)*Cot[c + d*x]^2 + a*(2*a^2 + 15*b^2)*Cot[c + d*
x]^3) + Cos[c + d*x]^4*((8*a^4 + 150*a^2*b^2 + 15*b^4)*Cot[c + d*x] + 60*a*
b*(a^2 + 2*b^2)*(Log[Cos[c + d*x]] - Log[Sin[c + d*x]])) - 5*b^2*(18*a^2 +
5*b^2)*Cos[c + d*x]^3*Sin[c + d*x] - (5*b^4*Sin[2*(c + d*x)])/2)*(a + b*Tan
[c + d*x])^4)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4)
```

Maple [A] (verified)

Time = 25.78 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{b^4\left(\frac{1}{3\sin(dx+c)\cos(dx+c)^3}+\frac{4}{3\sin(dx+c)\cos(dx+c)}-\frac{8\cot(dx+c)}{3}\right)+4a\,b^3\left(\frac{1}{2\sin(dx+c)^2\cos(dx+c)^2}-\frac{1}{\sin(dx+c)^2}+2\ln(\tan(dx+c)\right.}{}$
default	$\frac{b^4\left(\frac{1}{3\sin(dx+c)\cos(dx+c)^3}+\frac{4}{3\sin(dx+c)\cos(dx+c)}-\frac{8\cot(dx+c)}{3}\right)+4a\,b^3\left(\frac{1}{2\sin(dx+c)^2\cos(dx+c)^2}-\frac{1}{\sin(dx+c)^2}+2\ln(\tan(dx+c)\right.}{}$
risch	$\frac{64ia^2b^2e^{4i(dx+c)}-32ia^2b^2e^{8i(dx+c)}+64ia^2b^2e^{10i(dx+c)}-128ia^2b^2e^{6i(dx+c)}-56a^3b\,e^{10i(dx+c)}+16a\,b^3e^{10i(dx+c)}+64ia^2b^2}{}$

```
[In] int(csc(d*x+c)^6*(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^4*(1/3/sin(d*x+c)/cos(d*x+c)^3+4/3/sin(d*x+c)/cos(d*x+c)-8/3*cot(d*x
+c))+4*a*b^3*(1/2/sin(d*x+c)^2/cos(d*x+c)^2-1/sin(d*x+c)^2+2*ln(tan(d*x+c))
)+6*a^2*b^2*(-1/3/sin(d*x+c)^3/cos(d*x+c)+4/3/sin(d*x+c)/cos(d*x+c)-8/3*cot
```

```
(d*x+c))+4*a^3*b*(-1/4/sin(d*x+c)^4-1/2/sin(d*x+c)^2+ln(tan(d*x+c)))+a^4*(-
8/15-1/5*csc(d*x+c)^4-4/15*csc(d*x+c)^2)*cot(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(188) = 376$.

Time = 0.29 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.99

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^4\,dx = -\frac{8\,(a^4+30\,a^2b^2+5\,b^4)\cos\left(dx+c\right)^8-20\,(a^4+30\,a^2b^2+5\,b^4)\cos\left(dx+c\right)^6+15\,(a^4+30\,a^2b^2+5\,b^4)\,\mathrm{co}}{}$$

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/15*(8*(a^4 + 30*a^2*b^2 + 5*b^4)*cos(d*x + c)^8 - 20*(a^4 + 30*a^2*b^2 +
 5*b^4)*cos(d*x + c)^6 + 15*(a^4 + 30*a^2*b^2 + 5*b^4)*cos(d*x + c)^4 - 5*b
^4 - 10*(9*a^2*b^2 + b^4)*cos(d*x + c)^2 + 30*((a^3*b + 2*a*b^3)*cos(d*x +
c)^7 - 2*(a^3*b + 2*a*b^3)*cos(d*x + c)^5 + (a^3*b + 2*a*b^3)*cos(d*x + c)^
3)*log(cos(d*x + c)^2)*sin(d*x + c) - 30*((a^3*b + 2*a*b^3)*cos(d*x + c)^7
- 2*(a^3*b + 2*a*b^3)*cos(d*x + c)^5 + (a^3*b + 2*a*b^3)*cos(d*x + c)^3)*lo
g(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) - 15*(2*(a^3*b + 2*a*b^3)*cos(d*x
 + c)^5 + 2*a*b^3*cos(d*x + c) - 3*(a^3*b + 2*a*b^3)*cos(d*x + c)^3)*sin(d*
x + c))/((d*cos(d*x + c)^7 - 2*d*cos(d*x + c)^5 + d*cos(d*x + c)^3)*sin(d*x
 + c))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^4\,dx = \text{Timed out}$$

```
[In] integrate(csc(d*x+c)**6*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.56 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.88

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^4\,dx$$

$$= \frac{5\,b^4\tan\left(dx+c\right)^3 + 30\,ab^3\tan\left(dx+c\right)^2 + 60\left(a^3b+2\,ab^3\right)\log\left(\tan\left(dx+c\right)\right) + 30\left(3\,a^2b^2+b^4\right)\tan\left(dx+c\right)}{15\,d}$$

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/15*(5*b^4*tan(d*x + c)^3 + 30*a*b^3*tan(d*x + c)^2 + 60*(a^3*b + 2*a*b^3)
*log(tan(d*x + c)) + 30*(3*a^2*b^2 + b^4)*tan(d*x + c) - (15*a^3*b*tan(d*x
+ c) + 15*(a^4 + 12*a^2*b^2 + b^4)*tan(d*x + c)^4 + 3*a^4 + 30*(2*a^3*b + a
*b^3)*tan(d*x + c)^3 + 10*(a^4 + 3*a^2*b^2)*tan(d*x + c)^2)/tan(d*x + c)^5)
/d
```

Giac [A] (verification not implemented)

none

Time = 1.18 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.21

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^4\,dx$$

$$= \frac{5\,b^4\tan\left(dx+c\right)^3 + 30\,ab^3\tan\left(dx+c\right)^2 + 90\,a^2b^2\tan\left(dx+c\right) + 30\,b^4\tan\left(dx+c\right) + 60\left(a^3b+2\,ab^3\right)\log\left(|\mathrm{t}\right.}{}$$

```
[In] integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/15*(5*b^4*tan(d*x + c)^3 + 30*a*b^3*tan(d*x + c)^2 + 90*a^2*b^2*tan(d*x +
 c) + 30*b^4*tan(d*x + c) + 60*(a^3*b + 2*a*b^3)*log(abs(tan(d*x + c))) - (
137*a^3*b*tan(d*x + c)^5 + 274*a*b^3*tan(d*x + c)^5 + 15*a^4*tan(d*x + c)^4
 + 180*a^2*b^2*tan(d*x + c)^4 + 15*b^4*tan(d*x + c)^4 + 60*a^3*b*tan(d*x +
c)^3 + 30*a*b^3*tan(d*x + c)^3 + 10*a^4*tan(d*x + c)^2 + 30*a^2*b^2*tan(d*x
 + c)^2 + 15*a^3*b*tan(d*x + c) + 3*a^4)/tan(d*x + c)^5)/d
```

Mupad [B] (verification not implemented)

Time = 4.62 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.93

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^4\,dx = \frac{\ln\left(\tan(c+d\,x)\right)\,\left(4\,a^3\,b+8\,a\,b^3\right)}{d}$$

$$-\frac{\cot(c+d\,x)^5\left(\tan(c+d\,x)^2\left(\frac{2\,a^4}{3}+2\,a^2\,b^2\right)+\tan(c+d\,x)^3\left(4\,a^3\,b+2\,a\,b^3\right)+\frac{a^4}{5}+\tan(c+d\,x)^4\left(a^4+\right.\right.}{d}$$

$$+\frac{b^4\tan(c+d\,x)^3}{3\,d}+\frac{\tan(c+d\,x)\,\left(6\,a^2\,b^2+2\,b^4\right)}{d}+\frac{2\,a\,b^3\tan(c+d\,x)^2}{d}$$

```
[In] int((a + b*tan(c + d*x))^4/sin(c + d*x)^6,x)
```

```
[Out] (log(tan(c + d*x))*(8*a*b^3 + 4*a^3*b))/d - (cot(c + d*x)^5*(tan(c + d*x)^2
*((2*a^4)/3 + 2*a^2*b^2) + tan(c + d*x)^3*(2*a*b^3 + 4*a^3*b) + a^4/5 + tan
(c + d*x)^4*(a^4 + b^4 + 12*a^2*b^2) + a^3*b*tan(c + d*x)))/d + (b^4*tan(c
+ d*x)^3)/(3*d) + (tan(c + d*x)*(2*b^4 + 6*a^2*b^2))/d + (2*a*b^3*tan(c + d
*x)^2)/d
```

3.50 $\int \csc^7(c+dx)(a+b\tan(c+dx))^4\,dx$

Optimal result

Integrand size = 21, antiderivative size = 402

$$\begin{aligned}\int \csc^7(c+dx)(a+b\tan(c+dx))^4\,dx =& -\frac{5a^4\text{arctanh}(\cos(c+dx))}{16d} \\ &-\frac{45a^2b^2\text{arctanh}(\cos(c+dx))}{4d} \\ &-\frac{5b^4\text{arctanh}(\cos(c+dx))}{2d} \\ &+\frac{4a^3b\text{arctanh}(\sin(c+dx))}{d} \\ &+\frac{10ab^3\text{arctanh}(\sin(c+dx))}{d}-\frac{4a^3b\csc(c+dx)}{d} \\ &-\frac{10ab^3\csc(c+dx)}{d}-\frac{5a^4\cot(c+dx)\csc(c+dx)}{16d} \\ &-\frac{4a^3b\csc^3(c+dx)}{3d}-\frac{10ab^3\csc^3(c+dx)}{3d} \\ &-\frac{5a^4\cot(c+dx)\csc^3(c+dx)}{24d}-\frac{4a^3b\csc^5(c+dx)}{5d} \\ &-\frac{a^4\cot(c+dx)\csc^5(c+dx)}{6d}+\frac{45a^2b^2\sec(c+dx)}{4d} \\ &+\frac{5b^4\sec(c+dx)}{2d}-\frac{15a^2b^2\csc^2(c+dx)\sec(c+dx)}{4d} \\ &-\frac{3a^2b^2\csc^4(c+dx)\sec(c+dx)}{2d} \\ &+\frac{2ab^3\csc^3(c+dx)\sec^2(c+dx)}{d} \\ &+\frac{5b^4\sec^3(c+dx)}{6d}-\frac{b^4\csc^2(c+dx)\sec^3(c+dx)}{2d}\end{aligned}$$

```
[Out] -5/16*a^4*arctanh(cos(d*x+c))/d-45/4*a^2*b^2*arctanh(cos(d*x+c))/d-5/2*b^4*
arctanh(cos(d*x+c))/d+4*a^3*b*arctanh(sin(d*x+c))/d+10*a*b^3*arctanh(sin(d*
x+c))/d-4*a^3*b*csc(d*x+c)/d-10*a*b^3*csc(d*x+c)/d-5/16*a^4*cot(d*x+c)*csc(
d*x+c)/d-4/3*a^3*b*csc(d*x+c)^3/d-10/3*a*b^3*csc(d*x+c)^3/d-5/24*a^4*cot(d*
x+c)*csc(d*x+c)^3/d-4/5*a^3*b*csc(d*x+c)^5/d-1/6*a^4*cot(d*x+c)*csc(d*x+c)^
5/d+45/4*a^2*b^2*sec(d*x+c)/d+5/2*b^4*sec(d*x+c)/d-15/4*a^2*b^2*csc(d*x+c)^
2*sec(d*x+c)/d-3/2*a^2*b^2*csc(d*x+c)^4*sec(d*x+c)/d+2*a*b^3*csc(d*x+c)^3*s
ec(d*x+c)^2/d+5/6*b^4*sec(d*x+c)^3/d-1/2*b^4*csc(d*x+c)^2*sec(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3598, 3853, 3855, 2701, 308, 213, 2702, 294, 327}

$$\begin{aligned}\int \csc^7(c+dx)(a+b\tan(c+dx))^4\,dx = &-\frac{5a^4\text{arctanh}(\cos(c+dx))}{16d}\\&-\frac{a^4\cot(c+dx)\csc^5(c+dx)}{6d}\\&-\frac{5a^4\cot(c+dx)\csc^3(c+dx)}{24d}\\&-\frac{5a^4\cot(c+dx)\csc(c+dx)}{16d}\\&+\frac{4a^3b\text{arctanh}(\sin(c+dx))}{d}-\frac{4a^3b\csc^5(c+dx)}{5d}\\&-\frac{4a^3b\csc^3(c+dx)}{3d}-\frac{4a^3b\csc(c+dx)}{d}\\&-\frac{45a^2b^2\text{arctanh}(\cos(c+dx))}{4d}+\frac{45a^2b^2\sec(c+dx)}{4d}\\&-\frac{3a^2b^2\csc^4(c+dx)\sec(c+dx)}{2d}\\&-\frac{15a^2b^2\csc^2(c+dx)\sec(c+dx)}{4d}\\&+\frac{10ab^3\text{arctanh}(\sin(c+dx))}{d}\\&-\frac{10ab^3\csc^3(c+dx)}{3d}-\frac{10ab^3\csc(c+dx)}{d}\\&+\frac{2ab^3\csc^3(c+dx)\sec^2(c+dx)}{d}\\&-\frac{5b^4\text{arctanh}(\cos(c+dx))}{2d}+\frac{5b^4\sec^3(c+dx)}{6d}\\&+\frac{5b^4\sec(c+dx)}{2d}-\frac{b^4\csc^2(c+dx)\sec^3(c+dx)}{2d}\end{aligned}$$

```
[In] Int[Csc[c + d*x]^7*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (-5*a^4*ArcTanh[Cos[c + d*x]])/(16*d) - (45*a^2*b^2*ArcTanh[Cos[c + d*x]])/
(4*d) - (5*b^4*ArcTanh[Cos[c + d*x]])/(2*d) + (4*a^3*b*ArcTanh[Sin[c + d*x]
])/d + (10*a*b^3*ArcTanh[Sin[c + d*x]])/d - (4*a^3*b*Csc[c + d*x])/d - (10*
a*b^3*Csc[c + d*x])/d - (5*a^4*Cot[c + d*x]*Csc[c + d*x])/(16*d) - (4*a^3*b
*Csc[c + d*x]^3)/(3*d) - (10*a*b^3*Csc[c + d*x]^3)/(3*d) - (5*a^4*Cot[c + d
*x]*Csc[c + d*x]^3)/(24*d) - (4*a^3*b*Csc[c + d*x]^5)/(5*d) - (a^4*Cot[c +
d*x]*Csc[c + d*x]^5)/(6*d) + (45*a^2*b^2*Sec[c + d*x])/(4*d) + (5*b^4*Sec[c
 + d*x])/(2*d) - (15*a^2*b^2*Csc[c + d*x]^2*Sec[c + d*x])/(4*d) - (3*a^2*b^
2*Csc[c + d*x]^4*Sec[c + d*x])/(2*d) + (2*a*b^3*Csc[c + d*x]^3*Sec[c + d*x]
^2)/d + (5*b^4*Sec[c + d*x]^3)/(6*d) - (b^4*Csc[c + d*x]^2*Sec[c + d*x]^3)/
(2*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3598

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^4 \csc^7(c+dx) + 4a^3 b \csc^6(c+dx) \sec(c+dx) + 6a^2 b^2 \csc^5(c+dx) \sec^2(c+dx)\right. \\
&\qquad \left. + 4ab^3 \csc^4(c+dx) \sec^3(c+dx) + b^4 \csc^3(c+dx) \sec^4(c+dx)\right) \, dx \\
&= a^4 \int \csc^7(c+dx) \, dx + \left(4a^3 b\right) \int \csc^6(c+dx) \sec(c+dx) \, dx \\
&\quad + \left(6a^2 b^2\right) \int \csc^5(c+dx) \sec^2(c+dx) \, dx \\
&\quad + \left(4ab^3\right) \int \csc^4(c+dx) \sec^3(c+dx) \, dx + b^4 \int \csc^3(c+dx) \sec^4(c+dx) \, dx
\end{aligned}$$

$$= -\frac{a^4 \cot(c+dx)\csc^5(c+dx)}{6d} + \frac{1}{6}\left(5a^4\right)\int \csc^5(c+dx)\,dx$$
$$- \frac{\left(4a^3b\right)\text{Subst}\left(\int \frac{x^6}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d}$$
$$+ \frac{\left(6a^2b^2\right)\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^3}\,dx, x, \sec(c+dx)\right)}{d}$$
$$- \frac{\left(4ab^3\right)\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^2}\,dx, x, \csc(c+dx)\right)}{d}$$
$$+ \frac{b^4\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^2}\,dx, x, \sec(c+dx)\right)}{d}$$
$$= -\frac{5a^4 \cot(c+dx)\csc^3(c+dx)}{24d} - \frac{a^4 \cot(c+dx)\csc^5(c+dx)}{6d}$$
$$- \frac{3a^2b^2\csc^4(c+dx)\sec(c+dx)}{2d} + \frac{2ab^3\csc^3(c+dx)\sec^2(c+dx)}{d}$$
$$- \frac{b^4\csc^2(c+dx)\sec^3(c+dx)}{2d} + \frac{1}{8}\left(5a^4\right)\int \csc^3(c+dx)\,dx$$
$$- \frac{\left(4a^3b\right)\text{Subst}\left(\int \left(1+x^2+x^4+\frac{1}{-1+x^2}\right)\,dx, x, \csc(c+dx)\right)}{d}$$
$$+ \frac{\left(15a^2b^2\right)\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx, x, \sec(c+dx)\right)}{2d}$$
$$- \frac{\left(10ab^3\right)\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d}$$
$$+ \frac{\left(5b^4\right)\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{2d}$$

$$= -\frac{4a^3b\csc(c+dx)}{d} - \frac{5a^4\cot(c+dx)\csc(c+dx)}{16d} - \frac{4a^3b\csc^3(c+dx)}{3d} - \frac{5a^4\cot(c+dx)\csc^3(c+dx)}{24d} - \frac{4a^3b\csc^5(c+dx)}{5d} - \frac{a^4\cot(c+dx)\csc^5(c+dx)}{6d} - \frac{15a^2b^2\csc^2(c+dx)\sec(c+dx)}{4d} - \frac{3a^2b^2\csc^4(c+dx)\sec(c+dx)}{2d} + \frac{2ab^3\csc^3(c+dx)\sec^2(c+dx)}{d} - \frac{b^4\csc^2(c+dx)\sec^3(c+dx)}{2d} + \frac{1}{16}\left(5a^4\right)\int\csc(c+dx)\,dx - \frac{(4a^3b)\,\text{Subst}\left(\int\frac{1}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d} + \frac{(45a^2b^2)\,\text{Subst}\left(\int\frac{x^2}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{4d} - \frac{(10ab^3)\,\text{Subst}\left(\int\left(1+x^2+\frac{1}{-1+x^2}\right)\,dx, x, \csc(c+dx)\right)}{d} + \frac{(5b^4)\,\text{Subst}\left(\int\left(1+x^2+\frac{1}{-1+x^2}\right)\,dx, x, \sec(c+dx)\right)}{2d}$$

$$= -\frac{5a^4\text{arctanh}(\cos(c+dx))}{16d} + \frac{4a^3b\text{arctanh}(\sin(c+dx))}{d} - \frac{4a^3b\csc(c+dx)}{d} - \frac{10ab^3\csc(c+dx)}{d} - \frac{5a^4\cot(c+dx)\csc(c+dx)}{16d} - \frac{4a^3b\csc^3(c+dx)}{3d} - \frac{10ab^3\csc^3(c+dx)}{3d} - \frac{5a^4\cot(c+dx)\csc^3(c+dx)}{24d} - \frac{4a^3b\csc^5(c+dx)}{5d} - \frac{a^4\cot(c+dx)\csc^5(c+dx)}{6d} + \frac{45a^2b^2\sec(c+dx)}{4d} + \frac{5b^4\sec(c+dx)}{2d} - \frac{15a^2b^2\csc^2(c+dx)\sec(c+dx)}{4d} - \frac{3a^2b^2\csc^4(c+dx)\sec(c+dx)}{2d} + \frac{2ab^3\csc^3(c+dx)\sec^2(c+dx)}{d} + \frac{5b^4\sec^3(c+dx)}{6d} - \frac{b^4\csc^2(c+dx)\sec^3(c+dx)}{2d} + \frac{(45a^2b^2)\,\text{Subst}\left(\int\frac{1}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{4d} - \frac{(10ab^3)\,\text{Subst}\left(\int\frac{1}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d} + \frac{(5b^4)\,\text{Subst}\left(\int\frac{1}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{2d}$$

$$
\begin{aligned}
&= -\frac{5a^4\text{arctanh}(\cos(c+dx))}{16d} - \frac{45a^2b^2\text{arctanh}(\cos(c+dx))}{4d} \\
&\quad - \frac{5b^4\text{arctanh}(\cos(c+dx))}{2d} + \frac{4a^3b\text{arctanh}(\sin(c+dx))}{d} \\
&\quad + \frac{10ab^3\text{arctanh}(\sin(c+dx))}{d} - \frac{4a^3b\csc(c+dx)}{d} - \frac{10ab^3\csc(c+dx)}{d} \\
&\quad - \frac{5a^4\cot(c+dx)\csc(c+dx)}{16d} - \frac{4a^3b\csc^3(c+dx)}{3d} - \frac{10ab^3\csc^3(c+dx)}{3d} \\
&\quad - \frac{5a^4\cot(c+dx)\csc^3(c+dx)}{24d} - \frac{4a^3b\csc^5(c+dx)}{5d} \\
&\quad - \frac{a^4\cot(c+dx)\csc^5(c+dx)}{6d} + \frac{45a^2b^2\sec(c+dx)}{4d} + \frac{5b^4\sec(c+dx)}{2d} \\
&\quad - \frac{15a^2b^2\csc^2(c+dx)\sec(c+dx)}{4d} - \frac{3a^2b^2\csc^4(c+dx)\sec(c+dx)}{2d} \\
&\quad + \frac{2ab^3\csc^3(c+dx)\sec^2(c+dx)}{d} + \frac{5b^4\sec^3(c+dx)}{6d} - \frac{b^4\csc^2(c+dx)\sec^3(c+dx)}{2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 8.99 (sec) , antiderivative size = 660, normalized size of antiderivative = 1.64

$$
\begin{aligned}
&\int \csc^7(c+dx)(a+b\tan(c+dx))^4\,dx \\
&= -\frac{5(a^4+36a^2b^2+8b^4)\cos^4(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{16d(a\cos(c+dx)+b\sin(c+dx))^4} \\
&\quad - \frac{2(2a^3b+5ab^3)\cos^4(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{d(a\cos(c+dx)+b\sin(c+dx))^4} \\
&\quad + \frac{5(a^4+36a^2b^2+8b^4)\cos^4(c+dx)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{16d(a\cos(c+dx)+b\sin(c+dx))^4} \\
&\quad + \frac{2(2a^3b+5ab^3)\cos^4(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{d(a\cos(c+dx)+b\sin(c+dx))^4} \\
&\quad + \frac{\cot(c+dx)\csc^5(c+dx)\left(-2545a^4+540a^2b^2+5240b^4-2760a^4\cos(2(c+dx))-7200a^2b^2\cos(2(c+dx\right.}{}
\end{aligned}
$$

```
[In] Integrate[Csc[c + d*x]^7*(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (-5*(a^4 + 36*a^2*b^2 + 8*b^4)*Cos[c + d*x]^4*Log[Cos[(c + d*x)/2]]*(a + b*
Tan[c + d*x])^4)/(16*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (2*(2*a^3*b +
 5*a*b^3)*Cos[c + d*x]^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Ta
n[c + d*x])^4)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (5*(a^4 + 36*a^2*b
^2 + 8*b^4)*Cos[c + d*x]^4*Log[Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(1
6*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (2*(2*a^3*b + 5*a*b^3)*Cos[c + d
*x]^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(d*(
a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cot[c + d*x]*Csc[c + d*x]^5*(-2545*a
```

```
^4 + 540*a^2*b^2 + 5240*b^4 - 2760*a^4*Cos[2*(c + d*x)] - 7200*a^2*b^2*Cos[
2*(c + d*x)] - 6720*b^4*Cos[2*(c + d*x)] + 60*a^4*Cos[4*(c + d*x)] + 2160*a
^2*b^2*Cos[4*(c + d*x)] + 480*b^4*Cos[4*(c + d*x)] + 200*a^4*Cos[6*(c + d*x
)] + 7200*a^2*b^2*Cos[6*(c + d*x)] + 1600*b^4*Cos[6*(c + d*x)] - 75*a^4*Cos
[8*(c + d*x)] - 2700*a^2*b^2*Cos[8*(c + d*x)] - 600*b^4*Cos[8*(c + d*x)] -
15744*a^3*b*Sin[2*(c + d*x)] - 8640*a*b^3*Sin[2*(c + d*x)] - 1152*a^3*b*Sin
[4*(c + d*x)] - 2880*a*b^3*Sin[4*(c + d*x)] + 3200*a^3*b*Sin[6*(c + d*x)] +
 8000*a*b^3*Sin[6*(c + d*x)] - 960*a^3*b*Sin[8*(c + d*x)] - 2400*a*b^3*Sin[
8*(c + d*x)])*(a + b*Tan[c + d*x])^4)/(30720*d*(a*Cos[c + d*x] + b*Sin[c +
d*x])^4)
```

Maple [A] (verified)

Time = 50.86 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{b^4\left(\frac{1}{3\sin(dx+c)^2\cos(dx+c)^3}-\frac{5}{6\sin(dx+c)^2\cos(dx+c)}+\frac{5}{2\cos(dx+c)}+\frac{5\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+4a\,b^3\left(-\frac{1}{3\sin(dx+c)^3\cos(dx-}\right.}{}$
default	$\frac{b^4\left(\frac{1}{3\sin(dx+c)^2\cos(dx+c)^3}-\frac{5}{6\sin(dx+c)^2\cos(dx+c)}+\frac{5}{2\cos(dx+c)}+\frac{5\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+4a\,b^3\left(-\frac{1}{3\sin(dx+c)^3\cos(dx-}\right.}{}$
risch	Expression too large to display

```
[In] int(csc(d*x+c)^7*(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^4*(1/3/sin(d*x+c)^2/cos(d*x+c)^3-5/6/sin(d*x+c)^2/cos(d*x+c)+5/2/cos
(d*x+c)+5/2*ln(csc(d*x+c)-cot(d*x+c)))+4*a*b^3*(-1/3/sin(d*x+c)^3/cos(d*x+c
)^2+5/6/sin(d*x+c)/cos(d*x+c)^2-5/2/sin(d*x+c)+5/2*ln(sec(d*x+c)+tan(d*x+c)
))+6*a^2*b^2*(-1/4/sin(d*x+c)^4/cos(d*x+c)-5/8/sin(d*x+c)^2/cos(d*x+c)+15/8
/cos(d*x+c)+15/8*ln(csc(d*x+c)-cot(d*x+c)))+4*a^3*b*(-1/5/sin(d*x+c)^5-1/3/
sin(d*x+c)^3-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a^4*((-1/6*csc(d*x+c)^
5-5/24*csc(d*x+c)^3-5/16*csc(d*x+c))*cot(d*x+c)+5/16*ln(csc(d*x+c)-cot(d*x+
c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 697, normalized size of antiderivative = 1.73

$$\int \csc^7(c+dx)(a+b\tan(c+dx))^4\,dx$$

$$=\frac{150\,(a^4+36\,a^2b^2+8\,b^4)\cos{(dx+c)}^8-400\,(a^4+36\,a^2b^2+8\,b^4)\cos{(dx+c)}^6+330\,(a^4+36\,a^2b^2+8\,b^4)}{}$$

```
[In] integrate(csc(d*x+c)^7*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/480*(150*(a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*x + c)^8 - 400*(a^4 + 36*a^2*b^
2 + 8*b^4)*cos(d*x + c)^6 + 330*(a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*x + c)^4 -
 160*b^4 - 480*(6*a^2*b^2 + b^4)*cos(d*x + c)^2 - 75*((a^4 + 36*a^2*b^2 + 8
*b^4)*cos(d*x + c)^9 - 3*(a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*x + c)^7 + 3*(a^4
 + 36*a^2*b^2 + 8*b^4)*cos(d*x + c)^5 - (a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*x
+ c)^3)*log(1/2*cos(d*x + c) + 1/2) + 75*((a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*
x + c)^9 - 3*(a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*x + c)^7 + 3*(a^4 + 36*a^2*b^
2 + 8*b^4)*cos(d*x + c)^5 - (a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*x + c)^3)*log(
-1/2*cos(d*x + c) + 1/2) + 480*((2*a^3*b + 5*a*b^3)*cos(d*x + c)^9 - 3*(2*a
^3*b + 5*a*b^3)*cos(d*x + c)^7 + 3*(2*a^3*b + 5*a*b^3)*cos(d*x + c)^5 - (2*
a^3*b + 5*a*b^3)*cos(d*x + c)^3)*log(sin(d*x + c) + 1) - 480*((2*a^3*b + 5*
a*b^3)*cos(d*x + c)^9 - 3*(2*a^3*b + 5*a*b^3)*cos(d*x + c)^7 + 3*(2*a^3*b +
 5*a*b^3)*cos(d*x + c)^5 - (2*a^3*b + 5*a*b^3)*cos(d*x + c)^3)*log(-sin(d*x
 + c) + 1) + 64*(15*(2*a^3*b + 5*a*b^3)*cos(d*x + c)^7 - 35*(2*a^3*b + 5*a*
b^3)*cos(d*x + c)^5 - 15*a*b^3*cos(d*x + c) + 23*(2*a^3*b + 5*a*b^3)*cos(d*
x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^9 - 3*d*cos(d*x + c)^7 + 3*d*cos(d*
x + c)^5 - d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^7(c+dx)(a+b\tan(c+dx))^4\,dx = \text{Timed out}$$

```
[In] integrate(csc(d*x+c)**7*(a+b*tan(d*x+c))**4,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.96

$$\int \csc^7(c+dx)(a+b\tan(c+dx))^4\,dx = \frac{5\,a^4\left(\frac{2\left(15\,\cos(dx+c)^5-40\,\cos(dx+c)^3+33\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)+40\,b^{}}{}$$

```
[In] integrate(csc(d*x+c)^7*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/480*(5*a^4*(2*(15*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 33*cos(d*x + c))/(
cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 15*log(cos(d*x
```

```
+ c) + 1) + 15*log(cos(d*x + c) - 1)) + 40*b^4*(2*(15*cos(d*x + c)^4 - 10*c
os(d*x + c)^2 - 2)/(cos(d*x + c)^5 - cos(d*x + c)^3) - 15*log(cos(d*x + c)
+ 1) + 15*log(cos(d*x + c) - 1)) + 180*a^2*b^2*(2*(15*cos(d*x + c)^4 - 25*c
os(d*x + c)^2 + 8)/(cos(d*x + c)^5 - 2*cos(d*x + c)^3 + cos(d*x + c)) - 15*
log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) - 160*a*b^3*(2*(15*sin(d*
x + c)^4 - 10*sin(d*x + c)^2 - 2)/(sin(d*x + c)^5 - sin(d*x + c)^3) - 15*lo
g(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 64*a^3*b*(2*(15*sin(d*x +
 c)^4 + 5*sin(d*x + c)^2 + 3)/sin(d*x + c)^5 - 15*log(sin(d*x + c) + 1) + 1
5*log(sin(d*x + c) - 1)))/d
```

Giac [A] (verification not implemented)

none

Time = 1.15 (sec) , antiderivative size = 647, normalized size of antiderivative = 1.61

$$\int \csc^7(c+dx)(a+b\tan(c+dx))^4\,dx$$

$$= \frac{5\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6 - 48\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 45\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 + 180\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 - 5}{}$$

```
[In] integrate(csc(d*x+c)^7*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/1920*(5*a^4*tan(1/2*d*x + 1/2*c)^6 - 48*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 45
*a^4*tan(1/2*d*x + 1/2*c)^4 + 180*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 - 560*a^3*
b*tan(1/2*d*x + 1/2*c)^3 - 320*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 225*a^4*tan(1
/2*d*x + 1/2*c)^2 + 2880*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 240*b^4*tan(1/2*d
*x + 1/2*c)^2 - 5280*a^3*b*tan(1/2*d*x + 1/2*c) - 8640*a*b^3*tan(1/2*d*x +
1/2*c) + 3840*(2*a^3*b + 5*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3840
*(2*a^3*b + 5*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 600*(a^4 + 36*a^2
*b^2 + 8*b^4)*log(abs(tan(1/2*d*x + 1/2*c))) + 1280*(6*a*b^3*tan(1/2*d*x +
1/2*c)^5 - 18*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 - 9*b^4*tan(1/2*d*x + 1/2*c)^4
 + 36*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 12*b^4*tan(1/2*d*x + 1/2*c)^2 - 6*a*
b^3*tan(1/2*d*x + 1/2*c) - 18*a^2*b^2 - 7*b^4)/(tan(1/2*d*x + 1/2*c)^2 - 1)
^3 - (1470*a^4*tan(1/2*d*x + 1/2*c)^6 + 52920*a^2*b^2*tan(1/2*d*x + 1/2*c)^
6 + 11760*b^4*tan(1/2*d*x + 1/2*c)^6 + 5280*a^3*b*tan(1/2*d*x + 1/2*c)^5 +
8640*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 225*a^4*tan(1/2*d*x + 1/2*c)^4 + 2880*a
^2*b^2*tan(1/2*d*x + 1/2*c)^4 + 240*b^4*tan(1/2*d*x + 1/2*c)^4 + 560*a^3*b*
tan(1/2*d*x + 1/2*c)^3 + 320*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 45*a^4*tan(1/2*
d*x + 1/2*c)^2 + 180*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 48*a^3*b*tan(1/2*d*x
+ 1/2*c) + 5*a^4)/tan(1/2*d*x + 1/2*c)^6)/d
```

Mupad [B] (verification not implemented)

Time = 4.70 (sec) , antiderivative size = 990, normalized size of antiderivative = 2.46

$$\int \csc^7(c+dx)(a+b\tan(c+dx))^4\,dx = \text{Too large to display}$$

```
[In] int((a + b*tan(c + d*x))^4/sin(c + d*x)^7,x)
```

```
[Out] (a^4*tan(c/2 + (d*x)/2)^6)/(384*d) - (atan(-((10*a*b^3 + 4*a^3*b)*(20*a*b^3
 + 8*a^3*b - 6*tan(c/2 + (d*x)/2)*(10*a*b^3 + 4*a^3*b) - tan(c/2 + (d*x)/2)
*((5*a^4)/8 + 5*b^4 + (45*a^2*b^2)/2))*1i + (10*a*b^3 + 4*a^3*b)*(20*a*b^3
+ 8*a^3*b + 6*tan(c/2 + (d*x)/2)*(10*a*b^3 + 4*a^3*b) - tan(c/2 + (d*x)/2)*
((5*a^4)/8 + 5*b^4 + (45*a^2*b^2)/2))*1i)/(2*tan(c/2 + (d*x)/2)*(400*a^2*b^
6 + 320*a^4*b^4 + 64*a^6*b^2) + (10*a*b^3 + 4*a^3*b)*(20*a*b^3 + 8*a^3*b -
6*tan(c/2 + (d*x)/2)*(10*a*b^3 + 4*a^3*b) - tan(c/2 + (d*x)/2)*((5*a^4)/8 +
 5*b^4 + (45*a^2*b^2)/2)) - (10*a*b^3 + 4*a^3*b)*(20*a*b^3 + 8*a^3*b + 6*ta
n(c/2 + (d*x)/2)*(10*a*b^3 + 4*a^3*b) - tan(c/2 + (d*x)/2)*((5*a^4)/8 + 5*b
^4 + (45*a^2*b^2)/2)) + 100*a*b^7 + 5*a^7*b + 490*a^3*b^5 + (385*a^5*b^3)/2
))*(a*b^3*20i + a^3*b*8i))/d + (tan(c/2 + (d*x)/2)^4*((a^2*(a^2 + 12*b^2))/
128 + a^4/64))/d + (tan(c/2 + (d*x)/2)^2*((a^2*(a^2 + 12*b^2))/16 + (7*a^4)
/128 + b^4/8 + (3*a^2*b^2)/4))/d - (tan(c/2 + (d*x)/2)*((9*a*b^3)/2 + (11*a
^3*b)/4))/d - (tan(c/2 + (d*x)/2)^4*((7*a^4)/2 + 8*b^4 + 78*a^2*b^2) - tan(
c/2 + (d*x)/2)^10*((15*a^4)/2 + 392*b^4 + 864*a^2*b^2) - tan(c/2 + (d*x)/2)
^6*((109*a^4)/6 + (968*b^4)/3 + 1038*a^2*b^2) + tan(c/2 + (d*x)/2)^8*(21*a^
4 + 536*b^4 + 1818*a^2*b^2) + tan(c/2 + (d*x)/2)^2*(a^4 + 6*a^2*b^2) + a^4/
6 - tan(c/2 + (d*x)/2)^11*(32*a*b^3 + 176*a^3*b) + tan(c/2 + (d*x)/2)^3*((3
2*a*b^3)/3 + (208*a^3*b)/15) + tan(c/2 + (d*x)/2)^5*(256*a*b^3 + (624*a^3*b
)/5) - tan(c/2 + (d*x)/2)^7*(1088*a*b^3 + (2368*a^3*b)/5) + tan(c/2 + (d*x)
/2)^9*((2560*a*b^3)/3 + (1528*a^3*b)/3) + (8*a^3*b*tan(c/2 + (d*x)/2))/5)/(
d*(64*tan(c/2 + (d*x)/2)^6 - 192*tan(c/2 + (d*x)/2)^8 + 192*tan(c/2 + (d*x)
/2)^10 - 64*tan(c/2 + (d*x)/2)^12)) - (tan(c/2 + (d*x)/2)^3*((a*b^3)/6 + (7
*a^3*b)/24))/d + (log(tan(c/2 + (d*x)/2))*((5*a^4)/16 + (5*b^4)/2 + (45*a^2
*b^2)/4))/d - (a^3*b*tan(c/2 + (d*x)/2)^5)/(40*d)
```

3.51 $\int \frac{\sin^5(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 274

$$\int \frac{\sin^5(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{a^5 b\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}\,d}+\frac{a^3b^2\cos(c+dx)}{(a^2+b^2)^3\,d}+\frac{ab^2\cos(c+dx)}{(a^2+b^2)^2\,d}-\frac{a\cos(c+dx)}{(a^2+b^2)\,d}-\frac{ab^2\cos^3(c+dx)}{3\,(a^2+b^2)^2\,d}+\frac{2a\cos^3(c+dx)}{3\,(a^2+b^2)\,d}-\frac{a\cos^5(c+dx)}{5\,(a^2+b^2)\,d}+\frac{a^4b\sin(c+dx)}{(a^2+b^2)^3\,d}+\frac{a^2b\sin^3(c+dx)}{3\,(a^2+b^2)^2\,d}+\frac{b\sin^5(c+dx)}{5\,(a^2+b^2)\,d}$$

```
[Out] a^5*b*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(7/2)/
d+a^3*b^2*cos(d*x+c)/(a^2+b^2)^3/d+a*b^2*cos(d*x+c)/(a^2+b^2)^2/d-a*cos(d*x
+c)/(a^2+b^2)/d-1/3*a*b^2*cos(d*x+c)^3/(a^2+b^2)^2/d+2/3*a*cos(d*x+c)^3/(a^
2+b^2)/d-1/5*a*cos(d*x+c)^5/(a^2+b^2)/d+a^4*b*sin(d*x+c)/(a^2+b^2)^3/d+1/3*
a^2*b*sin(d*x+c)^3/(a^2+b^2)^2/d+1/5*b*sin(d*x+c)^5/(a^2+b^2)/d
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used

= {3599, 3188, 2644, 30, 2713, 3178, 3153, 212, 2718}

$$\int \frac{\sin^5(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{b\sin^5(c+dx)}{5d\,(a^2+b^2)} + \frac{a^2b\sin^3(c+dx)}{3d\,(a^2+b^2)^2} - \frac{a\cos^5(c+dx)}{5d\,(a^2+b^2)} + \frac{2a\cos^3(c+dx)}{3d\,(a^2+b^2)} - \frac{ab^2\cos^3(c+dx)}{3d\,(a^2+b^2)^2} - \frac{a\cos(c+dx)}{d\,(a^2+b^2)} + \frac{ab^2\cos(c+dx)}{d\,(a^2+b^2)^2} + \frac{a^5b\,\mathrm{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\,(a^2+b^2)^{7/2}} + \frac{a^4b\sin(c+dx)}{d\,(a^2+b^2)^3} + \frac{a^3b^2\cos(c+dx)}{d\,(a^2+b^2)^3}$$

```
[In] Int[Sin[c + d*x]^5/(a + b*Tan[c + d*x]),x]
```

```
[Out] (a^5*b*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/((a^2 +
b^2)^(7/2)*d) + (a^3*b^2*Cos[c + d*x])/((a^2 + b^2)^3*d) + (a*b^2*Cos[c + d
*x])/((a^2 + b^2)^2*d) - (a*Cos[c + d*x])/((a^2 + b^2)*d) - (a*b^2*Cos[c +
d*x]^3)/(3*(a^2 + b^2)^2*d) + (2*a*Cos[c + d*x]^3)/(3*(a^2 + b^2)*d) - (a*C
os[c + d*x]^5)/(5*(a^2 + b^2)*d) + (a^4*b*Sin[c + d*x])/((a^2 + b^2)^3*d) +
 (a^2*b*Sin[c + d*x]^3)/(3*(a^2 + b^2)^2*d) + (b*Sin[c + d*x]^5)/(5*(a^2 +
b^2)*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3178

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-a)*(Sin[c + d*x]^(m - 1)/(d*(a^2
 + b^2)*(m - 1))), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*
Cos[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*
x]^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m
, 1]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cos(c+dx)\sin^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx \\
&= \frac{a\int \sin^5(c+dx)\,dx}{a^2+b^2}+\frac{b\int \cos(c+dx)\sin^4(c+dx)\,dx}{a^2+b^2}-\frac{(ab)\int \frac{\sin^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{a^2+b^2}
\end{aligned}
$$

$$= \frac{a^2 b \sin^3(c+dx)}{3(a^2+b^2)^2 d} - \frac{(a^3 b) \int \frac{\sin^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)} dx}{(a^2+b^2)^2} - \frac{(ab^2) \int \sin^3(c+dx) dx}{(a^2+b^2)^2} - \frac{a\text{Subst}\left(\int (1-2x^2+x^4) dx, x, \cos(c+dx)\right)}{(a^2+b^2) d} + \frac{b\text{Subst}\left(\int x^4 dx, x, \sin(c+dx)\right)}{(a^2+b^2) d}$$

$$= -\frac{a\cos(c+dx)}{(a^2+b^2) d} + \frac{2a\cos^3(c+dx)}{3(a^2+b^2) d} - \frac{a\cos^5(c+dx)}{5(a^2+b^2) d} + \frac{a^4 b \sin(c+dx)}{(a^2+b^2)^3 d} + \frac{a^2 b \sin^3(c+dx)}{3(a^2+b^2)^2 d} + \frac{b\sin^5(c+dx)}{5(a^2+b^2) d} - \frac{(a^5 b) \int \frac{1}{a\cos(c+dx)+b\sin(c+dx)} dx}{(a^2+b^2)^3} - \frac{(a^3 b^2) \int \sin(c+dx) dx}{(a^2+b^2)^3} + \frac{(ab^2) \text{Subst}\left(\int (1-x^2) dx, x, \cos(c+dx)\right)}{(a^2+b^2)^2 d}$$

$$= \frac{a^3 b^2 \cos(c+dx)}{(a^2+b^2)^3 d} + \frac{ab^2 \cos(c+dx)}{(a^2+b^2)^2 d} - \frac{a\cos(c+dx)}{(a^2+b^2) d} - \frac{ab^2 \cos^3(c+dx)}{3(a^2+b^2)^2 d} + \frac{2a\cos^3(c+dx)}{3(a^2+b^2) d} - \frac{a\cos^5(c+dx)}{5(a^2+b^2) d} + \frac{a^4 b \sin(c+dx)}{(a^2+b^2)^3 d} + \frac{a^2 b \sin^3(c+dx)}{3(a^2+b^2)^2 d} + \frac{b\sin^5(c+dx)}{5(a^2+b^2) d} + \frac{(a^5 b) \text{Subst}\left(\int \frac{1}{a^2+b^2-x^2} dx, x, b\cos(c+dx) - a\sin(c+dx)\right)}{(a^2+b^2)^3 d}$$

$$= \frac{a^5 b \text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2} d} + \frac{a^3 b^2 \cos(c+dx)}{(a^2+b^2)^3 d} + \frac{ab^2 \cos(c+dx)}{(a^2+b^2)^2 d} - \frac{a\cos(c+dx)}{(a^2+b^2) d} - \frac{ab^2 \cos^3(c+dx)}{3(a^2+b^2)^2 d} + \frac{2a\cos^3(c+dx)}{3(a^2+b^2) d} - \frac{a\cos^5(c+dx)}{5(a^2+b^2) d} + \frac{a^4 b \sin(c+dx)}{(a^2+b^2)^3 d} + \frac{a^2 b \sin^3(c+dx)}{3(a^2+b^2)^2 d} + \frac{b\sin^5(c+dx)}{5(a^2+b^2) d}$$

Mathematica [A] (verified)

Time = 3.80 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.05

$$\int \frac{\sin^5(c+dx)}{a+b\tan(c+dx)} dx = \frac{-480a^5 b \text{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right) + \sqrt{a^2+b^2}\left(-30a(5a^4-4a^2b^2-b^4)\cos(c+dx) + 5a(5a^4+6a^2b^2+b^4\right.}{}$$

```
[In] Integrate[Sin[c + d*x]^5/(a + b*Tan[c + d*x]),x]
```

```
[Out] (-480*a^5*b*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + Sqrt[a^2 +
 b^2]*(-30*a*(5*a^4 - 4*a^2*b^2 - b^4)*Cos[c + d*x] + 5*a*(5*a^4 + 6*a^2*b^
2 + b^4)*Cos[3*(c + d*x)] - 3*a^5*Cos[5*(c + d*x)] - 6*a^3*b^2*Cos[5*(c + d
*x)] - 3*a*b^4*Cos[5*(c + d*x)] + 330*a^4*b*Sin[c + d*x] + 120*a^2*b^3*Sin[
```

c + d*x] + 30*b^5*Sin[c + d*x] - 35*a^4*b*Sin[3*(c + d*x)] - 50*a^2*b^3*Sin [3*(c + d*x)] - 15*b^5*Sin[3*(c + d*x)] + 3*a^4*b*Sin[5*(c + d*x)] + 6*a^2* b^3*Sin[5*(c + d*x)] + 3*b^5*Sin[5*(c + d*x)]))/(240*(a^2 + b^2)^(7/2)*d)

Maple [A] (verified)

Time = 9.86 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.31

method	result
derivativedivides	$\frac{-\frac{64a^5 b \,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(32a^6+96a^4b^2+96a^2b^4+32b^6\right)\sqrt{a^2+b^2}}+\frac{2a^4b\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a^3b^2\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\frac{16}{3}a^4b+\frac{4}{3}a^2b^3\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2(6}{}}{}$
default	$\frac{-\frac{64a^5 b \,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(32a^6+96a^4b^2+96a^2b^4+32b^6\right)\sqrt{a^2+b^2}}+\frac{2a^4b\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a^3b^2\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\frac{16}{3}a^4b+\frac{4}{3}a^2b^3\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2(6}{}}{}$
risch	$-\frac{i e^{3i(dx+c)} b}{32(-2iab+a^2-b^2)d}+\frac{5\,e^{3i(dx+c)} a}{96(-2iab+a^2-b^2)d}+\frac{i e^{i(dx+c)} a b}{4(-3ib\,a^2+ib^3+a^3-3a\,b^2)d}-\frac{5\,e^{i(dx+c)} a^2}{16(-3ib\,a^2+ib^3+a^3-3a\,b^2)d}+\frac{}{16}$

[In] int(sin(d*x+c)^5/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(-64*a^5*b/(32*a^6+96*a^4*b^2+96*a^2*b^4+32*b^6)/(a^2+b^2)^(1/2)*arctan h(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))+2/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*(a^4*b*tan(1/2*d*x+1/2*c)^9+a^3*b^2*tan(1/2*d*x+1/2*c)^8+(16/3*a^4 *b+4/3*a^2*b^3)*tan(1/2*d*x+1/2*c)^7+(6*a^3*b^2+2*a*b^4)*tan(1/2*d*x+1/2*c) ^6+(178/15*a^4*b+136/15*a^2*b^3+16/5*b^5)*tan(1/2*d*x+1/2*c)^5+(-16/3*a^5-2 /3*a*b^4)*tan(1/2*d*x+1/2*c)^4+(16/3*a^4*b+4/3*a^2*b^3)*tan(1/2*d*x+1/2*c)^ 3+(2*a^3*b^2-8/3*a^5+2/3*a*b^4)*tan(1/2*d*x+1/2*c)^2+a^4*b*tan(1/2*d*x+1/2* c)-8/15*a^5+3/5*a^3*b^2+2/15*a*b^4)/(1+tan(1/2*d*x+1/2*c)^2)^5)

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.35

$$\int \frac{\sin^5(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{15\sqrt{a^2+b^2}a^5b\log\left(\frac{2ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2a^2-b^2-2\sqrt{a^2+b^2}(b\cos(dx+c)-a\sin(dx+c))}{2ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)-6\left(a^7+3a\right.}{}$$

[In] integrate(sin(d*x+c)^5/(a+b*tan(d*x+c)),x, algorithm="fricas")

[Out] 1/30*(15*sqrt(a^2 + b^2)*a^5*b*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a

```
*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)
^2 + b^2)) - 6*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(d*x + c)^5 + 10*(2
*a^7 + 5*a^5*b^2 + 4*a^3*b^4 + a*b^6)*cos(d*x + c)^3 - 30*(a^7 + a^5*b^2)*c
os(d*x + c) + 2*(23*a^6*b + 34*a^4*b^3 + 14*a^2*b^5 + 3*b^7 + 3*(a^6*b + 3*
a^4*b^3 + 3*a^2*b^5 + b^7)*cos(d*x + c)^4 - (11*a^6*b + 28*a^4*b^3 + 23*a^2
*b^5 + 6*b^7)*cos(d*x + c)^2)*sin(d*x + c))/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
 4*a^2*b^6 + b^8)*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(c+dx)}{a+b\tan(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate(sin(d*x+c)**5/(a+b*tan(d*x+c)),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(260) = 520$.

Time = 0.31 (sec) , antiderivative size = 658, normalized size of antiderivative = 2.40

$$\int \frac{\sin^5(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{15\,a^5 b\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}} - \frac{2\left(8\,a^5-9\,a^3b^2-2\,ab^4-\frac{15\,a^4b\sin(dx+c)}{\cos(dx+c)+1}-\frac{15\,a^3b^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{15\,a^4b\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{10\left(4\,a^5-3\,a^3b^2-ab^4\right)\sin\cdots}{(\cos(dx+c)+1)^2}\cdots\right.}{a^6+3\,a^4b^2+3\,a^2b^4+b^6+\frac{5\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10\left(a^6+3\,a^4b^2+3\,a\cdots\right.}{(\cos(d\cdots}\cdots}}{\cdots}$$

```
[In] integrate(sin(d*x+c)^5/(a+b*tan(d*x+c)),x, algorithm="maxima")
[Out] 1/15*(15*a^5*b*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2)
)/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a^6 + 3*a^4*
b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) - 2*(8*a^5 - 9*a^3*b^2 - 2*a*b^4 -
15*a^4*b*sin(d*x + c)/(cos(d*x + c) + 1) - 15*a^3*b^2*sin(d*x + c)^8/(cos(d
*x + c) + 1)^8 - 15*a^4*b*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 10*(4*a^5 -
 3*a^3*b^2 - a*b^4)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 20*(4*a^4*b + a^2
*b^3)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 10*(8*a^5 + a*b^4)*sin(d*x + c)
^4/(cos(d*x + c) + 1)^4 - 2*(89*a^4*b + 68*a^2*b^3 + 24*b^5)*sin(d*x + c)^5
/(cos(d*x + c) + 1)^5 - 30*(3*a^3*b^2 + a*b^4)*sin(d*x + c)^6/(cos(d*x + c)
 + 1)^6 - 20*(4*a^4*b + a^2*b^3)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/(a^6
+ 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sin
```

```
(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*s
in(d*x + c)^6/(cos(d*x + c) + 1)^6 + 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*
sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*s
in(d*x + c)^10/(cos(d*x + c) + 1)^10))/d
```

Giac [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.69

$$\int \frac{\sin^5(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{15\,a^5 b \log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}} + \frac{2\left(15\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+15\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+80\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+20\,a^2b^3\tan\left(\frac{1}{2}\,c\right.\right.}{}}{}$$

```
[In] integrate(sin(d*x+c)^5/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/15*(15*a^5*b*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/
abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 2*(15*a^4*b*tan(1/2*d*x + 1/2*c)^9 +
15*a^3*b^2*tan(1/2*d*x + 1/2*c)^8 + 80*a^4*b*tan(1/2*d*x + 1/2*c)^7 + 20*a^
2*b^3*tan(1/2*d*x + 1/2*c)^7 + 90*a^3*b^2*tan(1/2*d*x + 1/2*c)^6 + 30*a*b^4
*tan(1/2*d*x + 1/2*c)^6 + 178*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 136*a^2*b^3*ta
n(1/2*d*x + 1/2*c)^5 + 48*b^5*tan(1/2*d*x + 1/2*c)^5 - 80*a^5*tan(1/2*d*x +
 1/2*c)^4 - 10*a*b^4*tan(1/2*d*x + 1/2*c)^4 + 80*a^4*b*tan(1/2*d*x + 1/2*c)
^3 + 20*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 40*a^5*tan(1/2*d*x + 1/2*c)^2 + 30
*a^3*b^2*tan(1/2*d*x + 1/2*c)^2 + 10*a*b^4*tan(1/2*d*x + 1/2*c)^2 + 15*a^4*
b*tan(1/2*d*x + 1/2*c) - 8*a^5 + 9*a^3*b^2 + 2*a*b^4)/((a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6)*(tan(1/2*d*x + 1/2*c)^2 + 1)^5))/d
```

Mupad [B] (verification not implemented)

Time = 8.30 (sec) , antiderivative size = 683, normalized size of antiderivative = 2.49

$$\int \frac{\sin^5(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{2\,(-8\,a^5+9\,a^3\,b^2+2\,a\,b^4)}{15\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)} + \frac{8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(4\,a^4\,b+a^2\,b^3)}{3\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)} + \frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,(3\,a^3\,b^2+a\,b^4)}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6} + \frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(-4\,a^5+3\,a^3\,b^2+a\,b^4)}{3\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)} + \cdots}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+5\tan\left(\frac{c}{2}+\cdots\right.\right.} + \frac{2\,a^5\,b\,\mathrm{atanh}\left(\frac{2\,a^6\,b+2\,b^7+6\,a^2\,b^5+6\,a^4\,b^3-2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)}{2\,(a^2+b^2)^{7/2}}\right)}{d\,(a^2+b^2)^{7/2}}$$

```
[In] int(sin(c + d*x)^5/(a + b*tan(c + d*x)),x)
```

```
[Out] ((2*(2*a*b^4 - 8*a^5 + 9*a^3*b^2))/(15*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))
 + (8*tan(c/2 + (d*x)/2)^3*(4*a^4*b + a^2*b^3))/(3*(a^6 + b^6 + 3*a^2*b^4 +
 3*a^4*b^2)) + (4*tan(c/2 + (d*x)/2)^6*(a*b^4 + 3*a^3*b^2))/(a^6 + b^6 + 3*
a^2*b^4 + 3*a^4*b^2) + (4*tan(c/2 + (d*x)/2)^2*(a*b^4 - 4*a^5 + 3*a^3*b^2))
/(3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (4*tan(c/2 + (d*x)/2)^5*(89*a^4*
b + 24*b^5 + 68*a^2*b^3))/(15*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (4*tan
(c/2 + (d*x)/2)^4*(a*b^4 + 8*a^5))/(3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))
+ (2*a^3*b^2*tan(c/2 + (d*x)/2)^8)/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (8
*b*tan(c/2 + (d*x)/2)^7*(4*a^4 + a^2*b^2))/(3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^
4*b^2)) + (2*a^4*b*tan(c/2 + (d*x)/2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)
+ (2*a^4*b*tan(c/2 + (d*x)/2)^9)/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))/(d*(5
*tan(c/2 + (d*x)/2)^2 + 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 +
 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 + 1)) + (2*a^5*b*atanh((2*a
^6*b + 2*b^7 + 6*a^2*b^5 + 6*a^4*b^3 - 2*a*tan(c/2 + (d*x)/2)*(a^6 + b^6 +
3*a^2*b^4 + 3*a^4*b^2))/(2*(a^2 + b^2)^(7/2))))/(d*(a^2 + b^2)^(7/2))
```

3.52 $\int \frac{\sin^4(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 158

$$\begin{aligned}\int \frac{\sin^4(c+dx)}{a+b\tan(c+dx)}\,dx &= \frac{a(3a^4-6a^2b^2-b^4)\,x}{8\,(a^2+b^2)^3}+\frac{a^4b\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^3\,d}\\&\quad+\frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4\,(a^2+b^2)\,d}\\&\quad-\frac{\cos^2(c+dx)\,(4b(2a^2+b^2)+a(5a^2+b^2)\tan(c+dx))}{8\,(a^2+b^2)^2\,d}\end{aligned}$$

```
[Out] 1/8*a*(3*a^4-6*a^2*b^2-b^4)*x/(a^2+b^2)^3+a^4*b*ln(a*cos(d*x+c)+b*sin(d*x+c
))/(a^2+b^2)^3/d+1/4*cos(d*x+c)^4*(b+a*tan(d*x+c))/(a^2+b^2)/d-1/8*cos(d*x+
c)^2*(4*b*(2*a^2+b^2)+a*(5*a^2+b^2)*tan(d*x+c))/(a^2+b^2)^2/d
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3597, 1661, 815, 649, 209, 266}

$$\begin{aligned}\int \frac{\sin^4(c+dx)}{a+b\tan(c+dx)}\,dx &= \frac{\cos^4(c+dx)(a\tan(c+dx)+b)}{4d\,(a^2+b^2)}\\&\quad-\frac{\cos^2(c+dx)\,(a(5a^2+b^2)\tan(c+dx)+4b(2a^2+b^2))}{8d\,(a^2+b^2)^2}\\&\quad+\frac{a^4b\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^3}+\frac{ax(3a^4-6a^2b^2-b^4)}{8\,(a^2+b^2)^3}\end{aligned}$$

```
[In] Int[Sin[c + d*x]^4/(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*(3*a^4 - 6*a^2*b^2 - b^4)*x)/(8*(a^2 + b^2)^3) + (a^4*b*Log[a*Cos[c + d*
x] + b*Sin[c + d*x]])/((a^2 + b^2)^3*d) + (Cos[c + d*x]^4*(b + a*Tan[c + d*
x]))/(4*(a^2 + b^2)*d) - (Cos[c + d*x]^2*(4*b*(2*a^2 + b^2) + a*(5*a^2 + b^
2)*Tan[c + d*x]))/(8*(a^2 + b^2)^2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{b\text{Subst}\left(\int \frac{x^4}{(a+x)(b^2+x^2)^3}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4(a^2+b^2)d} - \frac{\text{Subst}\left(\int \frac{\frac{a^2b^4}{a^2+b^2} - \frac{3ab^4x}{a^2+b^2} - 4b^2x^2}{(a+x)(b^2+x^2)^2}\,dx, x, b\tan(c+dx)\right)}{4bd}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4(a^2+b^2)d} - \frac{\cos^2(c+dx)\left(4b(2a^2+b^2)+a(5a^2+b^2)\tan(c+dx)\right)}{8(a^2+b^2)^2 d} + \frac{\text{Subst}\left(\int \frac{\frac{a^2b^4(3a^2-b^2)}{(a^2+b^2)^2} - \frac{ab^4(5a^2+b^2)x}{(a^2+b^2)^2}}{(a+x)(b^2+x^2)}\,dx, x, b\tan(c+dx)\right)}{8b^3d}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4(a^2+b^2)d} - \frac{\cos^2(c+dx)\left(4b(2a^2+b^2)+a(5a^2+b^2)\tan(c+dx)\right)}{8(a^2+b^2)^2 d} + \frac{\text{Subst}\left(\int \left(\frac{8a^4b^4}{(a^2+b^2)^3(a+x)} + \frac{ab^4(3a^4-6a^2b^2-b^4-8a^3x)}{(a^2+b^2)^3(b^2+x^2)}\right)dx, x, b\tan(c+dx)\right)}{8b^3d}$$

$$= \frac{a^4b\log(a+b\tan(c+dx))}{(a^2+b^2)^3 d} + \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4(a^2+b^2)d} - \frac{\cos^2(c+dx)\left(4b(2a^2+b^2)+a(5a^2+b^2)\tan(c+dx)\right)}{8(a^2+b^2)^2 d} + \frac{(ab)\text{Subst}\left(\int \frac{3a^4-6a^2b^2-b^4-8a^3x}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{8(a^2+b^2)^3 d}$$

$$= \frac{a^4b\log(a+b\tan(c+dx))}{(a^2+b^2)^3 d} + \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4(a^2+b^2)d} - \frac{\cos^2(c+dx)\left(4b(2a^2+b^2)+a(5a^2+b^2)\tan(c+dx)\right)}{8(a^2+b^2)^2 d} - \frac{(a^4b)\,\text{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{(a^2+b^2)^3 d} + \frac{(ab(3a^4-6a^2b^2-b^4))\,\text{Subst}\left(\int \frac{1}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{8(a^2+b^2)^3 d}$$

$$= \frac{a(3a^4-6a^2b^2-b^4)x}{8(a^2+b^2)^3} + \frac{a^4b\log(\cos(c+dx))}{(a^2+b^2)^3 d} + \frac{a^4b\log(a+b\tan(c+dx))}{(a^2+b^2)^3 d} + \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4(a^2+b^2)d} - \frac{\cos^2(c+dx)\left(4b(2a^2+b^2)+a(5a^2+b^2)\tan(c+dx)\right)}{8(a^2+b^2)^2 d}$$

Mathematica [A] (verified)

Time = 4.17 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.58

$$\int \frac{\sin^4(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{2ab(5a^4+6a^2b^2+b^4)\arctan(\tan(c+dx))+8b^2(2a^4+3a^2b^2+b^4)\cos^2(c+dx)-4b^2(a^2+b^2)^2\cos^4(c+\cdots}{\cdots}$$

[In] Integrate[Sin[c + d*x]^4/(a + b*Tan[c + d*x]),x]

[Out] -1/16*(2*a*b*(5*a^4 + 6*a^2*b^2 + b^4)*ArcTan[Tan[c + d*x]] + 8*b^2*(2*a^4 + 3*a^2*b^2 + b^4)*Cos[c + d*x]^2 - 4*b^2*(a^2 + b^2)^2*Cos[c + d*x]^4 + 8*a^4*((b^2 + a*Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]] - 2*b^2*Log[a + b*Tan[c + d*x]] + (b^2 - a*Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]]) - 4*a*b*(a^2 + b^2)^2*Cos[c + d*x]^3*Sin[c + d*x] + a*(5*a^4*b + 6*a^2*b^3 + b^5)*Sin[2*(c + d*x)])/(b*(a^2 + b^2)^3*d)

Maple [A] (verified)

Time = 4.69 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.32

method	result
derivativedivides	$\frac{\frac{\frac{\left(-\frac{5}{8}a^5-\frac{3}{4}a^3b^2-\frac{1}{8}a\,b^4\right)\left(\tan^3(dx+c)\right)+\left(-a^4b-\frac{3}{2}a^2b^3-\frac{1}{2}b^5\right)\left(\tan^2(dx+c)\right)+\left(-\frac{3}{8}a^5-\frac{1}{4}a^3b^2+\frac{1}{8}a\,b^4\right)\tan(dx+c)-\frac{3a^4b}{4}-a^2b^3-\frac{b^5}{4}}{\left(1+\tan^2(dx+c)\right)^2}\cdots}{\left(a^2+b^2\right)^3}\cdots}{d}$
default	$\frac{\frac{\frac{\left(-\frac{5}{8}a^5-\frac{3}{4}a^3b^2-\frac{1}{8}a\,b^4\right)\left(\tan^3(dx+c)\right)+\left(-a^4b-\frac{3}{2}a^2b^3-\frac{1}{2}b^5\right)\left(\tan^2(dx+c)\right)+\left(-\frac{3}{8}a^5-\frac{1}{4}a^3b^2+\frac{1}{8}a\,b^4\right)\tan(dx+c)-\frac{3a^4b}{4}-a^2b^3-\frac{b^5}{4}}{\left(1+\tan^2(dx+c)\right)^2}\cdots}{\left(a^2+b^2\right)^3}\cdots}{d}$
risch	$\frac{iaxb}{24ib\,a^2-8ib^3-8a^3+24a\,b^2}-\frac{3a^2x}{8(3ib\,a^2-ib^3-a^3+3a\,b^2)}+\frac{\mathrm{e}^{2i(dx+c)}b}{16(-2iab+a^2-b^2)d}+\frac{i\mathrm{e}^{2i(dx+c)}a}{8(-2iab+a^2-b^2)d}+\frac{\mathrm{e}^{-2i(dx+c)}b}{16(ib+a)^2d}\cdots$

[In] int(sin(d*x+c)^4/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(1/(a^2+b^2)^3*(((-5/8*a^5-3/4*a^3*b^2-1/8*a*b^4)*tan(d*x+c)^3+(-a^4*b-3/2*a^2*b^3-1/2*b^5)*tan(d*x+c)^2+(-3/8*a^5-1/4*a^3*b^2+1/8*a*b^4)*tan(d*x+c)-3/4*a^4*b-a^2*b^3-1/4*b^5)/(1+tan(d*x+c)^2)^2+1/8*a*(-4*a^3*b*ln(1+tan(d*x+c)^2)+(3*a^4-6*a^2*b^2-b^4)*arctan(tan(d*x+c))))+a^4*b/(a^2+b^2)^3*ln(a+b*tan(d*x+c)))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.37

$$\int \frac{\sin^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{4\,a^4b\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)+2\left(a^4b+2\,a^2b^3+b^5\right)\cos\left(dx+c\right)^4}{}$$

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*a^4*b*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)
^2 + b^2) + 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^4 + (3*a^5 - 6*a^3*b^2
 - a*b^4)*d*x - 4*(2*a^4*b + 3*a^2*b^3 + b^5)*cos(d*x + c)^2 + (2*(a^5 + 2*
a^3*b^2 + a*b^4)*cos(d*x + c)^3 - (5*a^5 + 6*a^3*b^2 + a*b^4)*cos(d*x + c))
*sin(d*x + c))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^4(c+dx)}{a+b\tan(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate(sin(d*x+c)**4/(a+b*tan(d*x+c)),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.77

$$\int \frac{\sin^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{8\,a^4b\log\left(b\tan\left(dx+c\right)+a\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{4\,a^4b\log\left(\tan\left(dx+c\right)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{\left(3\,a^5-6\,a^3b^2-ab^4\right)\left(dx+c\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{\left(5\,a^3+ab^2\right)\tan\left(dx+c\right)^3+6\,a^2b+2\,b^3+4\left(2\,a^2b+b^3\right)\tan}{\left(a^4+2\,a^2b^2+b^4\right)\tan\left(dx+c\right)^4+a^4+2\,a^2b^2+b^4+2}}{8\,d}$$

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/8*(8*a^4*b*log(b*tan(d*x + c) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) -
4*a^4*b*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (3*a^
5 - 6*a^3*b^2 - a*b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - ((5*
a^3 + a*b^2)*tan(d*x + c)^3 + 6*a^2*b + 2*b^3 + 4*(2*a^2*b + b^3)*tan(d*x +
 c)^2 + (3*a^3 - a*b^2)*tan(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*tan(d*x + c)
^4 + a^4 + 2*a^2*b^2 + b^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*tan(d*x + c)^2))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. 2(152) = 304.

Time = 0.40 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.11

$$\int \frac{\sin^4(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{\frac{8\,a^4b^2\log(|b\tan(dx+c)+a|)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7} - \frac{4\,a^4b\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{(3\,a^5-6\,a^3b^2-ab^4)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{6\,a^4b\tan(dx+c)^4-5\,a^5\tan(dx+c)^3-6\,a^3b^2\tan(dx+}{}}{8\,d}$$

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(8*a^4*b^2*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5
+ b^7) - 4*a^4*b*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
) + (3*a^5 - 6*a^3*b^2 - a*b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6) + (6*a^4*b*tan(d*x + c)^4 - 5*a^5*tan(d*x + c)^3 - 6*a^3*b^2*tan(d*x + c
)^3 - a*b^4*tan(d*x + c)^3 + 4*a^4*b*tan(d*x + c)^2 - 12*a^2*b^3*tan(d*x +
c)^2 - 4*b^5*tan(d*x + c)^2 - 3*a^5*tan(d*x + c) - 2*a^3*b^2*tan(d*x + c) +
 a*b^4*tan(d*x + c) - 8*a^2*b^3 - 2*b^5)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6)*(tan(d*x + c)^2 + 1)^2))/d
```

Mupad [B] (verification not implemented)

Time = 5.20 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.98

$$\int \frac{\sin^4(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{a^4\,b\,\ln\left(a+b\tan(c+d\,x)\right)}{d\,(a^2+b^2)^3} - \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(a\,b-a^2\,3\mathrm{i}\right)}{16\,d\,\left(-a^3-a^2\,b\,3\mathrm{i}+3\,a\,b^2+b^3\,1\mathrm{i}\right)}$$
$$- \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(-3\,a^2+a\,b\,1\mathrm{i}\right)}{16\,d\,\left(-a^3\,1\mathrm{i}-3\,a^2\,b+a\,b^2\,3\mathrm{i}+b^3\right)}$$
$$- \frac{\frac{3\,a^2\,b+b^3}{4\,(a^4+2\,a^2\,b^2+b^4)} + \frac{\tan(c+d\,x)^3\,(5\,a^3+a\,b^2)}{8\,(a^4+2\,a^2\,b^2+b^4)} + \frac{\tan(c+d\,x)^2\,(2\,a^2\,b+b^3)}{2\,(a^4+2\,a^2\,b^2+b^4)} + \frac{a\tan(c+d\,x)\,(3\,a^2-b^2)}{8\,(a^4+2\,a^2\,b^2+b^4)}}{d\,\left(\tan\left(c+d\,x\right)^4+2\tan\left(c+d\,x\right)^2+1\right)}$$

```
[In] int(sin(c + d*x)^4/(a + b*tan(c + d*x)),x)
```

```
[Out] (a^4*b*log(a + b*tan(c + d*x)))/(d*(a^2 + b^2)^3) - (log(tan(c + d*x) - 1i)
*(a*b - a^2*3i))/(16*d*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)) - (log(tan(c +
d*x) + 1i)*(a*b*1i - 3*a^2))/(16*d*(a*b^2*3i - 3*a^2*b - a^3*1i + b^3)) - (
(3*a^2*b + b^3)/(4*(a^4 + b^4 + 2*a^2*b^2)) + (tan(c + d*x)^3*(a*b^2 + 5*a^
3))/(8*(a^4 + b^4 + 2*a^2*b^2)) + (tan(c + d*x)^2*(2*a^2*b + b^3))/(2*(a^4
+ b^4 + 2*a^2*b^2)) + (a*tan(c + d*x)*(3*a^2 - b^2))/(8*(a^4 + b^4 + 2*a^2*
b^2)))/(d*(2*tan(c + d*x)^2 + tan(c + d*x)^4 + 1))
```

3.53 $\int \frac{\sin^3(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 168

$$\int \frac{\sin^3(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{a^3 b\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}\,d} + \frac{ab^2\cos(c+dx)}{(a^2+b^2)^2\,d} - \frac{a\cos(c+dx)}{(a^2+b^2)\,d} + \frac{a\cos^3(c+dx)}{3\,(a^2+b^2)\,d} + \frac{a^2 b\sin(c+dx)}{(a^2+b^2)^2\,d} + \frac{b\sin^3(c+dx)}{3\,(a^2+b^2)\,d}$$

```
[Out] a^3*b*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)/
d+a*b^2*cos(d*x+c)/(a^2+b^2)^2/d-a*cos(d*x+c)/(a^2+b^2)/d+1/3*a*cos(d*x+c)^
3/(a^2+b^2)/d+a^2*b*sin(d*x+c)/(a^2+b^2)^2/d+1/3*b*sin(d*x+c)^3/(a^2+b^2)/d
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3599, 3188, 2644, 30, 2713, 3178, 3153, 212, 2718}

$$\int \frac{\sin^3(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{b\sin^3(c+dx)}{3d\,(a^2+b^2)} + \frac{a^2 b\sin(c+dx)}{d\,(a^2+b^2)^2} + \frac{a\cos^3(c+dx)}{3d\,(a^2+b^2)} - \frac{a\cos(c+dx)}{d\,(a^2+b^2)} + \frac{ab^2\cos(c+dx)}{d\,(a^2+b^2)^2} + \frac{a^3 b\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\,(a^2+b^2)^{5/2}}$$

```
[In] Int[Sin[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

```
[Out] (a^3*b*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/((a^2 +
b^2)^(5/2)*d) + (a*b^2*Cos[c + d*x])/((a^2 + b^2)^2*d) - (a*Cos[c + d*x])/(
```

```
(a^2 + b^2)*d) + (a*Cos[c + d*x]^3)/(3*(a^2 + b^2)*d) + (a^2*b*Sin[c + d*x]
)/((a^2 + b^2)^2*d) + (b*Sin[c + d*x]^3)/(3*(a^2 + b^2)*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3178

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-a)*(Sin[c + d*x]^(m - 1)/(d*(a^2
 + b^2)*(m - 1))), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*
Cos[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*
x]^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m
, 1]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rubi steps

$$\text{integral} = \int \frac{\cos(c+dx)\sin^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$

$$= \frac{a\int \sin^3(c+dx)\,dx}{a^2+b^2}+\frac{b\int \cos(c+dx)\sin^2(c+dx)\,dx}{a^2+b^2}-\frac{(ab)\int \frac{\sin^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{a^2+b^2}$$

$$= \frac{a^2b\sin(c+dx)}{(a^2+b^2)^2 d}-\frac{(a^3b)\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{(a^2+b^2)^2}-\frac{(ab^2)\int \sin(c+dx)\,dx}{(a^2+b^2)^2} - \frac{a\text{Subst}\left(\int (1-x^2)\ dx, x, \cos(c+dx)\right)}{(a^2+b^2)\,d}+\frac{b\text{Subst}\left(\int x^2\,dx, x, \sin(c+dx)\right)}{(a^2+b^2)\,d}$$

$$= \frac{ab^2\cos(c+dx)}{(a^2+b^2)^2 d}-\frac{a\cos(c+dx)}{(a^2+b^2)\,d}+\frac{a\cos^3(c+dx)}{3(a^2+b^2)\,d}+\frac{a^2b\sin(c+dx)}{(a^2+b^2)^2 d} + \frac{b\sin^3(c+dx)}{3(a^2+b^2)\,d}+\frac{(a^3b)\,\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx, x, b\cos(c+dx)-a\sin(c+dx)\right)}{(a^2+b^2)^2 d}$$

$$= \frac{a^3b\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2} d}+\frac{ab^2\cos(c+dx)}{(a^2+b^2)^2 d}-\frac{a\cos(c+dx)}{(a^2+b^2)\,d} + \frac{a\cos^3(c+dx)}{3(a^2+b^2)\,d}+\frac{a^2b\sin(c+dx)}{(a^2+b^2)^2 d}+\frac{b\sin^3(c+dx)}{3(a^2+b^2)\,d}$$

Mathematica [A] (verified)

Time = 1.80 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.83

$$\int \frac{\sin^3(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{-24a^3b\,\mathrm{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)+\sqrt{a^2+b^2}\left((-9a^3+3ab^2)\cos(c+dx)+a(a^2+b^2)\cos(3(c+dx))-2b(\ldots\right.}{12\left(a^2+b^2\right)^{5/2}d}$$

```
[In] Integrate[Sin[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

```
[Out] (-24*a^3*b*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + Sqrt[a^2 +
b^2]*((-9*a^3 + 3*a*b^2)*Cos[c + d*x] + a*(a^2 + b^2)*Cos[3*(c + d*x)] - 2*
b*(-7*a^2 - b^2 + (a^2 + b^2)*Cos[2*(c + d*x)])*Sin[c + d*x]))/(12*(a^2 + b
^2)^(5/2)*d)
```

Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.20

method	result
derivativedivides	$\frac{-\frac{16a^3b\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(8a^4+16a^2b^2+8b^4\right)\sqrt{a^2+b^2}}+\frac{2a^2b\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\,b^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\frac{10}{3}a^2b+\frac{4}{3}b^3\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a^3\left(\tan^2\left(\ldots\right.\right.}{\left(a^4+2a^2b^2+b^4\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}}{d}$
default	$\frac{-\frac{16a^3b\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(8a^4+16a^2b^2+8b^4\right)\sqrt{a^2+b^2}}+\frac{2a^2b\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\,b^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\frac{10}{3}a^2b+\frac{4}{3}b^3\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a^3\left(\tan^2\left(\ldots\right.\right.}{\left(a^4+2a^2b^2+b^4\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}}{d}$
risch	$\frac{i\mathrm{e}^{i(dx+c)}b}{8(-2iab+a^2-b^2)d}-\frac{3\,\mathrm{e}^{i(dx+c)}a}{8(-2iab+a^2-b^2)d}-\frac{i\mathrm{e}^{-i(dx+c)}b}{8(ib+a)^2d}-\frac{3\,\mathrm{e}^{-i(dx+c)}a}{8(ib+a)^2d}-\frac{ib\,a^3\ln\left(\mathrm{e}^{i(dx+c)}-\frac{ib+a}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}\,(a^2+b^2)^2d}+\frac{ib\,a^3\ldots}{\ldots}$

```
[In] int(sin(d*x+c)^3/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-16*a^3*b/(8*a^4+16*a^2*b^2+8*b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*ta
n(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))+2/(a^4+2*a^2*b^2+b^4)*(a^2*b*tan(1/2
*d*x+1/2*c)^5+a*b^2*tan(1/2*d*x+1/2*c)^4+(10/3*a^2*b+4/3*b^3)*tan(1/2*d*x+1
/2*c)^3-2*a^3*tan(1/2*d*x+1/2*c)^2+a^2*b*tan(1/2*d*x+1/2*c)-2/3*a^3+1/3*a*b
^2)/(1+tan(1/2*d*x+1/2*c)^2)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.55

$$\int \frac{\sin^3(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{3\sqrt{a^2+b^2}a^3b\log\left(\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b^2-2\,\sqrt{a^2+b^2}(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)+2\,(a^5+2\,a^3}{6\,(a^6}$$

```
[In] integrate(sin(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(a^2 + b^2)*a^3*b*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 -
b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*s
in(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
 + b^2)) + 2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(d*x + c)^3 - 6*(a^5 + a^3*b^2)*c
os(d*x + c) + 2*(4*a^4*b + 5*a^2*b^3 + b^5 - (a^4*b + 2*a^2*b^3 + b^5)*cos(
d*x + c)^2)*sin(d*x + c))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d)
```

Sympy [F]

$$\int \frac{\sin^3(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\sin^3(c+dx)}{a+b\tan(c+dx)}\,dx$$

```
[In] integrate(sin(d*x+c)**3/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**3/(a + b*tan(c + d*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 364 vs. 2(160) = 320.

Time = 0.30 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.17

$$\int \frac{\sin^3(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{3\,a^3b\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}}-\frac{2\left(2\,a^3-ab^2-\frac{3\,a^2b\sin(dx+c)}{\cos(dx+c)+1}+\frac{6\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{3\,ab^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{3\,a^2b\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{2\,(5\,a^2b+2\,b^3)\sin(dx}{(\cos(dx+c)+1)^3}\right.}{a^4+2\,a^2b^2+b^4+\frac{3\,(a^4+2\,a^2b^2+b^4)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\,(a^4+2\,a^2b^2+b^4)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{(a^4+2\,a^2b^2+b^4)\sin(dx+}{(\cos(dx+c)+1)^6}}}{3\,d}$$

```
[In] integrate(sin(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*(3*a^3*b*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/
(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^
2 + b^4)*sqrt(a^2 + b^2)) - 2*(2*a^3 - a*b^2 - 3*a^2*b*sin(d*x + c)/(cos(d*
x + c) + 1) + 6*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 3*a*b^2*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4 - 3*a^2*b*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 -
2*(5*a^2*b + 2*b^3)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a^4 + 2*a^2*b^2 +
 b^4 + 3*(a^4 + 2*a^2*b^2 + b^4)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*(a
^4 + 2*a^2*b^2 + b^4)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (a^4 + 2*a^2*b^
2 + b^4)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6))/d
```

Giac [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.43

$$\int \frac{\sin^3(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{3\,a^3b\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} + \frac{2\left(3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+10\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\right.}{(a^4+2\,a^2b^2+b^4)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{3\,d}$$

```
[In] integrate(sin(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/3*(3*a^3*b*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/ab
s(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 +
b^4)*sqrt(a^2 + b^2)) + 2*(3*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 3*a*b^2*tan(1/2
*d*x + 1/2*c)^4 + 10*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 4*b^3*tan(1/2*d*x + 1/2
*c)^3 - 6*a^3*tan(1/2*d*x + 1/2*c)^2 + 3*a^2*b*tan(1/2*d*x + 1/2*c) - 2*a^3
 + a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*c)^2 + 1)^3))/d
```

Mupad [B] (verification not implemented)

Time = 7.21 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.93

$$\int \frac{\sin^3(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{\frac{2\,a\,b^2}{3}-\frac{4\,a^3}{3}}{a^4+2\,a^2\,b^2+b^4}-\frac{4\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{a^4+2\,a^2\,b^2+b^4}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{20\,a^2\,b}{3}+\frac{8\,b^3}{3}\right)}{a^4+2\,a^2\,b^2+b^4}+\frac{2\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^4+2\,a^2\,b^2+b^4}+\frac{2\,a\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{a^4+2\,a^2\,b^2+b^4}+\frac{2\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{a^4+2\,a^2\,b^2+b^4}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)} + \frac{2\,a^3\,b\,\mathrm{atanh}\left(\frac{a^4\,b+b^5+2\,a^2\,b^3-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}{{\left(a^2+b^2\right)}^{5/2}}\right)}{d\,{\left(a^2+b^2\right)}^{5/2}}$$

```
[In] int(sin(c + d*x)^3/(a + b*tan(c + d*x)),x)
```

```
[Out] (((2*a*b^2)/3 - (4*a^3)/3)/(a^4 + b^4 + 2*a^2*b^2) - (4*a^3*tan(c/2 + (d*x)
/2)^2)/(a^4 + b^4 + 2*a^2*b^2) + (tan(c/2 + (d*x)/2)^3*((20*a^2*b)/3 + (8*b
^3)/3))/(a^4 + b^4 + 2*a^2*b^2) + (2*a^2*b*tan(c/2 + (d*x)/2))/(a^4 + b^4 +
 2*a^2*b^2) + (2*a*b^2*tan(c/2 + (d*x)/2)^4)/(a^4 + b^4 + 2*a^2*b^2) + (2*a
^2*b*tan(c/2 + (d*x)/2)^5)/(a^4 + b^4 + 2*a^2*b^2))/(d*(3*tan(c/2 + (d*x)/2
)^2 + 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 + 1)) + (2*a^3*b*atanh(
(a^4*b + b^5 + 2*a^2*b^3 - a*tan(c/2 + (d*x)/2)*(a^4 + b^4 + 2*a^2*b^2))/(a
^2 + b^2)^(5/2)))/(d*(a^2 + b^2)^(5/2))
```

3.54 $\int \frac{\sin^2(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 94

$$\int \frac{\sin^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{a(a^2-b^2)\,x}{2\,(a^2+b^2)^2}+\frac{a^2b\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^2\,d} - \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\,(a^2+b^2)\,d}$$

```
[Out] 1/2*a*(a^2-b^2)*x/(a^2+b^2)^2+a^2*b*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)
^2/d-1/2*cos(d*x+c)^2*(b+a*tan(d*x+c))/(a^2+b^2)/d
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3597, 1661, 815, 649, 209, 266}

$$\int \frac{\sin^2(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{\cos^2(c+dx)(a\tan(c+dx)+b)}{2d\,(a^2+b^2)} + \frac{a^2b\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^2}+\frac{ax(a^2-b^2)}{2\,(a^2+b^2)^2}$$

```
[In] Int[Sin[c + d*x]^2/(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*(a^2 - b^2)*x)/(2*(a^2 + b^2)^2) + (a^2*b*Log[a*Cos[c + d*x] + b*Sin[c +
 d*x]])/((a^2 + b^2)^2*d) - (Cos[c + d*x]^2*(b + a*Tan[c + d*x]))/(2*(a^2 +
 b^2)*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{b\text{Subst}\left(\int \frac{x^2}{(a+x)(b^2+x^2)^2}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\left(a^2+b^2\right)d} - \frac{\text{Subst}\left(\int \frac{-\frac{a^2b^2}{a^2+b^2}+\frac{ab^2x}{a^2+b^2}}{(a+x)(b^2+x^2)}\,dx, x, b\tan(c+dx)\right)}{2bd}$$

$$= -\frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2(a^2+b^2)d} - \frac{\text{Subst}\left(\int \left(-\frac{2a^2b^2}{(a^2+b^2)^2(a+x)} - \frac{ab^2(a^2-b^2-2ax)}{(a^2+b^2)^2(b^2+x^2)}\right) dx, x, b\tan(c+dx)\right)}{2bd}$$

$$= \frac{a^2b\log(a+b\tan(c+dx))}{(a^2+b^2)^2 d} - \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2(a^2+b^2)d} + \frac{(ab)\text{Subst}\left(\int \frac{a^2-b^2-2ax}{b^2+x^2} dx, x, b\tan(c+dx)\right)}{2(a^2+b^2)^2 d}$$

$$= \frac{a^2b\log(a+b\tan(c+dx))}{(a^2+b^2)^2 d} - \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2(a^2+b^2)d} - \frac{(a^2b)\,\text{Subst}\left(\int \frac{x}{b^2+x^2} dx, x, b\tan(c+dx)\right)}{(a^2+b^2)^2 d} + \frac{(ab(a^2-b^2))\,\text{Subst}\left(\int \frac{1}{b^2+x^2} dx, x, b\tan(c+dx)\right)}{2(a^2+b^2)^2 d}$$

$$= \frac{a(a^2-b^2)x}{2(a^2+b^2)^2} + \frac{a^2b\log(\cos(c+dx))}{(a^2+b^2)^2 d} + \frac{a^2b\log(a+b\tan(c+dx))}{(a^2+b^2)^2 d} - \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2(a^2+b^2)d}$$

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.81

$$\int \frac{\sin^2(c+dx)}{a+b\tan(c+dx)} dx = -\frac{2ab(a^2+b^2)\arctan(\tan(c+dx)) + 2b^2(a^2+b^2)\cos^2(c+dx) + a\left(2a\left(\left(b^2+a\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}-b\tan(c\right.\right.\right.}{4b(}$$

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Tan[c + d*x]),x]
```

```
[Out] -1/4*(2*a*b*(a^2 + b^2)*ArcTan[Tan[c + d*x]] + 2*b^2*(a^2 + b^2)*Cos[c + d*
x]^2 + a*(2*a*((b^2 + a*Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]] - 2*b^
2*Log[a + b*Tan[c + d*x]] + (b^2 - a*Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c +
 d*x]]) + b*(a^2 + b^2)*Sin[2*(c + d*x)]))/(b*(a^2 + b^2)^2*d)
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.30

method	result
derivativedivides	$\frac{\frac{\frac{\left(-\frac{1}{2}a^3-\frac{1}{2}a\,b^2\right)\tan(dx+c)-\frac{a^2b}{2}-\frac{b^3}{2}}{1+\tan^2(dx+c)}+\frac{a\left(-ab\ln\left(1+\tan^2(dx+c)\right)+\left(a^2-b^2\right)\arctan(\tan(dx+c))\right)}{2}}{\left(a^2+b^2\right)^2}+\frac{a^2b\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^2}}{d}$
default	$\frac{\frac{\frac{\left(-\frac{1}{2}a^3-\frac{1}{2}a\,b^2\right)\tan(dx+c)-\frac{a^2b}{2}-\frac{b^3}{2}}{1+\tan^2(dx+c)}+\frac{a\left(-ab\ln\left(1+\tan^2(dx+c)\right)+\left(a^2-b^2\right)\arctan(\tan(dx+c))\right)}{2}}{\left(a^2+b^2\right)^2}+\frac{a^2b\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^2}}{d}$
risch	$-\frac{ax}{2(2iab-a^2+b^2)}+\frac{i\mathrm{e}^{2i(dx+c)}}{8(-ib+a)d}-\frac{i\mathrm{e}^{-2i(dx+c)}}{8(ib+a)d}-\frac{2ia^2bx}{a^4+2a^2b^2+b^4}-\frac{2ia^2bc}{d(a^4+2a^2b^2+b^4)}+\frac{a^2b\ln\left(\mathrm{e}^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)}{d(a^4+2a^2b^2+b^4)}$

```
[In] int(sin(d*x+c)^2/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a^2+b^2)^2*(((-1/2*a^3-1/2*a*b^2)*tan(d*x+c)-1/2*a^2*b-1/2*b^3)/(1+
tan(d*x+c)^2)+1/2*a*(-a*b*ln(1+tan(d*x+c)^2)+(a^2-b^2)*arctan(tan(d*x+c))))
+a^2*b/(a^2+b^2)^2*ln(a+b*tan(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.30

$$\int \frac{\sin^2(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$=\frac{a^2b\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)+\left(a^3-ab^2\right)dx-\left(a^2b+b^3\right)\cos\left(dx+\right.}{2\left(a^4+2\,a^2b^2+b^4\right)d}$$

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*b*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
 + b^2) + (a^3 - a*b^2)*d*x - (a^2*b + b^3)*cos(d*x + c)^2 - (a^3 + a*b^2)*
cos(d*x + c)*sin(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d)
```

Sympy [F]

$$\int \frac{\sin^2(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\sin^2(c+dx)}{a+b\tan(c+dx)}\,dx$$

```
[In] integrate(sin(d*x+c)**2/(a+b*tan(d*x+c)),x)
[Out] Integral(sin(c + d*x)**2/(a + b*tan(c + d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.53

$$\int \frac{\sin^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{2a^2b\log(b\tan(dx+c)+a)}{a^4+2a^2b^2+b^4} - \frac{a^2b\log\left(\tan(dx+c)^2+1\right)}{a^4+2a^2b^2+b^4} + \frac{(a^3-ab^2)(dx+c)}{a^4+2a^2b^2+b^4} - \frac{a\tan(dx+c)+b}{(a^2+b^2)\tan(dx+c)^2+a^2+b^2}}{2d}$$

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="maxima")
[Out] 1/2*(2*a^2*b*log(b*tan(d*x + c) + a)/(a^4 + 2*a^2*b^2 + b^4) - a^2*b*log(ta
n(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^3 - a*b^2)*(d*x + c)/(a^4 +
2*a^2*b^2 + b^4) - (a*tan(d*x + c) + b)/((a^2 + b^2)*tan(d*x + c)^2 + a^2 +
 b^2))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(90) = 180.

Time = 0.39 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.96

$$\int \frac{\sin^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{2a^2b^2\log(|b\tan(dx+c)+a|)}{a^4b+2a^2b^3+b^5} - \frac{a^2b\log\left(\tan(dx+c)^2+1\right)}{a^4+2a^2b^2+b^4} + \frac{(a^3-ab^2)(dx+c)}{a^4+2a^2b^2+b^4} + \frac{a^2b\tan(dx+c)^2-a^3\tan(dx+c)-ab^2\tan(dx+c)-b^3}{(a^4+2a^2b^2+b^4)\left(\tan(dx+c)^2+1\right)}}{2d}$$

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="giac")
[Out] 1/2*(2*a^2*b^2*log(abs(b*tan(d*x + c) + a))/(a^4*b + 2*a^2*b^3 + b^5) - a^2
*b*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^3 - a*b^2)*(d*x + c
)/(a^4 + 2*a^2*b^2 + b^4) + (a^2*b*tan(d*x + c)^2 - a^3*tan(d*x + c) - a*b^
2*tan(d*x + c) - b^3)/((a^4 + 2*a^2*b^2 + b^4)*(tan(d*x + c)^2 + 1)))/d
```

Mupad [B] (verification not implemented)

Time = 5.01 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.56

$$\int \frac{\sin^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{a^2\,b\,\ln\left(a+b\tan(c+d\,x)\right)}{d\,\left(a^2+b^2\right)^2} - \frac{a\,\ln\left(\tan(c+d\,x)-\mathrm{i}\right)}{4\,d\,\left(-a^2\,1\mathrm{i}+2\,a\,b+b^2\,1\mathrm{i}\right)} - \frac{\cos\left(c+d\,x\right)^2\,\left(\frac{b}{2\,\left(a^2+b^2\right)}+\frac{a\tan(c+d\,x)}{2\,\left(a^2+b^2\right)}\right)}{d} - \frac{a\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{4\,d\,\left(-a^2+a\,b\,2\mathrm{i}+b^2\right)}$$

```
[In] int(sin(c + d*x)^2/(a + b*tan(c + d*x)),x)
```

```
[Out] (a^2*b*log(a + b*tan(c + d*x)))/(d*(a^2 + b^2)^2) - (a*log(tan(c + d*x) + 1
i)*1i)/(4*d*(a*b*2i - a^2 + b^2)) - (a*log(tan(c + d*x) - 1i))/(4*d*(2*a*b
- a^2*1i + b^2*1i)) - (cos(c + d*x)^2*(b/(2*(a^2 + b^2)) + (a*tan(c + d*x))
/(2*(a^2 + b^2))))/d
```

3.55 $\int \frac{\sin(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 90

$$\int \frac{\sin(c+dx)}{a+b\tan(c+dx)}\,dx=\frac{ab\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d}-\frac{a\cos(c+dx)}{(a^2+b^2)\,d}+\frac{b\sin(c+dx)}{(a^2+b^2)\,d}$$

```
[Out] a*b*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)/d-
a*cos(d*x+c)/(a^2+b^2)/d+b*sin(d*x+c)/(a^2+b^2)/d
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3599, 3188, 2717, 2718, 3153, 212}

$$\int \frac{\sin(c+dx)}{a+b\tan(c+dx)}\,dx=\frac{ab\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\,(a^2+b^2)^{3/2}}+\frac{b\sin(c+dx)}{d\,(a^2+b^2)}-\frac{a\cos(c+dx)}{d\,(a^2+b^2)}$$

```
[In] Int[Sin[c + d*x]/(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*b*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/((a^2 + b^
2)^(3/2)*d) - (a*Cos[c + d*x])/((a^2 + b^2)*d) + (b*Sin[c + d*x])/((a^2 + b
^2)*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\cos(c+dx)\sin(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx \\
&= \frac{a\int \sin(c+dx)\,dx}{a^2+b^2}+\frac{b\int \cos(c+dx)\,dx}{a^2+b^2}-\frac{(ab)\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{a^2+b^2} \\
&= -\frac{a\cos(c+dx)}{(a^2+b^2)\,d}+\frac{b\sin(c+dx)}{(a^2+b^2)\,d}+\frac{(ab)\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx,x,b\cos(c+dx)-a\sin(c+dx)\right)}{(a^2+b^2)\,d} \\
&= \frac{ab\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d}-\frac{a\cos(c+dx)}{(a^2+b^2)\,d}+\frac{b\sin(c+dx)}{(a^2+b^2)\,d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.88

$$\int \frac{\sin(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{-2ab\text{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)+\sqrt{a^2+b^2}(-a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^{3/2}\,d}$$

```
[In] Integrate[Sin[c + d*x]/(a + b*Tan[c + d*x]),x]
```

```
[Out] (-2*a*b*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + Sqrt[a^2 + b^2
]*(-(a*Cos[c + d*x]) + b*Sin[c + d*x]))/((a^2 + b^2)^(3/2)*d)
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.12

method	result	size
derivativedivides	$\frac{\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2a}{\left(a^2+b^2\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{4ab\ \text{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(2a^2+2b^2\right)\sqrt{a^2+b^2}}}{d}$	101
default	$\frac{\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2a}{\left(a^2+b^2\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{4ab\ \text{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(2a^2+2b^2\right)\sqrt{a^2+b^2}}}{d}$	101
risch	$-\frac{e^{i(dx+c)}}{2(-ib+a)d}-\frac{e^{-i(dx+c)}}{2(ib+a)d}+\frac{iba\ln\left(e^{i(dx+c)}+\frac{ib+a}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}\,(a^2+b^2)d}-\frac{iba\ln\left(e^{i(dx+c)}-\frac{ib+a}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}\,(a^2+b^2)d}$	169

```
[In] int(sin(d*x+c)/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/(a^2+b^2)*(b*tan(1/2*d*x+1/2*c)-a)/(1+tan(1/2*d*x+1/2*c)^2)-4*a*b/(2
*a^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b
^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(86) = 172.

Time = 0.28 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.06

$$\int \frac{\sin(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\sqrt{a^2+b^2}ab\log\left(\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b^2-2\,\sqrt{a^2+b^2}(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)-2\,(a^3+ab^2)\,\mathrm{co}}{2\,(a^4+2\,a^2b^2+b^4)d}$$

```
[In] integrate(sin(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^2 + b^2)*a*b*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)
*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d
*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b
^2)) - 2*(a^3 + a*b^2)*cos(d*x + c) + 2*(a^2*b + b^3)*sin(d*x + c))/((a^4 +
 2*a^2*b^2 + b^4)*d)
```

Sympy [F]

$$\int \frac{\sin(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\sin{(c+dx)}}{a+b\tan{(c+dx)}}\,dx$$

```
[In] integrate(sin(d*x+c)/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)/(a + b*tan(c + d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.57

$$\int \frac{\sin(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{ab\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}}-\frac{2\left(a-\frac{b\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2+b^2+\frac{(a^2+b^2)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}}{d}$$

```
[In] integrate(sin(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] (a*b*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*s
in(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(a
 - b*sin(d*x + c)/(cos(d*x + c) + 1))/(a^2 + b^2 + (a^2 + b^2)*sin(d*x + c)
^2/(cos(d*x + c) + 1)^2))/d
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.31

$$\int \frac{\sin(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{ab\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}}+\frac{2\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)}{(a^2+b^2)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)}}{d}$$

```
[In] integrate(sin(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*b*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*ta
n(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) + 2*(b*tan
(1/2*d*x + 1/2*c) - a)/((a^2 + b^2)*(tan(1/2*d*x + 1/2*c)^2 + 1)))/d
```

Mupad [B] (verification not implemented)

Time = 4.45 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.22

$$\int \frac{\sin(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{2\,a\,b\,\mathrm{atanh}\left(\frac{a^2\,b+b^3-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a^2+b^2)}{(a^2+b^2)^{3/2}}\right)}{d\,(a^2+b^2)^{3/2}}-\frac{\frac{2\,a}{a^2+b^2}-\frac{2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2+b^2}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

```
[In] int(sin(c + d*x)/(a + b*tan(c + d*x)),x)
```

```
[Out] (2*a*b*atanh((a^2*b + b^3 - a*tan(c/2 + (d*x)/2)*(a^2 + b^2))/(a^2 + b^2)^(
3/2)))/(d*(a^2 + b^2)^(3/2)) - ((2*a)/(a^2 + b^2) - (2*b*tan(c/2 + (d*x)/2)
)/(a^2 + b^2))/(d*(tan(c/2 + (d*x)/2)^2 + 1))
```

3.56 $\int \frac{\csc(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 66

$$\int \frac{\csc(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{\operatorname{arctanh}(\cos(c+dx))}{ad}+\frac{b\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d}$$

```
[Out] -arctanh(cos(d*x+c))/a/d+b*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1
/2))/a/d/(a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3599, 3189, 3855, 3153, 212}

$$\int \frac{\csc(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{b\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{ad\sqrt{a^2+b^2}}-\frac{\operatorname{arctanh}(\cos(c+dx))}{ad}$$

```
[In] Int[Csc[c + d*x]/(a + b*Tan[c + d*x]),x]
```

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a*d)) + (b*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x
])/Sqrt[a^2 + b^2]])/(a*Sqrt[a^2 + b^2]*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3189

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[cos[c + d*x]^m*(sin[c + d*x]^n/(a*cos[c + d*x] + b*sin[c + d*x])),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cot(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx \\
&= \int \left(\frac{\csc(c+dx)}{a}-\frac{b}{a(a\cos(c+dx)+b\sin(c+dx))}\right)dx \\
&= \frac{\int \csc(c+dx)\,dx}{a}-\frac{b\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{a} \\
&= -\frac{\operatorname{arctanh}(\cos(c+dx))}{ad}+\frac{b\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx,x,b\cos(c+dx)-a\sin(c+dx)\right)}{ad} \\
&= -\frac{\operatorname{arctanh}(\cos(c+dx))}{ad}+\frac{b\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.14

$$\int \frac{\csc(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{-\frac{2b\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} - \log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + \log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{ad}$$

```
[In] Integrate[Csc[c + d*x]/(a + b*Tan[c + d*x]),x]
```

```
[Out] ((-2*b*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2]
- Log[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]])/(a*d)
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$\frac{\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2b\ \operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}}{d}$	63
default	$\frac{\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2b\ \operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}}{d}$	63
risch	$-\frac{ib\ln\left(\mathrm{e}^{i(dx+c)}-\frac{ib+a}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}\,da}+\frac{ib\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ib+a}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}\,da}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{ad}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}+1\right)}{ad}$	150

```
[In] int(csc(d*x+c)/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a*ln(tan(1/2*d*x+1/2*c))-2*b/a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(
1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(62) = 124.

Time = 0.31 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.77

$$\int \frac{\csc(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\sqrt{a^2+b^2}b\log\left(\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b^2-2\,\sqrt{a^2+b^2}(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)-(a^2+b^2)\log\left(\frac{1}{2}\right.}{2\,(a^3+ab^2)d}$$

```
[In] integrate(csc(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^2 + b^2)*b*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*c
os(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x
 + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2
)) - (a^2 + b^2)*log(1/2*cos(d*x + c) + 1/2) + (a^2 + b^2)*log(-1/2*cos(d*x
 + c) + 1/2))/((a^3 + a*b^2)*d)
```

Sympy [F]

$$\int \frac{\csc(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\csc\left(c+dx\right)}{a+b\tan\left(c+dx\right)}\,dx$$

```
[In] integrate(csc(d*x+c)/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)/(a + b*tan(c + d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.62

$$\int \frac{\csc(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{b\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}a}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{d}$$

```
[In] integrate(csc(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] (b*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin
(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a) + log(
sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.42

$$\int \frac{\csc(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{b\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a}+\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}}{d}$$

```
[In] integrate(csc(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] (b*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(
1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a) + log(abs(
tan(1/2*d*x + 1/2*c)))/a)/d
```

Mupad [B] (verification not implemented)

Time = 4.91 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.64

$$\int \frac{\csc(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$=\frac{\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a\,d}-\frac{2\,b\,\mathrm{atanh}\left(\frac{\sqrt{a^2+b^2}\,\left(1\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+2\mathrm{i}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b+4\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^2\right)}{b^3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,4\mathrm{i}+a\,b^2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}+a^2\,b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,3\mathrm{i}+a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2+b^2\right)\,1\mathrm{i}}\right)}{a\,d\,\sqrt{a^2+b^2}}$$

```
[In] int(1/(sin(c + d*x)*(a + b*tan(c + d*x))),x)
```

```
[Out] log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))/(a*d) - (2*b*atanh(((a^2 + b^2)^
(1/2)*(a^2*sin(c/2 + (d*x)/2)*1i + b^2*sin(c/2 + (d*x)/2)*4i + a*b*cos(c/2
+ (d*x)/2)*2i))/(b^3*sin(c/2 + (d*x)/2)*4i + a*b^2*cos(c/2 + (d*x)/2)*1i +
a^2*b*sin(c/2 + (d*x)/2)*3i + a*cos(c/2 + (d*x)/2)*(a^2 + b^2)*1i)))/(a*d*(
a^2 + b^2)^(1/2))
```

3.57 $\int \frac{\csc^2(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 50

$$\int \frac{\csc^2(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{\cot(c+dx)}{ad} - \frac{b\log(\tan(c+dx))}{a^2d} + \frac{b\log(a+b\tan(c+dx))}{a^2d}$$

```
[Out] -cot(d*x+c)/a/d-b*ln(tan(d*x+c))/a^2/d+b*ln(a+b*tan(d*x+c))/a^2/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3597, 46}

$$\int \frac{\csc^2(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{b\log(\tan(c+dx))}{a^2d} + \frac{b\log(a+b\tan(c+dx))}{a^2d} - \frac{\cot(c+dx)}{ad}$$

```
[In] Int[Csc[c + d*x]^2/(a + b*Tan[c + d*x]),x]
```

```
[Out] -(Cot[c + d*x]/(a*d)) - (b*Log[Tan[c + d*x]])/(a^2*d) + (b*Log[a + b*Tan[c
+ d*x]])/(a^2*d)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{b\text{Subst}\left(\int \frac{1}{x^2(a+x)}\,dx, x, b\tan(c+dx)\right)}{d}\\ &= \frac{b\text{Subst}\left(\int \left(\frac{1}{ax^2}-\frac{1}{a^2x}+\frac{1}{a^2(a+x)}\right)\,dx, x, b\tan(c+dx)\right)}{d}\\ &= -\frac{\cot(c+dx)}{ad}-\frac{b\log(\tan(c+dx))}{a^2d}+\frac{b\log(a+b\tan(c+dx))}{a^2d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \frac{\csc^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{-a\cot(c+dx)+b(-\log(\sin(c+dx))+\log(a\cos(c+dx)+b\sin(c+dx)))}{a^2d}$$

```
[In] Integrate[Csc[c + d*x]^2/(a + b*Tan[c + d*x]),x]
[Out] (-(a*Cot[c + d*x]) + b*(-Log[Sin[c + d*x]] + Log[a*Cos[c + d*x] + b*Sin[c +
 d*x]]))/(a^2*d)
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{-\frac{1}{a\tan(dx+c)}-\frac{b\ln(\tan(dx+c))}{a^2}+\frac{b\ln(a+b\tan(dx+c))}{a^2}}{d}$	48
default	$\frac{-\frac{1}{a\tan(dx+c)}-\frac{b\ln(\tan(dx+c))}{a^2}+\frac{b\ln(a+b\tan(dx+c))}{a^2}}{d}$	48
risch	$-\frac{2i}{da\left(\mathrm{e}^{2i(dx+c)}-1\right)}-\frac{b\ln\left(\mathrm{e}^{2i(dx+c)}-1\right)}{a^2d}+\frac{b\ln\left(\mathrm{e}^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)}{a^2d}$	82

```
[In] int(csc(d*x+c)^2/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/a/tan(d*x+c)-1/a^2*b*ln(tan(d*x+c))+1/a^2*b*ln(a+b*tan(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.90

$$\int \frac{\csc^2(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{b\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)\sin\left(dx+c\right)-b\log\left(-\frac{1}{4}\,\cos\left(dx+c\right)^2+\right.}{2\,a^2d\sin\left(dx+c\right)}$$

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b
^2)*sin(d*x + c) - b*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) - 2*a*cos(
d*x + c))/(a^2*d*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\csc^2(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\csc^2\left(c+dx\right)}{a+b\tan\left(c+dx\right)}\,dx$$

```
[In] integrate(csc(d*x+c)**2/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**2/(a + b*tan(c + d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \frac{\csc^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{b\log(b\tan(dx+c)+a)}{a^2}-\frac{b\log(\tan(dx+c))}{a^2}-\frac{1}{a\tan(dx+c)}}{d}$$

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] (b*log(b*tan(d*x + c) + a)/a^2 - b*log(tan(d*x + c))/a^2 - 1/(a*tan(d*x + c
)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.20

$$\int \frac{\csc^2(c+dx)}{a+b\tan(c+dx)}\,dx=\frac{\frac{b\log(|b\tan(dx+c)+a|)}{a^2}-\frac{b\log(|\tan(dx+c)|)}{a^2}+\frac{b\tan(dx+c)-a}{a^2\tan(dx+c)}}{d}$$

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] (b*log(abs(b*tan(d*x + c) + a))/a^2 - b*log(abs(tan(d*x + c)))/a^2 + (b*tan
(d*x + c) - a)/(a^2*tan(d*x + c)))/d
```

Mupad [B] (verification not implemented)

Time = 4.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int \frac{\csc^2(c+dx)}{a+b\tan(c+dx)}\,dx=\frac{2\,b\,\mathrm{atanh}\left(\frac{2\,b\tan(c+d\,x)}{a}+1\right)}{a^2\,d}-\frac{\cot(c+d\,x)}{a\,d}$$

```
[In] int(1/(sin(c + d*x)^2*(a + b*tan(c + d*x))),x)
```

```
[Out] (2*b*atanh((2*b*tan(c + d*x))/a + 1))/(a^2*d) - cot(c + d*x)/(a*d)
```

3.58 $\int \frac{\csc^3(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 122

$$\int \frac{\csc^3(c+dx)}{a+b\tan(c+dx)}\,dx=-\frac{\operatorname{arctanh}(\cos(c+dx))}{2ad}-\frac{b^2\operatorname{arctanh}(\cos(c+dx))}{a^3d}+\frac{b\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{a^3d}+\frac{b\csc(c+dx)}{a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

```
[Out] -1/2*arctanh(cos(d*x+c))/a/d-b^2*arctanh(cos(d*x+c))/a^3/d+b*csc(d*x+c)/a^2
/d-1/2*cot(d*x+c)*csc(d*x+c)/a/d+b*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2
+b^2)^(1/2))*(a^2+b^2)^(1/2)/a^3/d
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {3599, 3189, 3853, 3855, 2701, 327, 213, 2702, 3183, 3153, 212}

$$\int \frac{\csc^3(c+dx)}{a+b\tan(c+dx)}\,dx=-\frac{b^2\operatorname{arctanh}(\cos(c+dx))}{a^3d}+\frac{b\csc(c+dx)}{a^2d}+\frac{b\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{a^3d}-\frac{\operatorname{arctanh}(\cos(c+dx))}{2ad}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

```
[In] Int[Csc[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

```
[Out] -1/2*ArcTanh[Cos[c + d*x]]/(a*d) - (b^2*ArcTanh[Cos[c + d*x]])/(a^3*d) + (b
*Sqrt[a^2 + b^2]*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]]
)/(a^3*d) + (b*Csc[c + d*x])/(a^2*d) - (Cot[c + d*x]*Csc[c + d*x])/(2*a*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3183

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
```

```
), x] + (-Dist[a/b^2, Int[Cos[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/b
^2, Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3189

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[cos[c + d*x]^m*(sin[c + d*x]^n/(a*cos[c + d*x] + b*sin[c + d*x])),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cot(c+dx)\csc^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx \\
&= \int \left(\frac{\csc^3(c+dx)}{a}-\frac{b\csc^2(c+dx)\sec(c+dx)}{a^2}+\frac{b^2\csc(c+dx)\sec^2(c+dx)}{a^3}\right. \\
&\qquad \left.-\frac{b^3\sec^2(c+dx)}{a^3(a\cos(c+dx)+b\sin(c+dx))}\right)dx \\
&= \frac{\int \csc^3(c+dx)\,dx}{a}-\frac{b\int \csc^2(c+dx)\sec(c+dx)\,dx}{a^2} \\
&\quad +\frac{b^2\int \csc(c+dx)\sec^2(c+dx)\,dx}{a^3}-\frac{b^3\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{a^3}
\end{aligned}
$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{2ad} - \frac{b^2\sec(c+dx)}{a^3d} + \frac{\int \csc(c+dx)\,dx}{2a} + \frac{b\int \sec(c+dx)\,dx}{a^2} - \frac{(b(a^2+b^2))\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{a^3} + \frac{b\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{a^2d} + \frac{b^2\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{a^3d}$$

$$= -\frac{\text{arctanh}(\cos(c+dx))}{2ad} + \frac{b\text{arctanh}(\sin(c+dx))}{a^2d} + \frac{b\csc(c+dx)}{a^2d} - \frac{\cot(c+dx)\csc(c+dx)}{2ad} + \frac{b\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{a^2d} + \frac{b^2\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{a^3d} + \frac{(b(a^2+b^2))\,\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx, x, b\cos(c+dx)-a\sin(c+dx)\right)}{a^3d}$$

$$= -\frac{\text{arctanh}(\cos(c+dx))}{2ad} - \frac{b^2\text{arctanh}(\cos(c+dx))}{a^3d} + \frac{b\sqrt{a^2+b^2}\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{a^3d} + \frac{b\csc(c+dx)}{a^2d} - \frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.47

$$\int \frac{\csc^3(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{-16b\sqrt{a^2+b^2}\text{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right) + 4ab\cot\left(\frac{1}{2}(c+dx)\right) - a^2\csc^2\left(\frac{1}{2}(c+dx)\right) - 4a^2\log\left(\cos\left(\frac{1}{2}(c\right.\right.}{}$$

```
[In] Integrate[Csc[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

```
[Out] (-16*b*Sqrt[a^2 + b^2]*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] +
 4*a*b*Cot[(c + d*x)/2] - a^2*Csc[(c + d*x)/2]^2 - 4*a^2*Log[Cos[(c + d*x)/
2]] - 8*b^2*Log[Cos[(c + d*x)/2]] + 4*a^2*Log[Sin[(c + d*x)/2]] + 8*b^2*Log
[Sin[(c + d*x)/2]] + a^2*Sec[(c + d*x)/2]^2 + 4*a*b*Tan[(c + d*x)/2])/(8*a^
3*d)
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{\frac{\left(\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}{2}+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2}-\frac{2b\sqrt{a^2+b^2}\,\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a^3}-\frac{1}{8a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\left(2a^2+4b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^3}+}{d}$
default	$\frac{\frac{\left(\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}{2}+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2}-\frac{2b\sqrt{a^2+b^2}\,\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a^3}-\frac{1}{8a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\left(2a^2+4b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^3}+}{d}$
risch	$\frac{ie^{i(dx+c)}\left(-ia\,e^{2i(dx+c)}+2b\,e^{2i(dx+c)}-ia-2b\right)}{d\,a^2\left(e^{2i(dx+c)}-1\right)^2}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{2ad}+\frac{\ln\left(e^{i(dx+c)}-1\right)b^2}{a^3d}-\frac{i\sqrt{-a^2-b^2}\,b\ln\left(e^{i(dx+c)}-\right.}{d\,a^3}$

```
[In] int(csc(d*x+c)^3/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4/a^2*(1/2*tan(1/2*d*x+1/2*c)^2*a+2*b*tan(1/2*d*x+1/2*c))-2*b*(a^2+b
^2)^(1/2)/a^3*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))-1/8
/a/tan(1/2*d*x+1/2*c)^2+1/4/a^3*(2*a^2+4*b^2)*ln(tan(1/2*d*x+1/2*c))+1/2/a^
2*b/tan(1/2*d*x+1/2*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. 2(114) = 228.

Time = 0.31 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.21

$$\int \frac{\csc^3(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$=\frac{2\,a^2\cos\left(dx+c\right)-4\,ab\sin\left(dx+c\right)+2\left(b\cos\left(dx+c\right)^2-b\right)\sqrt{a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+}{2\,ab\cos(dx+c)\sin(dx-}\right.}{}$$

```
[In] integrate(csc(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^2*cos(d*x + c) - 4*a*b*sin(d*x + c) + 2*(b*cos(d*x + c)^2 - b)*sqr
t(a^2 + b^2)*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c
)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2
*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - ((a^2
 + 2*b^2)*cos(d*x + c)^2 - a^2 - 2*b^2)*log(1/2*cos(d*x + c) + 1/2) + ((a^2
 + 2*b^2)*cos(d*x + c)^2 - a^2 - 2*b^2)*log(-1/2*cos(d*x + c) + 1/2))/(a^3*
d*cos(d*x + c)^2 - a^3*d)
```

Sympy [F]

$$\int \frac{\csc^3(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\csc^3{(c+dx)}}{a+b\tan{(c+dx)}}\,dx$$

```
[In] integrate(csc(d*x+c)**3/(a+b*tan(d*x+c)),x)
[Out] Integral(csc(c + d*x)**3/(a + b*tan(c + d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.76

$$\int \frac{\csc^3(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{\frac{4\,b\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}{a^2}+\frac{4\,(a^2+2\,b^2)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{\left(a-\frac{4\,b\sin(dx+c)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)^2}{a^2\sin(dx+c)^2}+\frac{8\,(a^2b+b^3)\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a}}\right.}{\sqrt{a^2+b^2}a^3}}{8\,d}$$

```
[In] integrate(csc(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="maxima")
[Out] 1/8*((4*b*sin(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2)/a^2 + 4*(a^2 + 2*b^2)*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - (a
 - 4*b*sin(d*x + c)/(cos(d*x + c) + 1))*(cos(d*x + c) + 1)^2/(a^2*sin(d*x +
 c)^2) + 8*(a^2*b + b^3)*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(
a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(sqr
t(a^2 + b^2)*a^3))/d
```

Giac [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.71

$$\int \frac{\csc^3(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+4\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2}+\frac{4\,(a^2+2\,b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}+\frac{8\,(a^2b+b^3)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a^3}-\frac{6\,a^2\tan}{}}{8\,d}$$

```
[In] integrate(csc(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*((a*tan(1/2*d*x + 1/2*c)^2 + 4*b*tan(1/2*d*x + 1/2*c))/a^2 + 4*(a^2 + 2
*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + 8*(a^2*b + b^3)*log(abs(2*a*tan(
1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) -
2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3) - (6*a^2*tan(1/2*d*x + 1/2*
c)^2 + 12*b^2*tan(1/2*d*x + 1/2*c)^2 - 4*a*b*tan(1/2*d*x + 1/2*c) + a^2)/(a
^3*tan(1/2*d*x + 1/2*c)^2))/d
```

Mupad [B] (verification not implemented)

Time = 5.55 (sec) , antiderivative size = 764, normalized size of antiderivative = 6.26

$$\int \frac{\csc^3(c+dx)}{a+b\tan(c+dx)}\,dx$$

$$= \frac{b^2\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{2\left(\frac{a^3d}{2}-\frac{a^3d\cos(2c+2dx)}{2}\right)} - \frac{a^2\left(\frac{\cos(c+dx)}{2}-\frac{\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{4}+\frac{\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos(2c+2dx)}{4}\right)}{\frac{a^3d}{2}-\frac{a^3d\cos(2c+2dx)}{2}}$$

$$+\frac{ab\sin(c+dx)}{\frac{a^3d}{2}-\frac{a^3d\cos(2c+2dx)}{2}} - \frac{b^2\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos(2c+2dx)}{2\left(\frac{a^3d}{2}-\frac{a^3d\cos(2c+2dx)}{2}\right)}$$

$$+\frac{b\,\mathrm{atan}\left(\frac{a^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2+b^2}\,1\mathrm{i}+b^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2+b^2}\,8\mathrm{i}+ab^3\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2+b^2}\,4\mathrm{i}+a^3b\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2+b^2}\,3\mathrm{i}+a^2b^2\sin\left(\frac{c}{2}+\frac{d}{\cdots}\right.}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^5+4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^4b+5\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^3b^2+12\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^2b^3+4\cos\left(\frac{c}{2}+\frac{dx}{2}\right)ab^4+8\sin\left(\frac{c}{2}+\frac{dx}{2}\right)b^5\cdots}\right.}{\frac{a^3d}{2}-\frac{a^3d\cos(2c+2dx)}{2}}$$

$$-\frac{b\cos(2c+2dx)\,\mathrm{atan}\left(\frac{a^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2+b^2}\,1\mathrm{i}+b^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2+b^2}\,8\mathrm{i}+ab^3\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2+b^2}\,4\mathrm{i}+a^3b\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2\cdots}\right.}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^5+4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^4b+5\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^3b^2+12\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^2b^3+4\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\cdots}$$

$$\frac{a^3d}{2}-\frac{a^3d\cos(2c+2dx)}{2}$$

```
[In] int(1/(sin(c + d*x)^3*(a + b*tan(c + d*x))),x)
```

```
[Out] (b^2*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(2*((a^3*d)/2 - (a^3*d*cos
(2*c + 2*d*x))/2)) - (a^2*(cos(c + d*x)/2 - log(sin(c/2 + (d*x)/2)/cos(c/2
+ (d*x)/2))/4 + (log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x
))/4))/((a^3*d)/2 - (a^3*d*cos(2*c + 2*d*x))/2) + (a*b*sin(c + d*x))/((a^3*
d)/2 - (a^3*d*cos(2*c + 2*d*x))/2) - (b^2*log(sin(c/2 + (d*x)/2)/cos(c/2 +
(d*x)/2))*cos(2*c + 2*d*x))/(2*((a^3*d)/2 - (a^3*d*cos(2*c + 2*d*x))/2)) +
(b*atan((a^4*sin(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2)*1i + b^4*sin(c/2 + (d*x)/
2)*(a^2 + b^2)^(1/2)*8i + a*b^3*cos(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2)*4i + a
^3*b*cos(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2)*3i + a^2*b^2*sin(c/2 + (d*x)/2)*(
a^2 + b^2)^(1/2)*8i)/(a^5*cos(c/2 + (d*x)/2) + 8*b^5*sin(c/2 + (d*x)/2) + 4
*a*b^4*cos(c/2 + (d*x)/2) + 4*a^4*b*sin(c/2 + (d*x)/2) + 5*a^3*b^2*cos(c/2
+ (d*x)/2) + 12*a^2*b^3*sin(c/2 + (d*x)/2)))*(a^2 + b^2)^(1/2)*1i)/((a^3*d)
```

```
/2 - (a^3*d*cos(2*c + 2*d*x))/2) - (b*cos(2*c + 2*d*x)*atan((a^4*sin(c/2 +
(d*x)/2)*(a^2 + b^2)^(1/2)*1i + b^4*sin(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2)*8i
 + a*b^3*cos(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2)*4i + a^3*b*cos(c/2 + (d*x)/2)
*(a^2 + b^2)^(1/2)*3i + a^2*b^2*sin(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2)*8i)/(a
^5*cos(c/2 + (d*x)/2) + 8*b^5*sin(c/2 + (d*x)/2) + 4*a*b^4*cos(c/2 + (d*x)/
2) + 4*a^4*b*sin(c/2 + (d*x)/2) + 5*a^3*b^2*cos(c/2 + (d*x)/2) + 12*a^2*b^3
*sin(c/2 + (d*x)/2)))*(a^2 + b^2)^(1/2)*1i)/((a^3*d)/2 - (a^3*d*cos(2*c + 2
*d*x))/2)
```

3.59 $\int \frac{\csc^4(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 108

$$\int \frac{\csc^4(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{(a^2+b^2)\cot(c+dx)}{a^3d}+\frac{b\cot^2(c+dx)}{2a^2d}-\frac{\cot^3(c+dx)}{3ad} -\frac{b(a^2+b^2)\log(\tan(c+dx))}{a^4d}+\frac{b(a^2+b^2)\log(a+b\tan(c+dx))}{a^4d}$$

```
[Out] -(a^2+b^2)*cot(d*x+c)/a^3/d+1/2*b*cot(d*x+c)^2/a^2/d-1/3*cot(d*x+c)^3/a/d-b
*(a^2+b^2)*ln(tan(d*x+c))/a^4/d+b*(a^2+b^2)*ln(a+b*tan(d*x+c))/a^4/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3597, 908}

$$\int \frac{\csc^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{b\cot^2(c+dx)}{2a^2d}-\frac{b(a^2+b^2)\log(\tan(c+dx))}{a^4d} +\frac{b(a^2+b^2)\log(a+b\tan(c+dx))}{a^4d} -\frac{(a^2+b^2)\cot(c+dx)}{a^3d}-\frac{\cot^3(c+dx)}{3ad}$$

```
[In] Int[Csc[c + d*x]^4/(a + b*Tan[c + d*x]),x]
```

```
[Out] -(((a^2 + b^2)*Cot[c + d*x])/(a^3*d)) + (b*Cot[c + d*x]^2)/(2*a^2*d) - Cot[
c + d*x]^3/(3*a*d) - (b*(a^2 + b^2)*Log[Tan[c + d*x]])/(a^4*d) + (b*(a^2 +
b^2)*Log[a + b*Tan[c + d*x]])/(a^4*d)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b\text{Subst}\left(\int \frac{b^2+x^2}{x^4(a+x)}\,dx, x, b\tan(c+dx)\right)}{d} \\
&= \frac{b\text{Subst}\left(\int \left(\frac{b^2}{ax^4} - \frac{b^2}{a^2x^3} + \frac{a^2+b^2}{a^3x^2} + \frac{-a^2-b^2}{a^4x} + \frac{a^2+b^2}{a^4(a+x)}\right)\,dx, x, b\tan(c+dx)\right)}{d} \\
&= -\frac{(a^2+b^2)\cot(c+dx)}{a^3d} + \frac{b\cot^2(c+dx)}{2a^2d} - \frac{\cot^3(c+dx)}{3ad} \\
&\quad - \frac{b(a^2+b^2)\log(\tan(c+dx))}{a^4d} + \frac{b(a^2+b^2)\log(a+b\tan(c+dx))}{a^4d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.94 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.88

$$\int \frac{\csc^4(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{3a^2b\csc^2(c+dx) - 2\cot(c+dx)\left(2a^3+3ab^2+a^3\csc^2(c+dx)\right) - 6b(a^2+b^2)\left(\log(\sin(c+dx)) - \log(a\,c\right.}{6a^4d}$$

```
[In] Integrate[Csc[c + d*x]^4/(a + b*Tan[c + d*x]),x]
```

```
[Out] (3*a^2*b*Csc[c + d*x]^2 - 2*Cot[c + d*x]*(2*a^3 + 3*a*b^2 + a^3*Csc[c + d*x
]^2) - 6*b*(a^2 + b^2)*(Log[Sin[c + d*x]] - Log[a*Cos[c + d*x] + b*Sin[c +
d*x]]))/(6*a^4*d)
```

Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\frac{(a^2+b^2)b\ln(a+b\tan(dx+c))}{a^4}-\frac{1}{3a\tan(dx+c)^3}-\frac{a^2+b^2}{a^3\tan(dx+c)}+\frac{b}{2a^2\tan(dx+c)^2}-\frac{(a^2+b^2)b\ln(\tan(dx+c))}{a^4}}{d}$
default	$\frac{\frac{(a^2+b^2)b\ln(a+b\tan(dx+c))}{a^4}-\frac{1}{3a\tan(dx+c)^3}-\frac{a^2+b^2}{a^3\tan(dx+c)}+\frac{b}{2a^2\tan(dx+c)^2}-\frac{(a^2+b^2)b\ln(\tan(dx+c))}{a^4}}{d}$
risch	$-\frac{2(3ib^2e^{4i(dx+c)}+3ab\,e^{4i(dx+c)}-6ia^2e^{2i(dx+c)}-6ib^2e^{2i(dx+c)}-3ab\,e^{2i(dx+c)}+2ia^2+3ib^2)}{3d\,a^3(e^{2i(dx+c)}-1)^3}+\frac{b\ln\left(e^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)}{a^2d}$ -

[In] int(csc(d*x+c)^4/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*((a^2+b^2)/a^4*b*ln(a+b*tan(d*x+c))-1/3/a/tan(d*x+c)^3-(a^2+b^2)/a^3/ta n(d*x+c)+1/2/a^2*b/tan(d*x+c)^2-(a^2+b^2)/a^4*b*ln(tan(d*x+c)))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.93

$$\int \frac{\csc^4(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{2\,(2\,a^3+3\,ab^2)\cos{(dx+c)}^3+3\,a^2b\sin{(dx+c)}+3\,\left(a^2b+b^3-(a^2b+b^3)\cos{(dx+c)}^2\right)\log\left(2\,ab\cos{(dx}\right.}{}$$

[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="fricas")

[Out] -1/6*(2*(2*a^3 + 3*a*b^2)*cos(d*x + c)^3 + 3*a^2*b*sin(d*x + c) + 3*(a^2*b + b^3 - (a^2*b + b^3)*cos(d*x + c)^2)*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)*sin(d*x + c) - 3*(a^2*b + b^3 - (a^2*b + b^3)*cos(d*x + c)^2)*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) - 6*(a^3 + a*b^2)*cos(d*x + c))/((a^4*d*cos(d*x + c)^2 - a^4*d)*sin(d*x + c))

Sympy [F]

$$\int \frac{\csc^4(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\csc^4{(c+dx)}}{a+b\tan{(c+dx)}}\,dx$$

```
[In] integrate(csc(d*x+c)**4/(a+b*tan(d*x+c)),x)
[Out] Integral(csc(c + d*x)**4/(a + b*tan(c + d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.90

$$\int \frac{\csc^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{6\,(a^2b+b^3)\log(b\tan(dx+c)+a)}{a^4} - \frac{6\,(a^2b+b^3)\log(\tan(dx+c))}{a^4} + \frac{3\,ab\tan(dx+c)-6\,(a^2+b^2)\tan(dx+c)^2-2\,a^2}{a^3\tan(dx+c)^3}}{6\,d}$$

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="maxima")
[Out] 1/6*(6*(a^2*b + b^3)*log(b*tan(d*x + c) + a)/a^4 - 6*(a^2*b + b^3)*log(tan(
d*x + c))/a^4 + (3*a*b*tan(d*x + c) - 6*(a^2 + b^2)*tan(d*x + c)^2 - 2*a^2)
/(a^3*tan(d*x + c)^3))/d
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.33

$$\int \frac{\csc^4(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{\frac{6\,(a^2b+b^3)\log(|\tan(dx+c)|)}{a^4} - \frac{6\,(a^2b^2+b^4)\log(|b\tan(dx+c)+a|)}{a^4b} - \frac{11\,a^2b\tan(dx+c)^3+11\,b^3\tan(dx+c)^3-6\,a^3\tan(dx+c)^2-6\,ab^2\tan(dx}{a^4\tan(dx+c)^3}}{6\,d}$$

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="giac")
[Out] -1/6*(6*(a^2*b + b^3)*log(abs(tan(d*x + c)))/a^4 - 6*(a^2*b^2 + b^4)*log(ab
s(b*tan(d*x + c) + a))/(a^4*b) - (11*a^2*b*tan(d*x + c)^3 + 11*b^3*tan(d*x
+ c)^3 - 6*a^3*tan(d*x + c)^2 - 6*a*b^2*tan(d*x + c)^2 + 3*a^2*b*tan(d*x +
c) - 2*a^3)/(a^4*tan(d*x + c)^3))/d
```

Mupad [B] (verification not implemented)

Time = 4.36 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int \frac{\csc^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{2\,b\,\mathrm{atanh}\left(\frac{b\,(a^2+b^2)\,(a+2\,b\tan(c+d\,x))}{a\,(a^2\,b+b^3)}\right)\,(a^2+b^2)}{a^4\,d} - \frac{\frac{1}{3\,a}+\frac{\tan(c+d\,x)^2\,(a^2+b^2)}{a^3}-\frac{b\tan(c+d\,x)}{2\,a^2}}{d\tan\left(c+d\,x\right)^3}$$

```
[In] int(1/(sin(c + d*x)^4*(a + b*tan(c + d*x))),x)
```

```
[Out] (2*b*atanh((b*(a^2 + b^2)*(a + 2*b*tan(c + d*x)))/(a*(a^2*b + b^3)))*(a^2 +
 b^2))/(a^4*d) - (1/(3*a) + (tan(c + d*x)^2*(a^2 + b^2))/a^3 - (b*tan(c + d
*x))/(2*a^2))/(d*tan(c + d*x)^3)
```

3.60 $\int \frac{\csc^6(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 169

$$\begin{aligned}\int \frac{\csc^6(c+dx)}{a+b\tan(c+dx)}\,dx = &-\frac{(a^2+b^2)^2\cot(c+dx)}{a^5d}+\frac{b(2a^2+b^2)\cot^2(c+dx)}{2a^4d}\\&-\frac{(2a^2+b^2)\cot^3(c+dx)}{3a^3d}+\frac{b\cot^4(c+dx)}{4a^2d}\\&-\frac{\cot^5(c+dx)}{5ad}-\frac{b(a^2+b^2)^2\log(\tan(c+dx))}{a^6d}\\&+\frac{b(a^2+b^2)^2\log(a+b\tan(c+dx))}{a^6d}\end{aligned}$$

```
[Out] -(a^2+b^2)^2*cot(d*x+c)/a^5/d+1/2*b*(2*a^2+b^2)*cot(d*x+c)^2/a^4/d-1/3*(2*a
^2+b^2)*cot(d*x+c)^3/a^3/d+1/4*b*cot(d*x+c)^4/a^2/d-1/5*cot(d*x+c)^5/a/d-b*
(a^2+b^2)^2*ln(tan(d*x+c))/a^6/d+b*(a^2+b^2)^2*ln(a+b*tan(d*x+c))/a^6/d
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3597, 908}

$$\begin{aligned}\int \frac{\csc^6(c+dx)}{a+b\tan(c+dx)}\,dx = &\frac{b\cot^4(c+dx)}{4a^2d}-\frac{b(a^2+b^2)^2\log(\tan(c+dx))}{a^6d}\\&+\frac{b(a^2+b^2)^2\log(a+b\tan(c+dx))}{a^6d}\\&-\frac{(a^2+b^2)^2\cot(c+dx)}{a^5d}+\frac{b(2a^2+b^2)\cot^2(c+dx)}{2a^4d}\\&-\frac{(2a^2+b^2)\cot^3(c+dx)}{3a^3d}-\frac{\cot^5(c+dx)}{5ad}\end{aligned}$$

```
[In] Int[Csc[c + d*x]^6/(a + b*Tan[c + d*x]),x]
```

```
[Out] -(((a^2 + b^2)^2*Cot[c + d*x])/(a^5*d)) + (b*(2*a^2 + b^2)*Cot[c + d*x]^2)/
(2*a^4*d) - ((2*a^2 + b^2)*Cot[c + d*x]^3)/(3*a^3*d) + (b*Cot[c + d*x]^4)/(
4*a^2*d) - Cot[c + d*x]^5/(5*a*d) - (b*(a^2 + b^2)^2*Log[Tan[c + d*x]])/(a^
6*d) + (b*(a^2 + b^2)^2*Log[a + b*Tan[c + d*x]])/(a^6*d)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{b\text{Subst}\left(\int \frac{(b^2+x^2)^2}{x^6(a+x)}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{b\text{Subst}\left(\int \left(\frac{b^4}{ax^6} - \frac{b^4}{a^2x^5} + \frac{2a^2b^2+b^4}{a^3x^4} + \frac{b^2(-2a^2-b^2)}{a^4x^3} + \frac{(a^2+b^2)^2}{a^5x^2} - \frac{(a^2+b^2)^2}{a^6x} + \frac{(a^2+b^2)^2}{a^6(a+x)}\right)dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{(a^2+b^2)^2\cot(c+dx)}{a^5d} + \frac{b(2a^2+b^2)\cot^2(c+dx)}{2a^4d} - \frac{(2a^2+b^2)\cot^3(c+dx)}{3a^3d} + \frac{b\cot^4(c+dx)}{4a^2d} - \frac{\cot^5(c+dx)}{5ad} - \frac{b(a^2+b^2)^2\log(\tan(c+dx))}{a^6d} + \frac{b(a^2+b^2)^2\log(a+b\tan(c+dx))}{a^6d}$$

Mathematica [A] (verified)

Time = 6.17 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.89

$$\int \frac{\csc^6(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{-4\cot(c+dx)\left(8a^5+25a^3b^2+15ab^4+a^3(4a^2+5b^2)\csc^2(c+dx)+3a^5\csc^4(c+dx)\right)+15b\Big(2a^2(a^2+b^2)}{60a^6d}$$

```
[In] Integrate[Csc[c + d*x]^6/(a + b*Tan[c + d*x]),x]
```

```
[Out] (-4*Cot[c + d*x]*(8*a^5 + 25*a^3*b^2 + 15*a*b^4 + a^3*(4*a^2 + 5*b^2)*Csc[c
 + d*x]^2 + 3*a^5*Csc[c + d*x]^4) + 15*b*(2*a^2*(a^2 + b^2)*Csc[c + d*x]^2
+ a^4*Csc[c + d*x]^4 - 4*(a^2 + b^2)^2*(Log[Sin[c + d*x]] - Log[a*Cos[c + d
*x] + b*Sin[c + d*x]])))/(60*a^6*d)
```

Maple [A] (verified)

Time = 4.08 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{-\frac{1}{5a\tan(dx+c)^5}-\frac{2a^2+b^2}{3a^3\tan(dx+c)^3}-\frac{a^4+2a^2b^2+b^4}{a^5\tan(dx+c)}+\frac{b}{4a^2\tan(dx+c)^4}+\frac{\left(2a^2+b^2\right)b}{2a^4\tan(dx+c)^2}-\frac{\left(a^4+2a^2b^2+b^4\right)b\ln(\tan(dx+c))}{a^6}+\frac{\left(a^4+\right.}{}}{d}$
default	$\frac{-\frac{1}{5a\tan(dx+c)^5}-\frac{2a^2+b^2}{3a^3\tan(dx+c)^3}-\frac{a^4+2a^2b^2+b^4}{a^5\tan(dx+c)}+\frac{b}{4a^2\tan(dx+c)^4}+\frac{\left(2a^2+b^2\right)b}{2a^4\tan(dx+c)^2}-\frac{\left(a^4+2a^2b^2+b^4\right)b\ln(\tan(dx+c))}{a^6}+\frac{\left(a^4+\right.}{}}{d}$
risch	$-\frac{2i\left(15ia^3b\,e^{2i(dx+c)}+15ia\,b^3e^{2i(dx+c)}+15a^2b^2e^{8i(dx+c)}+15b^4e^{8i(dx+c)}-45ia\,b^3e^{4i(dx+c)}-15ia^3b\,e^{8i(dx+c)}-90a^2b^2e\right.}{}$

```
[In] int(csc(d*x+c)^6/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/5/a/tan(d*x+c)^5-1/3*(2*a^2+b^2)/a^3/tan(d*x+c)^3-(a^4+2*a^2*b^2+b^
4)/a^5/tan(d*x+c)+1/4/a^2*b/tan(d*x+c)^4+1/2*(2*a^2+b^2)/a^4*b/tan(d*x+c)^2
-(a^4+2*a^2*b^2+b^4)/a^6*b*ln(tan(d*x+c))+(a^4+2*a^2*b^2+b^4)/a^6*b*ln(a+b*
tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(161) = 322$.

Time = 0.29 (sec) , antiderivative size = 385, normalized size of antiderivative = 2.28

$$\int \frac{\csc^6(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{4\left(8\,a^5+25\,a^3b^2+15\,ab^4\right)\cos\left(dx+c\right)^5-20\left(4\,a^5+11\,a^3b^2+6\,ab^4\right)\cos\left(dx+c\right)^3-30\left(a^4b+2\,a^2b^3+\right.}{}$$

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(4*(8*a^5 + 25*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^5 - 20*(4*a^5 + 11*a^
3*b^2 + 6*a*b^4)*cos(d*x + c)^3 - 30*(a^4*b + 2*a^2*b^3 + b^5 + (a^4*b + 2*
a^2*b^3 + b^5)*cos(d*x + c)^4 - 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^2)
*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)*si
n(d*x + c) + 30*(a^4*b + 2*a^2*b^3 + b^5 + (a^4*b + 2*a^2*b^3 + b^5)*cos(d*
x + c)^4 - 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^2)*log(-1/4*cos(d*x + c
)^2 + 1/4)*sin(d*x + c) + 60*(a^5 + 2*a^3*b^2 + a*b^4)*cos(d*x + c) - 15*(3
*a^4*b + 2*a^2*b^3 - 2*(a^4*b + a^2*b^3)*cos(d*x + c)^2)*sin(d*x + c))/((a^
6*d*cos(d*x + c)^4 - 2*a^6*d*cos(d*x + c)^2 + a^6*d)*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\csc^6(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\csc^6(c+dx)}{a+b\tan(c+dx)}\,dx$$

```
[In] integrate(csc(d*x+c)**6/(a+b*tan(d*x+c)),x)
[Out] Integral(csc(c + d*x)**6/(a + b*tan(c + d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.99

$$\int \frac{\csc^6(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{60\,(a^4b+2\,a^2b^3+b^5)\log(b\tan(dx+c)+a)}{a^6} - \frac{60\,(a^4b+2\,a^2b^3+b^5)\log(\tan(dx+c))}{a^6} + \frac{15\,a^3b\tan(dx+c)-60\,(a^4+2\,a^2b^2+b^4)\tan(dx+c)^4-12\,a^4+3}{a^5\tan(dx}}{60\,d}$$

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c)),x, algorithm="maxima")
[Out] 1/60*(60*(a^4*b + 2*a^2*b^3 + b^5)*log(b*tan(d*x + c) + a)/a^6 - 60*(a^4*b
+ 2*a^2*b^3 + b^5)*log(tan(d*x + c))/a^6 + (15*a^3*b*tan(d*x + c) - 60*(a^4
 + 2*a^2*b^2 + b^4)*tan(d*x + c)^4 - 12*a^4 + 30*(2*a^3*b + a*b^3)*tan(d*x
+ c)^3 - 20*(2*a^4 + a^2*b^2)*tan(d*x + c)^2)/(a^5*tan(d*x + c)^5))/d
```

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.49

$$\int \frac{\csc^6(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{\frac{60\,(a^4b+2\,a^2b^3+b^5)\log(|\tan(dx+c)|)}{a^6} - \frac{60\,(a^4b^2+2\,a^2b^4+b^6)\log(|b\tan(dx+c)+a|)}{a^6b} - \frac{137\,a^4b\tan(dx+c)^5+274\,a^2b^3\tan(dx+c)^5+137\,b^5\tan}{}}{}$$

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c)),x, algorithm="giac")
[Out] -1/60*(60*(a^4*b + 2*a^2*b^3 + b^5)*log(abs(tan(d*x + c)))/a^6 - 60*(a^4*b^
2 + 2*a^2*b^4 + b^6)*log(abs(b*tan(d*x + c) + a))/(a^6*b) - (137*a^4*b*tan(
d*x + c)^5 + 274*a^2*b^3*tan(d*x + c)^5 + 137*b^5*tan(d*x + c)^5 - 60*a^5*t
an(d*x + c)^4 - 120*a^3*b^2*tan(d*x + c)^4 - 60*a*b^4*tan(d*x + c)^4 + 60*a
^4*b*tan(d*x + c)^3 + 30*a^2*b^3*tan(d*x + c)^3 - 40*a^5*tan(d*x + c)^2 - 2
0*a^3*b^2*tan(d*x + c)^2 + 15*a^4*b*tan(d*x + c) - 12*a^5)/(a^6*tan(d*x + c
)^5))/d
```

Mupad [B] (verification not implemented)

Time = 5.69 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.99

$$\int \frac{\csc^6(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{2\,b\,\mathrm{atanh}\left(\frac{b\,{\left(a^2+b^2\right)}^2\,\left(a+2\,b\,\tan(c+d\,x)\right)}{a\,\left(a^4\,b+2\,a^2\,b^3+b^5\right)}\right)\,{\left(a^2+b^2\right)}^2}{a^6\,d} - \frac{\frac{1}{5\,a}+\frac{{\tan(c+d\,x)}^2\,\left(2\,a^2+b^2\right)}{3\,a^3}+\frac{{\tan(c+d\,x)}^4\,\left(a^4+2\,a^2\,b^2+b^4\right)}{a^5}-\frac{b\tan(c+d\,x)}{4\,a^2}-\frac{b\,{\tan(c+d\,x)}^3\,\left(2\,a^2+b^2\right)}{2\,a^4}}{d\,{\tan\left(c+d\,x\right)}^5}$$

```
[In] int(1/(sin(c + d*x)^6*(a + b*tan(c + d*x))),x)
```

```
[Out] (2*b*atanh((b*(a^2 + b^2)^2*(a + 2*b*tan(c + d*x)))/(a*(a^4*b + b^5 + 2*a^2
*b^3)))*(a^2 + b^2)^2)/(a^6*d) - (1/(5*a) + (tan(c + d*x)^2*(2*a^2 + b^2))/
(3*a^3) + (tan(c + d*x)^4*(a^4 + b^4 + 2*a^2*b^2))/a^5 - (b*tan(c + d*x))/(
4*a^2) - (b*tan(c + d*x)^3*(2*a^2 + b^2))/(2*a^4))/(d*tan(c + d*x)^5)
```

3.61 $\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 297

$$\begin{aligned}\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx &= \frac{(5a^8-80a^6b^2+50a^4b^4+8a^2b^6+b^8)\,x}{16\,(a^2+b^2)^5} \\ &+ \frac{2a^5b(a^2-3b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^5\,d} \\ &- \frac{a^6b}{(a^2+b^2)^4\,d(a+b\tan(c+dx))} - \frac{\cos^6(c+dx)\,(2ab+(a^2-b^2)\tan(c+dx))}{6\,(a^2+b^2)^2\,d} \\ &+ \frac{\cos^4(c+dx)\,(12ab(3a^2+b^2)+(13a^4-18a^2b^2-7b^4)\tan(c+dx))}{24\,(a^2+b^2)^3\,d} \\ &- \frac{\cos^2(c+dx)\,(48a^5b+(11a^6-43a^4b^2-7a^2b^4-b^6)\tan(c+dx))}{16\,(a^2+b^2)^4\,d}\end{aligned}$$

```
[Out] 1/16*(5*a^8-80*a^6*b^2+50*a^4*b^4+8*a^2*b^6+b^8)*x/(a^2+b^2)^5+2*a^5*b*(a^2
-3*b^2)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^5/d-a^6*b/(a^2+b^2)^4/d/(a+
b*tan(d*x+c))-1/6*cos(d*x+c)^6*(2*a*b+(a^2-b^2)*tan(d*x+c))/(a^2+b^2)^2/d+1
/24*cos(d*x+c)^4*(12*a*b*(3*a^2+b^2)+(13*a^4-18*a^2*b^2-7*b^4)*tan(d*x+c))/
(a^2+b^2)^3/d-1/16*cos(d*x+c)^2*(48*a^5*b+(11*a^6-43*a^4*b^2-7*a^2*b^4-b^6)
*tan(d*x+c))/(a^2+b^2)^4/d
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3597, 1661, 1643, 649, 209, 266}

$$\begin{aligned}\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx =& -\frac{\cos^6(c+dx)\left((a^2-b^2)\tan(c+dx)+2ab\right)}{6d\left(a^2+b^2\right)^2}-\frac{a^6b}{d\left(a^2+b^2\right)^4(a+b\tan(c+dx))}\\&+\frac{2a^5b(a^2-3b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)^5}\\&+\frac{\cos^4(c+dx)\left(12ab(3a^2+b^2)+(13a^4-18a^2b^2-7b^4)\tan(c+dx)\right)}{24d\left(a^2+b^2\right)^3}\\&+\frac{x(5a^8-80a^6b^2+50a^4b^4+8a^2b^6+b^8)}{16\left(a^2+b^2\right)^5}\\&-\frac{\cos^2(c+dx)\left(48a^5b+(11a^6-43a^4b^2-7a^2b^4-b^6)\tan(c+dx)\right)}{16d\left(a^2+b^2\right)^4}\end{aligned}$$

```
[In] Int[Sin[c + d*x]^6/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((5*a^8 - 80*a^6*b^2 + 50*a^4*b^4 + 8*a^2*b^6 + b^8)*x)/(16*(a^2 + b^2)^5)
+ (2*a^5*b*(a^2 - 3*b^2)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)
^5*d) - (a^6*b)/((a^2 + b^2)^4*d*(a + b*Tan[c + d*x])) - (Cos[c + d*x]^6*(2
*a*b + (a^2 - b^2)*Tan[c + d*x]))/(6*(a^2 + b^2)^2*d) + (Cos[c + d*x]^4*(12
*a*b*(3*a^2 + b^2) + (13*a^4 - 18*a^2*b^2 - 7*b^4)*Tan[c + d*x]))/(24*(a^2
+ b^2)^3*d) - (Cos[c + d*x]^2*(48*a^5*b + (11*a^6 - 43*a^4*b^2 - 7*a^2*b^4
- b^6)*Tan[c + d*x]))/(16*(a^2 + b^2)^4*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{b\text{Subst}\left(\int \frac{x^6}{(a+x)^2(b^2+x^2)^4}\,dx, x, b\tan(c+dx)\right)}{d} \\
&= -\frac{\cos^6(c+dx)\left(2ab+(a^2-b^2)\tan(c+dx)\right)}{6\left(a^2+b^2\right)^2 d} \\
&\quad -\frac{\text{Subst}\left(\int \frac{-\frac{a^2b^6\left(a^2-b^2\right)}{\left(a^2+b^2\right)^2}+\frac{2ab^6\left(5a^2+b^2\right)x}{\left(a^2+b^2\right)^2}+\frac{b^4\left(6a^4+17a^2b^2+b^4\right)x^2}{\left(a^2+b^2\right)^2}-6b^2x^4}{(a+x)^2(b^2+x^2)^3}\,dx, x, b\tan(c+dx)\right)}{6bd} \\
&= -\frac{\cos^6(c+dx)\left(2ab+(a^2-b^2)\tan(c+dx)\right)}{6\left(a^2+b^2\right)^2 d} \\
&\quad +\frac{\cos^4(c+dx)\left(12ab(3a^2+b^2)+(13a^4-18a^2b^2-7b^4)\tan(c+dx)\right)}{24\left(a^2+b^2\right)^3 d} \\
&\quad +\frac{\text{Subst}\left(\int \frac{-\frac{3a^2b^6\left(3a^4-6a^2b^2-b^4\right)}{\left(a^2+b^2\right)^3}+\frac{6ab^6\left(13a^4+6a^2b^2+b^4\right)x}{\left(a^2+b^2\right)^3}+\frac{3b^4\left(8a^6+37a^4b^2+6a^2b^4+b^6\right)x^2}{\left(a^2+b^2\right)^3}}{(a+x)^2(b^2+x^2)^2}\,dx, x, b\tan(c+dx)\right)}{24b^3d}
\end{aligned}
$$

$$= -\frac{\cos^6(c+dx)\left(2ab+(a^2-b^2)\tan(c+dx)\right)}{6\left(a^2+b^2\right)^2 d} + \frac{\cos^4(c+dx)\left(12ab(3a^2+b^2)+(13a^4-18a^2b^2-7b^4)\tan(c+dx)\right)}{24\left(a^2+b^2\right)^3 d} - \frac{\cos^2(c+dx)\left(48a^5b+(11a^6-43a^4b^2-7a^2b^4-b^6)\tan(c+dx)\right)}{16\left(a^2+b^2\right)^4 d} - \frac{\text{Subst}\left(\int \frac{-\frac{3a^2b^6\left(5a^6-37a^4b^2+7a^2b^4+b^6\right)}{\left(a^2+b^2\right)^4}+\frac{6ab^6\left(11a^4-6a^2b^2-b^4\right)x}{\left(a^2+b^2\right)^3}+\frac{3b^6\left(11a^6-43a^4b^2-7a^2b^4-b^6\right)x^2}{\left(a^2+b^2\right)^4}}{(a+x)^2(b^2+x^2)}\, dx, x, b\tan(c+dx\right)}{48b^5d}$$

$$= -\frac{\cos^6(c+dx)\left(2ab+(a^2-b^2)\tan(c+dx)\right)}{6\left(a^2+b^2\right)^2 d} + \frac{\cos^4(c+dx)\left(12ab(3a^2+b^2)+(13a^4-18a^2b^2-7b^4)\tan(c+dx)\right)}{24\left(a^2+b^2\right)^3 d} - \frac{\cos^2(c+dx)\left(48a^5b+(11a^6-43a^4b^2-7a^2b^4-b^6)\tan(c+dx)\right)}{16\left(a^2+b^2\right)^4 d} - \frac{\text{Subst}\left(\int \left(-\frac{48a^6b^6}{(a^2+b^2)^4(a+x)^2}-\frac{96a^5b^6(a^2-3b^2)}{(a^2+b^2)^5(a+x)}+\frac{3b^6\left(-5a^8+80a^6b^2-50a^4b^4-8a^2b^6-b^8+32a^5(a^2-3b^2)x\right)}{(a^2+b^2)^5(b^2+x^2)}\right) dx, x, b\right.}{48b^5d}$$

$$= \frac{2a^5b(a^2-3b^2)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^5 d} - \frac{a^6b}{\left(a^2+b^2\right)^4 d(a+b\tan(c+dx))} - \frac{\cos^6(c+dx)\left(2ab+(a^2-b^2)\tan(c+dx)\right)}{6\left(a^2+b^2\right)^2 d} + \frac{\cos^4(c+dx)\left(12ab(3a^2+b^2)+(13a^4-18a^2b^2-7b^4)\tan(c+dx)\right)}{24\left(a^2+b^2\right)^3 d} - \frac{\cos^2(c+dx)\left(48a^5b+(11a^6-43a^4b^2-7a^2b^4-b^6)\tan(c+dx)\right)}{16\left(a^2+b^2\right)^4 d} - \frac{b\text{Subst}\left(\int \frac{-5a^8+80a^6b^2-50a^4b^4-8a^2b^6-b^8+32a^5(a^2-3b^2)x}{b^2+x^2}\, dx, x, b\tan(c+dx)\right)}{16\left(a^2+b^2\right)^5 d}$$

$$= \frac{2a^5b(a^2-3b^2)\log(a+b\tan(c+dx))}{(a^2+b^2)^5 d} - \frac{a^6b}{(a^2+b^2)^4 d(a+b\tan(c+dx))} - \frac{\cos^6(c+dx)\left(2ab+(a^2-b^2)\tan(c+dx)\right)}{6\left(a^2+b^2\right)^2 d} + \frac{\cos^4(c+dx)\left(12ab(3a^2+b^2)+(13a^4-18a^2b^2-7b^4)\tan(c+dx)\right)}{24\left(a^2+b^2\right)^3 d} - \frac{\cos^2(c+dx)\left(48a^5b+(11a^6-43a^4b^2-7a^2b^4-b^6)\tan(c+dx)\right)}{16\left(a^2+b^2\right)^4 d} - \frac{(2a^5b(a^2-3b^2))\,\mathrm{Subst}\left(\int\frac{x}{b^2+x^2}\,dx,x,b\tan(c+dx)\right)}{(a^2+b^2)^5 d} + \frac{(b(5a^8-80a^6b^2+50a^4b^4+8a^2b^6+b^8))\,\mathrm{Subst}\left(\int\frac{1}{b^2+x^2}\,dx,x,b\tan(c+dx)\right)}{16\left(a^2+b^2\right)^5 d}$$

$$= \frac{(5a^8-80a^6b^2+50a^4b^4+8a^2b^6+b^8)\,x}{16\left(a^2+b^2\right)^5} + \frac{2a^5b(a^2-3b^2)\log(\cos(c+dx))}{(a^2+b^2)^5 d} + \frac{2a^5b(a^2-3b^2)\log(a+b\tan(c+dx))}{(a^2+b^2)^5 d} - \frac{a^6b}{(a^2+b^2)^4 d(a+b\tan(c+dx))} - \frac{\cos^6(c+dx)\left(2ab+(a^2-b^2)\tan(c+dx)\right)}{6\left(a^2+b^2\right)^2 d} + \frac{\cos^4(c+dx)\left(12ab(3a^2+b^2)+(13a^4-18a^2b^2-7b^4)\tan(c+dx)\right)}{24\left(a^2+b^2\right)^3 d} - \frac{\cos^2(c+dx)\left(48a^5b+(11a^6-43a^4b^2-7a^2b^4-b^6)\tan(c+dx)\right)}{16\left(a^2+b^2\right)^4 d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 664 vs. $2(297) = 594$.

Time = 6.72 (sec) , antiderivative size = 664, normalized size of antiderivative = 2.24

$$\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

$$= \frac{b\left(-\frac{5(a^2-b^2)\arctan(\tan(c+dx))}{16b(a^2+b^2)^2} + \frac{3(3a^4-3a^2b^2-2b^4)\arctan(\tan(c+dx))}{8b(a^2+b^2)^3} - \frac{(3a^6-6a^4b^2-4a^2b^4-b^6)\arctan(\tan(c+dx))}{2b(a^2+b^2)^4} - \frac{3a^5\cos^2(c+d}{(a^2+b^2)^4}\right.}{}$$

```
[In] Integrate[Sin[c + d*x]^6/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (b*((-5*(a^2 - b^2)*ArcTan[Tan[c + d*x]])/(16*b*(a^2 + b^2)^2) + (3*(3*a^4
- 3*a^2*b^2 - 2*b^4)*ArcTan[Tan[c + d*x]])/(8*b*(a^2 + b^2)^3) - ((3*a^6 -
6*a^4*b^2 - 4*a^2*b^4 - b^6)*ArcTan[Tan[c + d*x]])/(2*b*(a^2 + b^2)^4) - (3
*a^5*Cos[c + d*x]^2)/(a^2 + b^2)^4 + (a*(3*a^2 + b^2)*Cos[c + d*x]^4)/(2*(a
```

```
^2 + b^2)^3) - (a*Cos[c + d*x]^6)/(3*(a^2 + b^2)^2) - (a^5*(2*a^2 - 6*b^2 -
 (a^3 - 7*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/(2*(a^2 + b^
2)^5) + (2*a^5*(a^2 - 3*b^2)*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^5 - (a^5*
(2*a^2 - 6*b^2 + (a^3 - 7*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x
]])/(2*(a^2 + b^2)^5) - (5*(a - b)*(a + b)*Cos[c + d*x]*Sin[c + d*x])/(16*b
*(a^2 + b^2)^2) + (3*(3*a^4 - 3*a^2*b^2 - 2*b^4)*Cos[c + d*x]*Sin[c + d*x])
/(8*b*(a^2 + b^2)^3) - ((3*a^6 - 6*a^4*b^2 - 4*a^2*b^4 - b^6)*Cos[c + d*x]*
Sin[c + d*x])/(2*b*(a^2 + b^2)^4) - (5*(a^2 - b^2)*Cos[c + d*x]^3*Sin[c + d
*x])/(24*b*(a^2 + b^2)^2) + ((3*a^4 - 3*a^2*b^2 - 2*b^4)*Cos[c + d*x]^3*Sin
[c + d*x])/(4*b*(a^2 + b^2)^3) - ((a^2 - b^2)*Cos[c + d*x]^5*Sin[c + d*x])/
(6*b*(a^2 + b^2)^2) - a^6/((a^2 + b^2)^4*(a + b*Tan[c + d*x]))))/d
```

Maple [A] (verified)

Time = 33.91 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{-\frac{b a^6}{(a^2+b^2)^4 (a+b\tan(dx+c))}+\frac{2b a^5 (a^2-3b^2)\ln(a+b\tan(dx+c))}{(a^2+b^2)^5}+\frac{\left(-\frac{11}{16}a^8+2a^6b^2+\frac{25}{8}a^4b^4+\frac{1}{2}b^6a^2+\frac{1}{16}b^8\right)\left(\tan^5(dx+c)\right)+\left(-3 \cdots\right.}{\cdots}}{\cdots}$
default	$\frac{-\frac{b a^6}{(a^2+b^2)^4 (a+b\tan(dx+c))}+\frac{2b a^5 (a^2-3b^2)\ln(a+b\tan(dx+c))}{(a^2+b^2)^5}+\frac{\left(-\frac{11}{16}a^8+2a^6b^2+\frac{25}{8}a^4b^4+\frac{1}{2}b^6a^2+\frac{1}{16}b^8\right)\left(\tan^5(dx+c)\right)+\left(-3 \cdots\right.}{\cdots}}{\cdots}$
risch	Expression too large to display

```
[In] int(sin(d*x+c)^6/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-b*a^6/(a^2+b^2)^4/(a+b*tan(d*x+c))+2*b*a^5*(a^2-3*b^2)/(a^2+b^2)^5*ln
(a+b*tan(d*x+c))+1/(a^2+b^2)^5*(((-11/16*a^8+2*a^6*b^2+25/8*a^4*b^4+1/2*b^6
*a^2+1/16*b^8)*tan(d*x+c)^5+(-3*a^7*b-3*a^5*b^3)*tan(d*x+c)^4+(-5/6*a^8+13/
3*a^6*b^2+5*a^4*b^4-1/3*b^6*a^2-1/6*b^8)*tan(d*x+c)^3+(-9/2*b*a^7-5/2*b^3*a
^5+5/2*a^3*b^5+1/2*a*b^7)*tan(d*x+c)^2+(-5/16*a^8+2*a^6*b^2+15/8*a^4*b^4-1/
2*b^6*a^2-1/16*b^8)*tan(d*x+c)-11/6*b*a^7-1/2*b^3*a^5+3/2*a^3*b^5+1/6*a*b^7
)/(1+tan(d*x+c)^2)^3+1/32*(-32*a^7*b+96*a^5*b^3)*ln(1+tan(d*x+c)^2)+1/16*(5
*a^8-80*a^6*b^2+50*a^4*b^4+8*a^2*b^6+b^8)*arctan(tan(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 619 vs. 2(289) = 578.

Time = 0.34 (sec) , antiderivative size = 619, normalized size of antiderivative = 2.08

$$\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{8\,(a^8b+4\,a^6b^3+6\,a^4b^5+4\,a^2b^7+b^9)\cos{(dx+c)}^7-2\,(19\,a^8b+68\,a^6b^3+90\,a^4b^5+52\,a^2b^7+11\,b^9)\cos}{}$$

```
[In] integrate(sin(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/48*(8*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cos(d*x + c)^7 -
 2*(19*a^8*b + 68*a^6*b^3 + 90*a^4*b^5 + 52*a^2*b^7 + 11*b^9)*cos(d*x + c)^
5 + (85*a^8*b + 224*a^6*b^3 + 210*a^4*b^5 + 88*a^2*b^7 + 17*b^9)*cos(d*x +
c)^3 - (17*a^8*b + 72*a^6*b^3 + 120*a^4*b^5 + 20*a^2*b^7 + 3*b^9 + 3*(5*a^9
 - 80*a^7*b^2 + 50*a^5*b^4 + 8*a^3*b^6 + a*b^8)*d*x)*cos(d*x + c) - 48*((a^
8*b - 3*a^6*b^3)*cos(d*x + c) + (a^7*b^2 - 3*a^5*b^4)*sin(d*x + c))*log(2*a
*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - (98*a^7*
b^2 + 24*a^5*b^4 - 30*a^3*b^6 - 4*a*b^8 - 8*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 +
4*a^3*b^6 + a*b^8)*cos(d*x + c)^6 + 2*(13*a^9 + 44*a^7*b^2 + 54*a^5*b^4 + 2
8*a^3*b^6 + 5*a*b^8)*cos(d*x + c)^4 + 3*(5*a^8*b - 80*a^6*b^3 + 50*a^4*b^5
+ 8*a^2*b^7 + b^9)*d*x - 3*(11*a^9 + 16*a^7*b^2 - 2*a^5*b^4 - 8*a^3*b^6 - a
*b^8)*cos(d*x + c)^2)*sin(d*x + c))/((a^11 + 5*a^9*b^2 + 10*a^7*b^4 + 10*a^
5*b^6 + 5*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^10*b + 5*a^8*b^3 + 10*a^6*b
^5 + 10*a^4*b^7 + 5*a^2*b^9 + b^11)*d*sin(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(sin(d*x+c)**6/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 799 vs. $2(289) = 578$.

Time = 0.32 (sec) , antiderivative size = 799, normalized size of antiderivative = 2.69

$$\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{3\,(5\,a^8-80\,a^6b^2+50\,a^4b^4+8\,a^2b^6+b^8)(dx+c)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}} + \frac{96\,(a^7b-3\,a^5b^3)\log(b\tan(dx+c)+a)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}} - \frac{48\,(a^7b-3\,a^5b^3)\log\left(\tan(dx+c)^2+1\right)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}} - }{}$$

```
[In] integrate(sin(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/48*(3*(5*a^8 - 80*a^6*b^2 + 50*a^4*b^4 + 8*a^2*b^6 + b^8)*(d*x + c)/(a^10
 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) + 96*(a^7*b - 3*
a^5*b^3)*log(b*tan(d*x + c) + a)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^
6 + 5*a^2*b^8 + b^10) - 48*(a^7*b - 3*a^5*b^3)*log(tan(d*x + c)^2 + 1)/(a^1
0 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) - (136*a^6*b -
64*a^4*b^3 - 8*a^2*b^5 + 3*(27*a^6*b - 43*a^4*b^3 - 7*a^2*b^5 - b^7)*tan(d*
x + c)^6 + 3*(11*a^7 + 5*a^5*b^2 - 7*a^3*b^4 - a*b^6)*tan(d*x + c)^5 + 8*(4
1*a^6*b - 31*a^4*b^3 + a^2*b^5 + b^7)*tan(d*x + c)^4 + 8*(5*a^7 - 4*a^5*b^2
 - 11*a^3*b^4 - 2*a*b^6)*tan(d*x + c)^3 + 3*(125*a^6*b - 69*a^4*b^3 - a^2*b
^5 + b^7)*tan(d*x + c)^2 + (15*a^7 - 23*a^5*b^2 - 43*a^3*b^4 - 5*a*b^6)*tan
(d*x + c))/(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8 + (a^8*b + 4*a^
6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*tan(d*x + c)^7 + (a^9 + 4*a^7*b^2 + 6*
a^5*b^4 + 4*a^3*b^6 + a*b^8)*tan(d*x + c)^6 + 3*(a^8*b + 4*a^6*b^3 + 6*a^4*
b^5 + 4*a^2*b^7 + b^9)*tan(d*x + c)^5 + 3*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*
a^3*b^6 + a*b^8)*tan(d*x + c)^4 + 3*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*
b^7 + b^9)*tan(d*x + c)^3 + 3*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*
b^8)*tan(d*x + c)^2 + (a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*tan
(d*x + c)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 735 vs. $2(289) = 578$.

Time = 0.54 (sec) , antiderivative size = 735, normalized size of antiderivative = 2.47

$$\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{3\,(5\,a^8-80\,a^6b^2+50\,a^4b^4+8\,a^2b^6+b^8)(dx+c)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}} - \frac{48\,(a^7b-3\,a^5b^3)\log\left(\tan(dx+c)^2+1\right)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}} + \frac{96\,(a^7b^2-3\,a^5b^4)\log(|b\tan(dx+c)+a|)}{a^{10}b+5\,a^8b^3+10\,a^6b^5+10\,a^4b^7+5\,a^2b^9+b^{11}} - }{}$$

```
[In] integrate(sin(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/48*(3*(5*a^8 - 80*a^6*b^2 + 50*a^4*b^4 + 8*a^2*b^6 + b^8)*(d*x + c)/(a^10
 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) - 48*(a^7*b - 3*
a^5*b^3)*log(tan(d*x + c)^2 + 1)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^
6 + 5*a^2*b^8 + b^10) + 96*(a^7*b^2 - 3*a^5*b^4)*log(abs(b*tan(d*x + c) + a
))/(a^10*b + 5*a^8*b^3 + 10*a^6*b^5 + 10*a^4*b^7 + 5*a^2*b^9 + b^11) - 48*(
2*a^7*b^2*tan(d*x + c) - 6*a^5*b^4*tan(d*x + c) + 3*a^8*b - 5*a^6*b^3)/((a^
10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*(b*tan(d*x + c
) + a)) + (88*a^7*b*tan(d*x + c)^6 - 264*a^5*b^3*tan(d*x + c)^6 - 33*a^8*ta
n(d*x + c)^5 + 96*a^6*b^2*tan(d*x + c)^5 + 150*a^4*b^4*tan(d*x + c)^5 + 24*
a^2*b^6*tan(d*x + c)^5 + 3*b^8*tan(d*x + c)^5 + 120*a^7*b*tan(d*x + c)^4 -
936*a^5*b^3*tan(d*x + c)^4 - 40*a^8*tan(d*x + c)^3 + 208*a^6*b^2*tan(d*x +
c)^3 + 240*a^4*b^4*tan(d*x + c)^3 - 16*a^2*b^6*tan(d*x + c)^3 - 8*b^8*tan(d
*x + c)^3 + 48*a^7*b*tan(d*x + c)^2 - 912*a^5*b^3*tan(d*x + c)^2 + 120*a^3*
b^5*tan(d*x + c)^2 + 24*a*b^7*tan(d*x + c)^2 - 15*a^8*tan(d*x + c) + 96*a^6
*b^2*tan(d*x + c) + 90*a^4*b^4*tan(d*x + c) - 24*a^2*b^6*tan(d*x + c) - 3*b
^8*tan(d*x + c) - 288*a^5*b^3 + 72*a^3*b^5 + 8*a*b^7)/((a^10 + 5*a^8*b^2 +
10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*(tan(d*x + c)^2 + 1)^3))/d
```

Mupad [B] (verification not implemented)

Time = 6.11 (sec) , antiderivative size = 757, normalized size of antiderivative = 2.55

$$\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

$$= \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(\frac{2\,a\,b}{{\left(a^2+b^2\right)}^2}-\frac{12\,a\,b^3}{{\left(a^2+b^2\right)}^3}+\frac{18\,a\,b^5}{{\left(a^2+b^2\right)}^4}-\frac{8\,a\,b^7}{{\left(a^2+b^2\right)}^5}\right)}{d}$$

$$+\frac{\frac{{\tan(c+d\,x)}^3\,\left(-5\,a^5+9\,a^3\,b^2+2\,a\,b^4\right)}{6\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{{\tan(c+d\,x)}^5\,\left(-11\,a^5+6\,a^3\,b^2+a\,b^4\right)}{16\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{{\tan(c+d\,x)}^6\,\left(-27\,a^6\,b+43\,a^4\,b^3+7\,a^2\,b^5+b^7\right)}{16\,\left(a^8+4\,a^6\,b^2+6\,a^4\,b^4+4\,a^2\,b^6+b^8\right)}+\frac{\tan(c+d\,x)\,(}{48\,(a^6+}}{d\,\left(b\tan\left(c+d\,x\right)^7+a\tan\left(c+d\,x\right)^6+3\,b\tan\left(c+d\,x\right)^5+3\,a\,\mathrm{t}\right.}$$

$$+\frac{\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)\,\left(a^3\,5{}\mathrm{i}-7\,a^2\,b+a\,b^2\,5{}\mathrm{i}+b^3\right)}{32\,d\,\left(a^5-a^4\,b\,5{}\mathrm{i}-10\,a^3\,b^2+a^2\,b^3\,10{}\mathrm{i}+5\,a\,b^4-b^5\,1{}\mathrm{i}\right)}$$

$$-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(a^3\,5{}\mathrm{i}+7\,a^2\,b+a\,b^2\,5{}\mathrm{i}-b^3\right)}{32\,d\,\left(a^5+a^4\,b\,5{}\mathrm{i}-10\,a^3\,b^2-a^2\,b^3\,10{}\mathrm{i}+5\,a\,b^4+b^5\,1{}\mathrm{i}\right)}$$

```
[In] int(sin(c + d*x)^6/(a + b*tan(c + d*x))^2,x)
```

```
[Out] (log(a + b*tan(c + d*x))*((2*a*b)/(a^2 + b^2)^2 - (12*a*b^3)/(a^2 + b^2)^3
+ (18*a*b^5)/(a^2 + b^2)^4 - (8*a*b^7)/(a^2 + b^2)^5))/d + ((tan(c + d*x)^3
*(2*a*b^4 - 5*a^5 + 9*a^3*b^2))/(6*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (
tan(c + d*x)^5*(a*b^4 - 11*a^5 + 6*a^3*b^2))/(16*(a^6 + b^6 + 3*a^2*b^4 + 3
*a^4*b^2)) + (tan(c + d*x)^6*(b^7 - 27*a^6*b + 7*a^2*b^5 + 43*a^4*b^3))/(16
*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (tan(c + d*x)*(5*a*b^4
- 15*a^5 + 38*a^3*b^2))/(48*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (a*(a*b^
5 - 17*a^5*b + 8*a^3*b^3))/(6*(a^2 + b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^
```

```
2)) - (tan(c + d*x)^4*(41*a^6*b + b^7 + a^2*b^5 - 31*a^4*b^3))/(6*(a^2 + b^
2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (tan(c + d*x)^2*(125*a^6*b + b^7
- a^2*b^5 - 69*a^4*b^3))/(16*(a^2 + b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2
)))/(d*(a + b*tan(c + d*x) + 3*a*tan(c + d*x)^2 + 3*a*tan(c + d*x)^4 + a*ta
n(c + d*x)^6 + 3*b*tan(c + d*x)^3 + 3*b*tan(c + d*x)^5 + b*tan(c + d*x)^7))
 + (log(tan(c + d*x) + 1i)*(a*b^2*5i - 7*a^2*b + a^3*5i + b^3))/(32*d*(5*a*
b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2)) - (log(tan(c + d
*x) - 1i)*(a*b^2*5i + 7*a^2*b + a^3*5i - b^3))/(32*d*(5*a*b^4 + a^4*b*5i +
a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2))
```

3.62 $\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 217

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{(3a^6-33a^4b^2+13a^2b^4+b^6)\,x}{8\,(a^2+b^2)^4} + \frac{2a^3b(a^2-2b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^4\,d} - \frac{a^4b}{(a^2+b^2)^3\,d(a+b\tan(c+dx))} + \frac{\cos^4(c+dx)\,(2ab+(a^2-b^2)\tan(c+dx))}{4\,(a^2+b^2)^2\,d} - \frac{\cos^2(c+dx)\,(16a^3b+(5a^4-12a^2b^2-b^4)\tan(c+dx))}{8\,(a^2+b^2)^3\,d}$$

```
[Out] 1/8*(3*a^6-33*a^4*b^2+13*a^2*b^4+b^6)*x/(a^2+b^2)^4+2*a^3*b*(a^2-2*b^2)*ln(
a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^4/d-a^4*b/(a^2+b^2)^3/d/(a+b*tan(d*x+c
))+1/4*cos(d*x+c)^4*(2*a*b+(a^2-b^2)*tan(d*x+c))/(a^2+b^2)^2/d-1/8*cos(d*x+
c)^2*(16*a^3*b+(5*a^4-12*a^2*b^2-b^4)*tan(d*x+c))/(a^2+b^2)^3/d
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used

$= \{3597, 1661, 1643, 649, 209, 266\}$

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\cos^4(c+dx)\left((a^2-b^2)\tan(c+dx)+2ab\right)}{4d\left(a^2+b^2\right)^2} - \frac{a^4b}{d\left(a^2+b^2\right)^3(a+b\tan(c+dx))} + \frac{2a^3b(a^2-2b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)^4} + \frac{x(3a^6-33a^4b^2+13a^2b^4+b^6)}{8\left(a^2+b^2\right)^4} - \frac{\cos^2(c+dx)\left(16a^3b+(5a^4-12a^2b^2-b^4)\tan(c+dx)\right)}{8d\left(a^2+b^2\right)^3}$$

```
[In] Int[Sin[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((3*a^6 - 33*a^4*b^2 + 13*a^2*b^4 + b^6)*x)/(8*(a^2 + b^2)^4) + (2*a^3*b*(a
^2 - 2*b^2)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^4*d) - (a^4*
b)/((a^2 + b^2)^3*d*(a + b*Tan[c + d*x])) + (Cos[c + d*x]^4*(2*a*b + (a^2 -
 b^2)*Tan[c + d*x]))/(4*(a^2 + b^2)^2*d) - (Cos[c + d*x]^2*(16*a^3*b + (5*a
^4 - 12*a^2*b^2 - b^4)*Tan[c + d*x]))/(8*(a^2 + b^2)^3*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{b\text{Subst}\left(\int \frac{x^4}{(a+x)^2(b^2+x^2)^3}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{\cos^4(c+dx)\left(2ab+(a^2-b^2)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^2 d} - \frac{\text{Subst}\left(\int \frac{\frac{a^2b^4\left(a^2-b^2\right)}{\left(a^2+b^2\right)^2}-\frac{2ab^4\left(3a^2+b^2\right)x}{\left(a^2+b^2\right)^2}-\frac{b^2\left(4a^4+11a^2b^2+b^4\right)x^2}{\left(a^2+b^2\right)^2}}{(a+x)^2(b^2+x^2)^2}\,dx, x, b\tan(c+dx)\right)}{4bd}$$

$$= \frac{\cos^4(c+dx)\left(2ab+(a^2-b^2)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^2 d} - \frac{\cos^2(c+dx)\left(16a^3b+(5a^4-12a^2b^2-b^4)\tan(c+dx)\right)}{8\left(a^2+b^2\right)^3 d} + \frac{\text{Subst}\left(\int \frac{\frac{a^2b^4\left(3a^4-12a^2b^2+b^4\right)}{\left(a^2+b^2\right)^3}-\frac{2ab^4\left(5a^2-b^2\right)x}{\left(a^2+b^2\right)^2}-\frac{b^4\left(5a^4-12a^2b^2-b^4\right)x^2}{\left(a^2+b^2\right)^3}}{(a+x)^2(b^2+x^2)}\,dx, x, b\tan(c+dx)\right)}{8b^3d}$$

$$= \frac{\cos^4(c+dx)\left(2ab+(a^2-b^2)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^2 d} - \frac{\cos^2(c+dx)\left(16a^3b+(5a^4-12a^2b^2-b^4)\tan(c+dx)\right)}{8\left(a^2+b^2\right)^3 d} + \frac{\text{Subst}\left(\int \left(\frac{8a^4b^4}{(a^2+b^2)^3(a+x)^2}+\frac{16a^3b^4(a^2-2b^2)}{(a^2+b^2)^4(a+x)}+\frac{b^4(3a^6-33a^4b^2+13a^2b^4+b^6-16a^3(a^2-2b^2)x)}{(a^2+b^2)^4(b^2+x^2)}\right)dx, x, b\tan(c+dx\right.}{8b^3d}$$

$$= \frac{2a^3b(a^2-2b^2)\log(a+b\tan(c+dx))}{(a^2+b^2)^4 d} - \frac{a^4b}{(a^2+b^2)^3 d(a+b\tan(c+dx))} + \frac{\cos^4(c+dx)\,(2ab+(a^2-b^2)\tan(c+dx))}{4\,(a^2+b^2)^2 d} - \frac{\cos^2(c+dx)\,(16a^3b+(5a^4-12a^2b^2-b^4)\tan(c+dx))}{8\,(a^2+b^2)^3 d} + \frac{b\text{Subst}\left(\int \frac{3a^6-33a^4b^2+13a^2b^4+b^6-16a^3(a^2-2b^2)x}{b^2+x^2}dx, x, b\tan(c+dx)\right)}{8\,(a^2+b^2)^4 d}$$

$$= \frac{2a^3b(a^2-2b^2)\log(a+b\tan(c+dx))}{(a^2+b^2)^4 d} - \frac{a^4b}{(a^2+b^2)^3 d(a+b\tan(c+dx))} + \frac{\cos^4(c+dx)\,(2ab+(a^2-b^2)\tan(c+dx))}{4\,(a^2+b^2)^2 d} - \frac{\cos^2(c+dx)\,(16a^3b+(5a^4-12a^2b^2-b^4)\tan(c+dx))}{8\,(a^2+b^2)^3 d} - \frac{(2a^3b(a^2-2b^2))\,\text{Subst}\left(\int \frac{x}{b^2+x^2}dx, x, b\tan(c+dx)\right)}{(a^2+b^2)^4 d} + \frac{(b(3a^6-33a^4b^2+13a^2b^4+b^6))\,\text{Subst}\left(\int \frac{1}{b^2+x^2}dx, x, b\tan(c+dx)\right)}{8\,(a^2+b^2)^4 d}$$

$$= \frac{(3a^6-33a^4b^2+13a^2b^4+b^6)\,x}{8\,(a^2+b^2)^4} + \frac{2a^3b(a^2-2b^2)\log(\cos(c+dx))}{(a^2+b^2)^4 d} + \frac{2a^3b(a^2-2b^2)\log(a+b\tan(c+dx))}{(a^2+b^2)^4 d} - \frac{a^4b}{(a^2+b^2)^3 d(a+b\tan(c+dx))} + \frac{\cos^4(c+dx)\,(2ab+(a^2-b^2)\tan(c+dx))}{4\,(a^2+b^2)^2 d} - \frac{\cos^2(c+dx)\,(16a^3b+(5a^4-12a^2b^2-b^4)\tan(c+dx))}{8\,(a^2+b^2)^3 d}$$

Mathematica [A] (verified)

Time = 4.47 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.81

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{b\left(\frac{3(a^2-b^2)(a^2+b^2)^2\arctan(\tan(c+dx))}{b} + \frac{4(a^2+b^2)(-2a^4+3a^2b^2+b^4)\arctan(\tan(c+dx))}{b} - 16a^3(a^2+b^2)\cos^2(c+dx) + 4a(\right.}{}$$

```
[In] Integrate[Sin[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (b*((3*(a^2 - b^2)*(a^2 + b^2)^2*ArcTan[Tan[c + d*x]])/b + (4*(a^2 + b^2)*(
-2*a^4 + 3*a^2*b^2 + b^4)*ArcTan[Tan[c + d*x]])/b - 16*a^3*(a^2 + b^2)*Cos[
c + d*x]^2 + 4*a*(a^2 + b^2)^2*Cos[c + d*x]^4 - 4*a^3*(2*a^2 - 4*b^2 + (-a^
3 + 5*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]] + 16*a^3*(a^2 - 2
*b^2)*Log[a + b*Tan[c + d*x]] - 4*a^3*(2*a^2 - 4*b^2 + (a^3 - 5*a*b^2)/Sqrt
[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + (2*(a^2 - b^2)*(a^2 + b^2)^2*Cos
[c + d*x]^3*Sin[c + d*x])/b + (3*(a - b)*(a + b)*(a^2 + b^2)^2*Sin[2*(c + d
*x)])/(2*b) + (2*(a^2 + b^2)*(-2*a^4 + 3*a^2*b^2 + b^4)*Sin[2*(c + d*x)])/b
 - (8*a^4*(a^2 + b^2))/(a + b*Tan[c + d*x])))/(8*(a^2 + b^2)^4*d)
```

Maple [A] (verified)

Time = 9.21 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.24

method	result
derivativedivides	$-\frac{a^4 b}{\left(a^2+b^2\right)^3 (a+b\tan(dx+c))}+\frac{2a^3b\left(a^2-2b^2\right)\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^4}+\frac{\left(-\frac{5}{8}a^6+\frac{7}{8}a^4b^2+\frac{13}{8}a^2b^4+\frac{1}{8}b^6\right)\left(\tan^3(dx+c)\right)+\left(-2a^5b-2a^3b^3\right)}{(1}$
default	$-\frac{a^4 b}{\left(a^2+b^2\right)^3 (a+b\tan(dx+c))}+\frac{2a^3b\left(a^2-2b^2\right)\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^4}+\frac{\left(-\frac{5}{8}a^6+\frac{7}{8}a^4b^2+\frac{13}{8}a^2b^4+\frac{1}{8}b^6\right)\left(\tan^3(dx+c)\right)+\left(-2a^5b-2a^3b^3\right)}{(1}$
risch	$-\frac{ixab}{2(4ia^3b-4iab^3-a^4+6a^2b^2-b^4)}-\frac{3x a^2}{8(4ia^3b-4iab^3-a^4+6a^2b^2-b^4)}-\frac{x b^2}{8(4ia^3b-4iab^3-a^4+6a^2b^2-b^4)}-\frac{ie^{4i}}{64(-2iab}$

```
[In] int(sin(d*x+c)^4/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^4*b/(a^2+b^2)^3/(a+b*tan(d*x+c))+2*a^3*b*(a^2-2*b^2)/(a^2+b^2)^4*ln
(a+b*tan(d*x+c))+1/(a^2+b^2)^4*(((-5/8*a^6+7/8*a^4*b^2+13/8*a^2*b^4+1/8*b^6
)*tan(d*x+c)^3+(-2*a^5*b-2*a^3*b^3)*tan(d*x+c)^2+(-3/8*a^6+9/8*a^4*b^2+11/8
*a^2*b^4-1/8*b^6)*tan(d*x+c)-3/2*a^5*b-a^3*b^3+1/2*a*b^5)/(1+tan(d*x+c)^2)^
2+1/16*(-16*a^5*b+32*a^3*b^3)*ln(1+tan(d*x+c)^2)+1/8*(3*a^6-33*a^4*b^2+13*a
^2*b^4+b^6)*arctan(tan(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. 2(211) = 422.

Time = 0.31 (sec) , antiderivative size = 444, normalized size of antiderivative = 2.05

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$=\frac{4\left(a^6b+3\,a^4b^3+3\,a^2b^5+b^7\right)\cos\left(dx+c\right)^5-6\left(3\,a^6b+7\,a^4b^3+5\,a^2b^5+b^7\right)\cos\left(dx+c\right)^3+\left(3\,a^6b+8\,a^4b^3\right.}{}$$

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cos(d*x + c)^5 - 6*(3*a^6*b +
 7*a^4*b^3 + 5*a^2*b^5 + b^7)*cos(d*x + c)^3 + (3*a^6*b + 8*a^4*b^3 + 23*a^
2*b^5 + 2*b^7 + 2*(3*a^7 - 33*a^5*b^2 + 13*a^3*b^4 + a*b^6)*d*x)*cos(d*x +
c) + 16*((a^6*b - 2*a^4*b^3)*cos(d*x + c) + (a^5*b^2 - 2*a^3*b^4)*sin(d*x +
 c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2
) + (29*a^5*b^2 + 10*a^3*b^4 - 3*a*b^6 + 4*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a
*b^6)*cos(d*x + c)^4 + 2*(3*a^6*b - 33*a^4*b^3 + 13*a^2*b^5 + b^7)*d*x - 2*
(5*a^7 + 9*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^9
 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*d*cos(d*x + c) + (a^8*b + 4*a
^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*d*sin(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(sin(d*x+c)**4/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 507 vs. 2(211) = 422.

Time = 0.66 (sec) , antiderivative size = 507, normalized size of antiderivative = 2.34

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{\frac{(3a^6-33a^4b^2+13a^2b^4+b^6)(dx+c)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8} + \frac{16(a^5b-2a^3b^3)\log(b\tan(dx+c)+a)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8} - \frac{8(a^5b-2a^3b^3)\log\left(\tan(dx+c)^2+1\right)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8} - \frac{\cdots}{a^7+3a^5b^2+3a^3b^4+ab\cdots}}{\cdots}$$

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*((3*a^6 - 33*a^4*b^2 + 13*a^2*b^4 + b^6)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6
*a^4*b^4 + 4*a^2*b^6 + b^8) + 16*(a^5*b - 2*a^3*b^3)*log(b*tan(d*x + c) + a
)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 8*(a^5*b - 2*a^3*b^3)*l
og(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - (2
0*a^4*b - 4*a^2*b^3 + (13*a^4*b - 12*a^2*b^3 - b^5)*tan(d*x + c)^4 + (5*a^5
 + 4*a^3*b^2 - a*b^4)*tan(d*x + c)^3 + (35*a^4*b - 12*a^2*b^3 + b^5)*tan(d*
x + c)^2 + 3*(a^5 - a*b^4)*tan(d*x + c))/(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b
^6 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*tan(d*x + c)^5 + (a^7 + 3*a^5*b^
2 + 3*a^3*b^4 + a*b^6)*tan(d*x + c)^4 + 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 +
b^7)*tan(d*x + c)^3 + 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*tan(d*x + c)^
2 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*tan(d*x + c)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 513 vs. 2(211) = 422.

Time = 0.53 (sec) , antiderivative size = 513, normalized size of antiderivative = 2.36

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{\frac{(3a^6-33a^4b^2+13a^2b^4+b^6)(dx+c)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8} - \frac{8(a^5b-2a^3b^3)\log\left(\tan(dx+c)^2+1\right)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8} + \frac{16(a^5b^2-2a^3b^4)\log(|b\tan(dx+c)+a|)}{a^8b+4a^6b^3+6a^4b^5+4a^2b^7+b^9} - \frac{8(2a^5b^2\tan(dx+c)-}{(a^8+4a^6b^2+6a^4b^4}}{}$$

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*((3*a^6 - 33*a^4*b^2 + 13*a^2*b^4 + b^6)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6
*a^4*b^4 + 4*a^2*b^6 + b^8) - 8*(a^5*b - 2*a^3*b^3)*log(tan(d*x + c)^2 + 1)
/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 16*(a^5*b^2 - 2*a^3*b^4)
*log(abs(b*tan(d*x + c) + a))/(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 +
b^9) - 8*(2*a^5*b^2*tan(d*x + c) - 4*a^3*b^4*tan(d*x + c) + 3*a^6*b - 3*a^4
*b^3)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(b*tan(d*x + c) + a)
) + (12*a^5*b*tan(d*x + c)^4 - 24*a^3*b^3*tan(d*x + c)^4 - 5*a^6*tan(d*x +
c)^3 + 7*a^4*b^2*tan(d*x + c)^3 + 13*a^2*b^4*tan(d*x + c)^3 + b^6*tan(d*x +
 c)^3 + 8*a^5*b*tan(d*x + c)^2 - 64*a^3*b^3*tan(d*x + c)^2 - 3*a^6*tan(d*x
+ c) + 9*a^4*b^2*tan(d*x + c) + 11*a^2*b^4*tan(d*x + c) - b^6*tan(d*x + c)
- 32*a^3*b^3 + 4*a*b^5)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(t
an(d*x + c)^2 + 1)^2))/d
```

Mupad [B] (verification not implemented)

Time = 5.82 (sec) , antiderivative size = 481, normalized size of antiderivative = 2.22

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{\frac{\tan(c+dx)^3(ab^2-5a^3)}{8(a^4+2a^2b^2+b^4)} + \frac{\tan(c+dx)^4(-13a^4b+12a^2b^3+b^5)}{8(a^6+3a^4b^2+3a^2b^4+b^6)} + \frac{3\tan(c+dx)(ab^2-a^3)}{8(a^4+2a^2b^2+b^4)} - \frac{\tan(c+dx)^2(35a^4b-12a^2b^3+b^5)}{8(a^2+b^2)(a^4+2a^2b^2+b^4)} + \frac{a(}{2(a^2+b^2}}{d\left(b\tan(c+dx)^5+a\tan(c+dx)^4+2b\tan(c+dx)^3+2a\tan(c+dx)^2+b\tan(c+dx)+a\right.}$$
$$+ \frac{\ln(a+b\tan(c+dx))\left(\frac{2ab}{(a^2+b^2)^2}-\frac{8ab^3}{(a^2+b^2)^3}+\frac{6ab^5}{(a^2+b^2)^4}\right)}{d}$$
$$+ \frac{\ln(\tan(c+dx)-\mathrm{i})\,(3a^2-ab\,4\mathrm{i}+b^2)}{16d\,(a^4\,1\mathrm{i}-4a^3b-a^2b^2\,6\mathrm{i}+4ab^3+b^4\,1\mathrm{i})}$$
$$- \frac{\ln(\tan(c+dx)+1\mathrm{i})\,(3a^2+ab\,4\mathrm{i}+b^2)}{16d\,(a^4\,1\mathrm{i}+4a^3b-a^2b^2\,6\mathrm{i}-4ab^3+b^4\,1\mathrm{i})}$$

```
[In] int(sin(c + d*x)^4/(a + b*tan(c + d*x))^2,x)
```

```
[Out] ((tan(c + d*x)^3*(a*b^2 - 5*a^3))/(8*(a^4 + b^4 + 2*a^2*b^2)) + (tan(c + d*
x)^4*(b^5 - 13*a^4*b + 12*a^2*b^3))/(8*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))
 + (3*tan(c + d*x)*(a*b^2 - a^3))/(8*(a^4 + b^4 + 2*a^2*b^2)) - (tan(c + d*
x)^2*(35*a^4*b + b^5 - 12*a^2*b^3))/(8*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2))
 + (a*(a*b^3 - 5*a^3*b))/(2*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)))/(d*(a + b
*tan(c + d*x) + 2*a*tan(c + d*x)^2 + a*tan(c + d*x)^4 + 2*b*tan(c + d*x)^3
+ b*tan(c + d*x)^5)) + (log(a + b*tan(c + d*x))*((2*a*b)/(a^2 + b^2)^2 - (8
*a*b^3)/(a^2 + b^2)^3 + (6*a*b^5)/(a^2 + b^2)^4))/d + (log(tan(c + d*x) - 1
i)*(3*a^2 - a*b*4i + b^2))/(16*d*(4*a*b^3 - 4*a^3*b + a^4*1i + b^4*1i - a^2
*b^2*6i)) - (log(tan(c + d*x) + 1i)*(a*b*4i + 3*a^2 + b^2))/(16*d*(4*a^3*b
- 4*a*b^3 + a^4*1i + b^4*1i - a^2*b^2*6i))
```

3.63 $\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 148

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{(a^4-6a^2b^2+b^4)\,x}{2\,(a^2+b^2)^3} + \frac{2ab(a^2-b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^3\,d} - \frac{a^2b}{(a^2+b^2)^2\,d(a+b\tan(c+dx))} - \frac{\cos^2(c+dx)\,(2ab+(a^2-b^2)\tan(c+dx))}{2\,(a^2+b^2)^2\,d}$$

```
[Out] 1/2*(a^4-6*a^2*b^2+b^4)*x/(a^2+b^2)^3+2*a*b*(a^2-b^2)*ln(a*cos(d*x+c)+b*sin
(d*x+c))/(a^2+b^2)^3/d-a^2*b/(a^2+b^2)^2/d/(a+b*tan(d*x+c))-1/2*cos(d*x+c)^
2*(2*a*b+(a^2-b^2)*tan(d*x+c))/(a^2+b^2)^2/d
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used

= {3597, 1661, 1643, 649, 209, 266}

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{a^2b}{d\,(a^2+b^2)^2\,(a+b\tan(c+dx))} - \frac{\cos^2(c+dx)\,((a^2-b^2)\tan(c+dx)+2ab)}{2d\,(a^2+b^2)^2} + \frac{2ab(a^2-b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^3} + \frac{x(a^4-6a^2b^2+b^4)}{2\,(a^2+b^2)^3}$$

```
[In] Int[Sin[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((a^4 - 6*a^2*b^2 + b^4)*x)/(2*(a^2 + b^2)^3) + (2*a*b*(a^2 - b^2)*Log[a*Co
s[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^3*d) - (a^2*b)/((a^2 + b^2)^2*d*
(a + b*Tan[c + d*x])) - (Cos[c + d*x]^2*(2*a*b + (a^2 - b^2)*Tan[c + d*x]))
/(2*(a^2 + b^2)^2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
```

```
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b\text{Subst}\left(\int \frac{x^2}{(a+x)^2(b^2+x^2)^2}\,dx, x, b\tan(c+dx)\right)}{d}\\
&= -\frac{\cos^2(c+dx)\left(2ab+(a^2-b^2)\tan(c+dx)\right)}{2\left(a^2+b^2\right)^2 d}\\
&\quad -\frac{\text{Subst}\left(\int \frac{-\frac{a^2b^2\left(a^2-b^2\right)}{\left(a^2+b^2\right)^2}+\frac{2ab^2x}{a^2+b^2}+\frac{b^2\left(a^2-b^2\right)x^2}{\left(a^2+b^2\right)^2}}{(a+x)^2(b^2+x^2)}\,dx, x, b\tan(c+dx)\right)}{2bd}\\
&= -\frac{\cos^2(c+dx)\left(2ab+(a^2-b^2)\tan(c+dx)\right)}{2\left(a^2+b^2\right)^2 d}\\
&\quad -\frac{\text{Subst}\left(\int \left(-\frac{2a^2b^2}{(a^2+b^2)^2(a+x)^2}+\frac{4ab^2(-a^2+b^2)}{(a^2+b^2)^3(a+x)}+\frac{b^2(-a^4+6a^2b^2-b^4+4a(a^2-b^2)x)}{(a^2+b^2)^3(b^2+x^2)}\right)dx, x, b\tan(c+dx)\right)}{2bd}\\
&= \frac{2ab(a^2-b^2)\log(a+b\tan(c+dx))}{(a^2+b^2)^3 d}-\frac{a^2b}{(a^2+b^2)^2 d(a+b\tan(c+dx))}\\
&\quad -\frac{\cos^2(c+dx)\left(2ab+(a^2-b^2)\tan(c+dx)\right)}{2\left(a^2+b^2\right)^2 d}\\
&\quad -\frac{b\text{Subst}\left(\int \frac{-a^4+6a^2b^2-b^4+4a(a^2-b^2)x}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{2\left(a^2+b^2\right)^3 d}
\end{aligned}$$

$$= \frac{2ab(a^2-b^2)\log(a+b\tan(c+dx))}{(a^2+b^2)^3 d} - \frac{a^2b}{(a^2+b^2)^2 d(a+b\tan(c+dx))} - \frac{\cos^2(c+dx)\,(2ab+(a^2-b^2)\tan(c+dx))}{2\,(a^2+b^2)^2 d} - \frac{(2ab(a^2-b^2))\,\text{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{(a^2+b^2)^3 d} + \frac{(b(a^4-6a^2b^2+b^4))\,\text{Subst}\left(\int \frac{1}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{2\,(a^2+b^2)^3 d}$$

$$= \frac{(a^4-6a^2b^2+b^4)\,x}{2\,(a^2+b^2)^3} + \frac{2ab(a^2-b^2)\log(\cos(c+dx))}{(a^2+b^2)^3 d} + \frac{2ab(a^2-b^2)\log(a+b\tan(c+dx))}{(a^2+b^2)^3 d} - \frac{a^2b}{(a^2+b^2)^2 d(a+b\tan(c+dx))} - \frac{\cos^2(c+dx)\,(2ab+(a^2-b^2)\tan(c+dx))}{2\,(a^2+b^2)^2 d}$$

Mathematica [A] (verified)

Time = 3.75 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.66

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{b\left(\frac{(a^2-b^2)\,(a^2+b^2)\arctan(\tan(c+dx))}{b} + 2a(a^2+b^2)\cos^2(c+dx) + a\left(2a^2-2b^2+\frac{-a^3+3ab^2}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}-b\tan\right.\right.}{}$$

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -1/2*(b*(((a^2 - b^2)*(a^2 + b^2)*ArcTan[Tan[c + d*x]])/b + 2*a*(a^2 + b^2)
*Cos[c + d*x]^2 + a*(2*a^2 - 2*b^2 + (-a^3 + 3*a*b^2)/Sqrt[-b^2])*Log[Sqrt[
-b^2] - b*Tan[c + d*x]] - 4*a*(a - b)*(a + b)*Log[a + b*Tan[c + d*x]] + a*(
2*a^2 - 2*b^2 + (a^3 - 3*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]
] + ((a - b)*(a + b)*(a^2 + b^2)*Sin[2*(c + d*x)])/(2*b) + (2*a^2*(a^2 + b^
2))/(a + b*Tan[c + d*x])))/((a^2 + b^2)^3*d)
```

Maple [A] (verified)

Time = 2.49 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.16

method	result
derivativedivides	$\frac{\frac{\left(-\frac{a^4}{2}+\frac{b^4}{2}\right)\tan(dx+c)-a^3b-a\,b^3}{1+\tan^2(dx+c)}+\frac{\left(-4a^3b+4a\,b^3\right)\ln\left(1+\tan^2(dx+c)\right)}{4}+\frac{\left(a^4-6a^2b^2+b^4\right)\arctan(\tan(dx+c))}{2}}{\left(a^2+b^2\right)^3}-\frac{a^2b}{\left(a^2+b^2\right)^2(a+b\tan(}}{d}$
default	$\frac{\frac{\left(-\frac{a^4}{2}+\frac{b^4}{2}\right)\tan(dx+c)-a^3b-a\,b^3}{1+\tan^2(dx+c)}+\frac{\left(-4a^3b+4a\,b^3\right)\ln\left(1+\tan^2(dx+c)\right)}{4}+\frac{\left(a^4-6a^2b^2+b^4\right)\arctan(\tan(dx+c))}{2}}{\left(a^2+b^2\right)^3}-\frac{a^2b}{\left(a^2+b^2\right)^2(a+b\tan(}}{d}$
risch	$-\frac{ixb}{2(3ib\,a^2-ib^3-a^3+3a\,b^2)}-\frac{xa}{2(3ib\,a^2-ib^3-a^3+3a\,b^2)}+\frac{i\mathrm{e}^{2i(dx+c)}}{8(-2iab+a^2-b^2)d}-\frac{i\mathrm{e}^{-2i(dx+c)}}{8(2iab+a^2-b^2)d}-\frac{4ia^3bx}{a^6+3a^4b^2+3a^2}$

```
[In] int(sin(d*x+c)^2/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a^2+b^2)^3*(((-1/2*a^4+1/2*b^4)*tan(d*x+c)-a^3*b-a*b^3)/(1+tan(d*x+
c)^2)+1/4*(-4*a^3*b+4*a*b^3)*ln(1+tan(d*x+c)^2)+1/2*(a^4-6*a^2*b^2+b^4)*arc
tan(tan(d*x+c)))-a^2*b/(a^2+b^2)^2/(a+b*tan(d*x+c))+2*a*b*(a^2-b^2)/(a^2+b^
2)^3*ln(a+b*tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(144) = 288$.

Time = 0.28 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.97

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{(a^4b+2\,a^2b^3+b^5)\cos{(dx+c)}^3+(a^2b^3-b^5-(a^5-6\,a^3b^2+ab^4)dx)\cos{(dx+c)}-2\,((a^4b-a^2b^3)\cos}{2}$$

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*((a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^3 + (a^2*b^3 - b^5 - (a^5 - 6*
a^3*b^2 + a*b^4)*d*x)*cos(d*x + c) - 2*((a^4*b - a^2*b^3)*cos(d*x + c) + (a
^3*b^2 - a*b^4)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 -
b^2)*cos(d*x + c)^2 + b^2) - (3*a^3*b^2 + a*b^4 + (a^4*b - 6*a^2*b^3 + b^5)
*d*x - (a^5 + 2*a^3*b^2 + a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/((a^7 + 3*a^
5*b^2 + 3*a^3*b^4 + a*b^6)*d*cos(d*x + c) + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5
+ b^7)*d*sin(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(sin(d*x+c)**2/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(144) = 288.

Time = 0.53 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.98

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{(a^4-6a^2b^2+b^4)(dx+c)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{4(a^3b-ab^3)\log(b\tan(dx+c)+a)}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{2(a^3b-ab^3)\log\left(\tan(dx+c)^2+1\right)}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{4a^2b+(3a^2b}{a^5+2a^3b^2+ab^4+(a^4b+2a^2b^3+b^5)\tan(d}}{2d}$$

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*((a^4 - 6*a^2*b^2 + b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
+ 4*(a^3*b - a*b^3)*log(b*tan(d*x + c) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
b^6) - 2*(a^3*b - a*b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b
^4 + b^6) - (4*a^2*b + (3*a^2*b - b^3)*tan(d*x + c)^2 + (a^3 + a*b^2)*tan(d
*x + c))/(a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*tan(d*x + c)^
3 + (a^5 + 2*a^3*b^2 + a*b^4)*tan(d*x + c)^2 + (a^4*b + 2*a^2*b^3 + b^5)*ta
n(d*x + c)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.51 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.78

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{(a^4-6a^2b^2+b^4)(dx+c)}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{2(a^3b-ab^3)\log\left(\tan(dx+c)^2+1\right)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{4(a^3b^2-ab^4)\log(|b\tan(dx+c)+a|)}{a^6b+3a^4b^3+3a^2b^5+b^7}-\frac{3a^2b\tan(dx+c)^2-b^3\tan(dx+c)^2+a^3}{(a^4+2a^2b^2+b^4)\left(b\tan(dx+c)^3+a\right.}}{2d}$$

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*((a^4 - 6*a^2*b^2 + b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
- 2*(a^3*b - a*b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
b^6) + 4*(a^3*b^2 - a*b^4)*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3
+ 3*a^2*b^5 + b^7) - (3*a^2*b*tan(d*x + c)^2 - b^3*tan(d*x + c)^2 + a^3*tan
(d*x + c) + a*b^2*tan(d*x + c) + 4*a^2*b)/((a^4 + 2*a^2*b^2 + b^4)*(b*tan(d
*x + c)^3 + a*tan(d*x + c)^2 + b*tan(d*x + c) + a)))/d
```

Mupad [B] (verification not implemented)

Time = 4.79 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.72

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(\frac{2\,a\,b}{{\left(a^2+b^2\right)}^2}-\frac{4\,a\,b^3}{{\left(a^2+b^2\right)}^3}\right)}{d} - \frac{\frac{{\tan(c+d\,x)}^2\,\left(3\,a^2\,b-b^3\right)}{2\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{a\tan(c+d\,x)}{2\,\left(a^2+b^2\right)}+\frac{2\,a^2\,b}{{\left(a^2+b^2\right)}^2}}{d\,\left(b\tan{\left(c+d\,x\right)}^3+a\tan{\left(c+d\,x\right)}^2+b\tan\left(c+d\,x\right)+a\right)} + \frac{\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)\,\left(a+b\,1{}\mathrm{i}\right)}{4\,d\,\left(-a^3\,1{}\mathrm{i}-3\,a^2\,b+a\,b^2\,3{}\mathrm{i}+b^3\right)} + \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(b+a\,1{}\mathrm{i}\right)}{4\,d\,\left(-a^3-a^2\,b\,3{}\mathrm{i}+3\,a\,b^2+b^3\,1{}\mathrm{i}\right)}$$

```
[In] int(sin(c + d*x)^2/(a + b*tan(c + d*x))^2,x)
```

```
[Out] (log(a + b*tan(c + d*x))*((2*a*b)/(a^2 + b^2)^2 - (4*a*b^3)/(a^2 + b^2)^3))
/d - ((tan(c + d*x)^2*(3*a^2*b - b^3))/(2*(a^4 + b^4 + 2*a^2*b^2)) + (a*tan
(c + d*x))/(2*(a^2 + b^2)) + (2*a^2*b)/(a^2 + b^2)^2)/(d*(a + b*tan(c + d*x
) + a*tan(c + d*x)^2 + b*tan(c + d*x)^3)) + (log(tan(c + d*x) + 1i)*(a + b*
1i))/(4*d*(a*b^2*3i - 3*a^2*b - a^3*1i + b^3)) + (log(tan(c + d*x) - 1i)*(a
*1i + b))/(4*d*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i))
```

3.64 $\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 72

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\cot(c+dx)}{a^2d} - \frac{2b\log(\tan(c+dx))}{a^3d} + \frac{2b\log(a+b\tan(c+dx))}{a^3d} - \frac{b}{a^2d(a+b\tan(c+dx))}$$

```
[Out] -cot(d*x+c)/a^2/d-2*b*ln(tan(d*x+c))/a^3/d+2*b*ln(a+b*tan(d*x+c))/a^3/d-b/a
^2/d/(a+b*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3597, 46}

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{2b\log(\tan(c+dx))}{a^3d} + \frac{2b\log(a+b\tan(c+dx))}{a^3d} - \frac{b}{a^2d(a+b\tan(c+dx))} - \frac{\cot(c+dx)}{a^2d}$$

```
[In] Int[Csc[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(Cot[c + d*x]/(a^2*d)) - (2*b*Log[Tan[c + d*x]])/(a^3*d) + (2*b*Log[a + b*
Tan[c + d*x]])/(a^3*d) - b/(a^2*d*(a + b*Tan[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
```

```
NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
n + 2, 0])
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{b\text{Subst}\left(\int \frac{1}{x^2(a+x)^2}\,dx, x, b\tan(c+dx)\right)}{d}\\ &= \frac{b\text{Subst}\left(\int \left(\frac{1}{a^2x^2}-\frac{2}{a^3x}+\frac{1}{a^2(a+x)^2}+\frac{2}{a^3(a+x)}\right)\,dx, x, b\tan(c+dx)\right)}{d}\\ &= -\frac{\cot(c+dx)}{a^2d}-\frac{2b\log(\tan(c+dx))}{a^3d}+\frac{2b\log(a+b\tan(c+dx))}{a^3d}-\frac{b}{a^2d(a+b\tan(c+dx))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.51

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{-a^2\cot^2(c+dx)-ab\cot(c+dx)(1+2\log(\sin(c+dx))-2\log(a\cos(c+dx)+b\sin(c+dx)))+b^2(1-2}{a^3d(b+a\cot(c+dx))}$$

```
[In] Integrate[Csc[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-(a^2*Cot[c + d*x]^2) - a*b*Cot[c + d*x]*(1 + 2*Log[Sin[c + d*x]] - 2*Log[
a*Cos[c + d*x] + b*Sin[c + d*x]]) + b^2*(1 - 2*Log[Sin[c + d*x]] + 2*Log[a*
Cos[c + d*x] + b*Sin[c + d*x]]))/(a^3*d*(b + a*Cot[c + d*x]))
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

method	result	siz
derivativedivides	$\frac{-\frac{b}{a^2(a+b\tan(dx+c))}+\frac{2b\ln(a+b\tan(dx+c))}{a^3}-\frac{1}{a^2\tan(dx+c)}-\frac{2b\ln(\tan(dx+c))}{a^3}}{d}$	67
default	$\frac{-\frac{b}{a^2(a+b\tan(dx+c))}+\frac{2b\ln(a+b\tan(dx+c))}{a^3}-\frac{1}{a^2\tan(dx+c)}-\frac{2b\ln(\tan(dx+c))}{a^3}}{d}$	67
risch	$-\frac{2i(2iab\,e^{2i(dx+c)}-a^2e^{2i(dx+c)}+2b^2e^{2i(dx+c)}-a^2-2b^2)}{(e^{2i(dx+c)}-1)(ia+b)(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia)a^2d}-\frac{2b\ln(e^{2i(dx+c)}-1)}{a^3d}+\frac{2b\ln\left(e^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)}{a^3d}$	17

```
[In] int(csc(d*x+c)^2/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a^2*b/(a+b*tan(d*x+c))+2/a^3*b*ln(a+b*tan(d*x+c))-1/a^2/tan(d*x+c)-
2/a^3*b*ln(tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(72) = 144.

Time = 0.28 (sec) , antiderivative size = 293, normalized size of antiderivative = 4.07

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{a^2b^2-(a^4+2\,a^2b^2)\cos{(dx+c)}^2-(a^3b+2\,ab^3)\cos{(dx+c)}\sin{(dx+c)}+\left(a^2b^2+b^4-(a^2b^2+b^4)\cos\right.}{}$$

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(a^2*b^2 - (a^4 + 2*a^2*b^2)*cos(d*x + c)^2 - (a^3*b + 2*a*b^3)*cos(d*x +
c)*sin(d*x + c) + (a^2*b^2 + b^4 - (a^2*b^2 + b^4)*cos(d*x + c)^2 + (a^3*b
+ a*b^3)*cos(d*x + c)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (
a^2 - b^2)*cos(d*x + c)^2 + b^2) - (a^2*b^2 + b^4 - (a^2*b^2 + b^4)*cos(d*x
 + c)^2 + (a^3*b + a*b^3)*cos(d*x + c)*sin(d*x + c))*log(-1/4*cos(d*x + c)^
2 + 1/4))/((a^5*b + a^3*b^3)*d*cos(d*x + c)^2 - (a^6 + a^4*b^2)*d*cos(d*x +
 c)*sin(d*x + c) - (a^5*b + a^3*b^3)*d)
```

Sympy [F]

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\csc^2{(c+dx)}}{(a+b\tan{(c+dx)})^2}\,dx$$

```
[In] integrate(csc(d*x+c)**2/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)**2/(a + b*tan(c + d*x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{2\,b\tan(dx+c)+a}{a^2b\tan(dx+c)^2+a^3\tan(dx+c)} - \frac{2\,b\log(b\tan(dx+c)+a)}{a^3} + \frac{2\,b\log(\tan(dx+c))}{a^3}}{d}$$

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -((2*b*tan(d*x + c) + a)/(a^2*b*tan(d*x + c)^2 + a^3*tan(d*x + c)) - 2*b*lo
g(b*tan(d*x + c) + a)/a^3 + 2*b*log(tan(d*x + c))/a^3)/d
```

Giac [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{2\,b\log(|b\tan(dx+c)+a|)}{a^3} - \frac{2\,b\log(|\tan(dx+c)|)}{a^3} - \frac{2\,b\tan(dx+c)+a}{\left(b\tan(dx+c)^2+a\tan(dx+c)\right)a^2}}{d}$$

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*b*log(abs(b*tan(d*x + c) + a))/a^3 - 2*b*log(abs(tan(d*x + c)))/a^3 - (2
*b*tan(d*x + c) + a)/((b*tan(d*x + c)^2 + a*tan(d*x + c))*a^2))/d
```

Mupad [B] (verification not implemented)

Time = 5.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.10

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{2\,b\,\ln\left(\frac{a+b\tan(c+d\,x)}{\tan(c+d\,x)}\right)}{a^3\,d} - \frac{2\,b}{a^2\,d\,\left(a+b\tan\left(c+d\,x\right)\right)} - \frac{1}{a\,d\tan\left(c+d\,x\right)\,\left(a+b\tan\left(c+d\,x\right)\right)}$$

```
[In] int(1/(sin(c + d*x)^2*(a + b*tan(c + d*x))^2),x)
```

```
[Out] (2*b*log((a + b*tan(c + d*x))/tan(c + d*x)))/(a^3*d) - (2*b)/(a^2*d*(a + b*
tan(c + d*x))) - 1/(a*d*tan(c + d*x)*(a + b*tan(c + d*x)))
```

3.65 $\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 140

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{(a^2+3b^2)\cot(c+dx)}{a^4d}+\frac{b\cot^2(c+dx)}{a^3d}-\frac{\cot^3(c+dx)}{3a^2d}-\frac{2b(a^2+2b^2)\log(\tan(c+dx))}{a^5d}+\frac{2b(a^2+2b^2)\log(a+b\tan(c+dx))}{a^5d}-\frac{b(a^2+b^2)}{a^4d(a+b\tan(c+dx))}$$

```
[Out] -(a^2+3*b^2)*cot(d*x+c)/a^4/d+b*cot(d*x+c)^2/a^3/d-1/3*cot(d*x+c)^3/a^2/d-2
*b*(a^2+2*b^2)*ln(tan(d*x+c))/a^5/d+2*b*(a^2+2*b^2)*ln(a+b*tan(d*x+c))/a^5/
d-b*(a^2+b^2)/a^4/d/(a+b*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3597, 908}

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{b\cot^2(c+dx)}{a^3d}-\frac{\cot^3(c+dx)}{3a^2d}-\frac{2b(a^2+2b^2)\log(\tan(c+dx))}{a^5d}+\frac{2b(a^2+2b^2)\log(a+b\tan(c+dx))}{a^5d}-\frac{b(a^2+b^2)}{a^4d(a+b\tan(c+dx))}-\frac{(a^2+3b^2)\cot(c+dx)}{a^4d}$$

```
[In] Int[Csc[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(((a^2 + 3*b^2)*Cot[c + d*x])/(a^4*d)) + (b*Cot[c + d*x]^2)/(a^3*d) - Cot[
c + d*x]^3/(3*a^2*d) - (2*b*(a^2 + 2*b^2)*Log[Tan[c + d*x]])/(a^5*d) + (2*b
*(a^2 + 2*b^2)*Log[a + b*Tan[c + d*x]])/(a^5*d) - (b*(a^2 + b^2))/(a^4*d*(a
 + b*Tan[c + d*x]))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b\text{Subst}\left(\int \frac{b^2+x^2}{x^4(a+x)^2}\,dx, x, b\tan(c+dx)\right)}{d}\\
&= \frac{b\text{Subst}\left(\int \left(\frac{b^2}{a^2x^4}-\frac{2b^2}{a^3x^3}+\frac{a^2+3b^2}{a^4x^2}-\frac{2(a^2+2b^2)}{a^5x}+\frac{a^2+b^2}{a^4(a+x)^2}+\frac{2(a^2+2b^2)}{a^5(a+x)}\right)dx, x, b\tan(c+dx)\right)}{d}\\
&= -\frac{(a^2+3b^2)\cot(c+dx)}{a^4d}+\frac{b\cot^2(c+dx)}{a^3d}\\
&\quad -\frac{\cot^3(c+dx)}{3a^2d}-\frac{2b(a^2+2b^2)\log(\tan(c+dx))}{a^5d}\\
&\quad +\frac{2b(a^2+2b^2)\log(a+b\tan(c+dx))}{a^5d}-\frac{b(a^2+b^2)}{a^4d(a+b\tan(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 4.10 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.74

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{-\cot^2(c+dx)\left(2a^4+9a^2b^2+a^4\csc^2(c+dx)\right)+3b^2(a^2+b^2+a^2\csc^2(c+dx)-2(a^2+2b^2)\log(\sin(c+dx}{}$$

```
[In] Integrate[Csc[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]
```

[Out] (-(Cot[c + d*x]^2*(2*a^4 + 9*a^2*b^2 + a^4*Csc[c + d*x]^2)) + 3*b^2*(a^2 + b^2 + a^2*Csc[c + d*x]^2 - 2*(a^2 + 2*b^2)*Log[Sin[c + d*x]] + 2*a^2*Log[a*Cos[c + d*x] + b*Sin[c + d*x]] + 4*b^2*Log[a*Cos[c + d*x] + b*Sin[c + d*x]]) + a*b*Cot[c + d*x]*(-2*a^2 - 9*b^2 + 2*a^2*Csc[c + d*x]^2 - 6*(a^2 + 2*b^2)*Log[Sin[c + d*x]] + 6*a^2*Log[a*Cos[c + d*x] + b*Sin[c + d*x]] + 12*b^2*Log[a*Cos[c + d*x] + b*Sin[c + d*x]]))/(3*a^5*d*(b + a*Cot[c + d*x]))

Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{-\frac{(a^2+b^2)b}{a^4(a+b\tan(dx+c))}+\frac{2b(a^2+2b^2)\ln(a+b\tan(dx+c))}{a^5}-\frac{1}{3a^2\tan(dx+c)^3}-\frac{a^2+3b^2}{a^4\tan(dx+c)}+\frac{b}{a^3\tan(dx+c)^2}-\frac{2b(a^2+2b^2)\ln(\tan(dx}{a^5}}{d}$
default	$\frac{-\frac{(a^2+b^2)b}{a^4(a+b\tan(dx+c))}+\frac{2b(a^2+2b^2)\ln(a+b\tan(dx+c))}{a^5}-\frac{1}{3a^2\tan(dx+c)^3}-\frac{a^2+3b^2}{a^4\tan(dx+c)}+\frac{b}{a^3\tan(dx+c)^2}-\frac{2b(a^2+2b^2)\ln(\tan(dx}{a^5}}{d}$
risch	$-\frac{4i(-12ia\,b^2e^{2i(dx+c)}-3a^2b\,e^{4i(dx+c)}+a^2b\,e^{2i(dx+c)}-3ia^3e^{4i(dx+c)}+6b^3e^{6i(dx+c)}-18b^3e^{4i(dx+c)}+3a^2b\,e^{6i(dx+c)}+i}{3(e^{2i(dx+c)}-1)^3(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia)}$

[In] int(csc(d*x+c)^4/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(-(a^2+b^2)*b/a^4/(a+b*tan(d*x+c))+2*b*(a^2+2*b^2)/a^5*ln(a+b*tan(d*x+c))-1/3/a^2/tan(d*x+c)^3-(a^2+3*b^2)/a^4/tan(d*x+c)+1/a^3*b/tan(d*x+c)^2-2*b*(a^2+2*b^2)/a^5*ln(tan(d*x+c)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(138) = 276$.

Time = 0.29 (sec) , antiderivative size = 442, normalized size of antiderivative = 3.16

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{2\left(a^4+6\,a^2b^2\right)\cos\left(dx+c\right)^4+6\,a^2b^2-3\left(a^4+6\,a^2b^2\right)\cos\left(dx+c\right)^2+3\left(\left(a^2b^2+2\,b^4\right)\cos\left(dx+c\right)^4+a^2b^2\right.}{}$$

[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="fricas")

[Out] 1/3*(2*(a^4 + 6*a^2*b^2)*cos(d*x + c)^4 + 6*a^2*b^2 - 3*(a^4 + 6*a^2*b^2)*cos(d*x + c)^2 + 3*((a^2*b^2 + 2*b^4)*cos(d*x + c)^4 + a^2*b^2 + 2*b^4 - 2*(a^2*b^2 + 2*b^4)*cos(d*x + c)^2 - ((a^3*b + 2*a*b^3)*cos(d*x + c)^3 - (a^3*b + 2*a*b^3)*cos(d*x + c))*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 3*((a^2*b^2 + 2*b^4)*cos(d*x + c)^4 + a^2*b^2 + 2*b^4 - 2*(a^2*b^2 + 2*b^4)*cos(d*x + c)^2 - ((a^3*b + 2*a*b^3

```
)*cos(d*x + c)^3 - (a^3*b + 2*a*b^3)*cos(d*x + c))*sin(d*x + c))*log(-1/4*c
os(d*x + c)^2 + 1/4) - 2*(6*a*b^3*cos(d*x + c) - (a^3*b + 6*a*b^3)*cos(d*x
+ c)^3)*sin(d*x + c))/(a^5*b*d*cos(d*x + c)^4 - 2*a^5*b*d*cos(d*x + c)^2 +
a^5*b*d - (a^6*d*cos(d*x + c)^3 - a^6*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

```
[In] integrate(csc(d*x+c)**4/(a+b*tan(d*x+c))**2,x)
[Out] Integral(csc(c + d*x)**4/(a + b*tan(c + d*x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.03

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{2\,a^2b\tan(dx+c)-6\,(a^2b+2\,b^3)\tan(dx+c)^3-a^3-3\,(a^3+2\,ab^2)\tan(dx+c)^2}{a^4b\tan(dx+c)^4+a^5\tan(dx+c)^3}+\frac{6\,(a^2b+2\,b^3)\log(b\tan(dx+c)+a)}{a^5}-\frac{6\,(a^2b+2\,b^3)\log(\tan(dx+c))}{a^5}}{3\,d}$$

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
[Out] 1/3*((2*a^2*b*tan(d*x + c) - 6*(a^2*b + 2*b^3)*tan(d*x + c)^3 - a^3 - 3*(a^
3 + 2*a*b^2)*tan(d*x + c)^2)/(a^4*b*tan(d*x + c)^4 + a^5*tan(d*x + c)^3) +
6*(a^2*b + 2*b^3)*log(b*tan(d*x + c) + a)/a^5 - 6*(a^2*b + 2*b^3)*log(tan(d
*x + c))/a^5)/d
```

Giac [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.45

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{6\,(a^2b+2\,b^3)\log(|\tan(dx+c)|)}{a^5}-\frac{6\,(a^2b^2+2\,b^4)\log(|b\tan(dx+c)+a|)}{a^5b}+\frac{3\,(2\,a^2b^2\tan(dx+c)+4\,b^4\tan(dx+c)+3\,a^3b+5\,ab^3)}{(b\tan(dx+c)+a)a^5}-\frac{11\,a^2b\tan(d}{}}{3\,d}$$

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(6*(a^2*b + 2*b^3)*log(abs(tan(d*x + c)))/a^5 - 6*(a^2*b^2 + 2*b^4)*lo
g(abs(b*tan(d*x + c) + a))/(a^5*b) + 3*(2*a^2*b^2*tan(d*x + c) + 4*b^4*tan(
d*x + c) + 3*a^3*b + 5*a*b^3)/((b*tan(d*x + c) + a)*a^5) - (11*a^2*b*tan(d*
x + c)^3 + 22*b^3*tan(d*x + c)^3 - 3*a^3*tan(d*x + c)^2 - 9*a*b^2*tan(d*x +
 c)^2 + 3*a^2*b*tan(d*x + c) - a^3)/(a^5*tan(d*x + c)^3))/d
```

Mupad [B] (verification not implemented)

Time = 4.31 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.07

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{4\,b\,\mathrm{atanh}\left(\frac{2\,b\,\left(a^2+2\,b^2\right)\,\left(a+2\,b\tan(c+d\,x)\right)}{a\,\left(2\,a^2\,b+4\,b^3\right)}\right)\,\left(a^2+2\,b^2\right)}{a^5\,d} - \frac{\frac{1}{3\,a}+\frac{\tan(c+d\,x)^2\,\left(a^2+2\,b^2\right)}{a^3}-\frac{2\,b\tan(c+d\,x)}{3\,a^2}+\frac{2\,b\tan(c+d\,x)^3\,\left(a^2+2\,b^2\right)}{a^4}}{d\,\left(b\tan\left(c+d\,x\right)^4+a\tan\left(c+d\,x\right)^3\right)}$$

```
[In] int(1/(sin(c + d*x)^4*(a + b*tan(c + d*x))^2),x)
```

```
[Out] (4*b*atanh((2*b*(a^2 + 2*b^2)*(a + 2*b*tan(c + d*x)))/(a*(2*a^2*b + 4*b^3))
)*(a^2 + 2*b^2))/(a^5*d) - (1/(3*a) + (tan(c + d*x)^2*(a^2 + 2*b^2))/a^3 -
(2*b*tan(c + d*x))/(3*a^2) + (2*b*tan(c + d*x)^3*(a^2 + 2*b^2))/a^4)/(d*(a*
tan(c + d*x)^3 + b*tan(c + d*x)^4))
```

3.66 $\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 219

$$\begin{aligned}\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^2} dx = &-\frac{(a^2+b^2)(a^2+5b^2)\cot(c+dx)}{a^6d}+\frac{2b(a^2+b^2)\cot^2(c+dx)}{a^5d}\\ &-\frac{(2a^2+3b^2)\cot^3(c+dx)}{3a^4d}+\frac{b\cot^4(c+dx)}{2a^3d}\\ &-\frac{\cot^5(c+dx)}{5a^2d}-\frac{2b(a^2+b^2)(a^2+3b^2)\log(\tan(c+dx))}{a^7d}\\ &+\frac{2b(a^2+b^2)(a^2+3b^2)\log(a+b\tan(c+dx))}{a^7d}\\ &-\frac{b(a^2+b^2)^2}{a^6d(a+b\tan(c+dx))}\end{aligned}$$

```
[Out] -(a^2+b^2)*(a^2+5*b^2)*cot(d*x+c)/a^6/d+2*b*(a^2+b^2)*cot(d*x+c)^2/a^5/d-1/
3*(2*a^2+3*b^2)*cot(d*x+c)^3/a^4/d+1/2*b*cot(d*x+c)^4/a^3/d-1/5*cot(d*x+c)^
5/a^2/d-2*b*(a^2+b^2)*(a^2+3*b^2)*ln(tan(d*x+c))/a^7/d+2*b*(a^2+b^2)*(a^2+3
*b^2)*ln(a+b*tan(d*x+c))/a^7/d-b*(a^2+b^2)^2/a^6/d/(a+b*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used

$= \{3597, 908\}$

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{b\cot^4(c+dx)}{2a^3d} - \frac{\cot^5(c+dx)}{5a^2d} - \frac{2b(a^2+b^2)(a^2+3b^2)\log(\tan(c+dx))}{a^7d} + \frac{2b(a^2+b^2)(a^2+3b^2)\log(a+b\tan(c+dx))}{a^7d} - \frac{b(a^2+b^2)^2}{a^6d(a+b\tan(c+dx))} - \frac{(a^2+b^2)(a^2+5b^2)\cot(c+dx)}{a^6d} + \frac{2b(a^2+b^2)\cot^2(c+dx)}{a^5d} - \frac{(2a^2+3b^2)\cot^3(c+dx)}{3a^4d}$$

```
[In] Int[Csc[c + d*x]^6/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(((a^2 + b^2)*(a^2 + 5*b^2)*Cot[c + d*x])/(a^6*d)) + (2*b*(a^2 + b^2)*Cot[
c + d*x]^2)/(a^5*d) - ((2*a^2 + 3*b^2)*Cot[c + d*x]^3)/(3*a^4*d) + (b*Cot[c
 + d*x]^4)/(2*a^3*d) - Cot[c + d*x]^5/(5*a^2*d) - (2*b*(a^2 + b^2)*(a^2 + 3
*b^2)*Log[Tan[c + d*x]])/(a^7*d) + (2*b*(a^2 + b^2)*(a^2 + 3*b^2)*Log[a + b
*Tan[c + d*x]])/(a^7*d) - (b*(a^2 + b^2)^2)/(a^6*d*(a + b*Tan[c + d*x]))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{b\text{Subst}\left(\int \frac{(b^2+x^2)^2}{x^6(a+x)^2}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{b\text{Subst}\left(\int \left(\frac{b^4}{a^2x^6} - \frac{2b^4}{a^3x^5} + \frac{2a^2b^2+3b^4}{a^4x^4} - \frac{4b^2(a^2+b^2)}{a^5x^3} + \frac{a^4+6a^2b^2+5b^4}{a^6x^2} - \frac{2(a^4+4a^2b^2+3b^4)}{a^7x} + \frac{(a^2+b^2)^2}{a^6(a+x)^2} + \frac{2(a^4+4a}{a^7(}\right.\right.}{d}$$

$$= -\frac{(a^2+b^2)(a^2+5b^2)\cot(c+dx)}{a^6d} + \frac{2b(a^2+b^2)\cot^2(c+dx)}{a^5d} - \frac{(2a^2+3b^2)\cot^3(c+dx)}{3a^4d} + \frac{b\cot^4(c+dx)}{2a^3d} - \frac{\cot^5(c+dx)}{5a^2d} - \frac{2b(a^2+b^2)(a^2+3b^2)\log(\tan(c+dx))}{a^7d} + \frac{2b(a^2+b^2)(a^2+3b^2)\log(a+b\tan(c+dx))}{a^7d} - \frac{b(a^2+b^2)^2}{a^6d(a+b\tan(c+dx))}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 589 vs. $2(219) = 438$.

Time = 7.71 (sec) , antiderivative size = 589, normalized size of antiderivative = 2.69

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\csc^5(c+dx)\sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{5a^2d(a+b\tan(c+dx))^2}$$
$$+ \frac{(-8a^4\cos(c+dx) - 75a^2b^2\cos(c+dx) - 75b^4\cos(c+dx))\csc(c+dx)\sec^2(c+dx)(a\cos(c+dx) + b\,\mathrm{si}}{15a^6d(a+b\tan(c+dx))^2}$$
$$+ \frac{b(a^2+2b^2)\csc^2(c+dx)\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{a^5d(a+b\tan(c+dx))^2}$$
$$+ \frac{(-4a^2\cos(c+dx) - 15b^2\cos(c+dx))\csc^3(c+dx)\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{15a^4d(a+b\tan(c+dx))^2}$$
$$+ \frac{b\csc^4(c+dx)\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{2a^3d(a+b\tan(c+dx))^2}$$
$$- \frac{2(a^4b+4a^2b^3+3b^5)\log(\sin(c+dx))\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{a^7d(a+b\tan(c+dx))^2}$$
$$+ \frac{2(a^4b+4a^2b^3+3b^5)\log(a\cos(c+dx)+b\sin(c+dx))\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{a^7d(a+b\tan(c+dx))^2}$$
$$+ \frac{\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))(a^4b^2\sin(c+dx)+2a^2b^4\sin(c+dx)+b^6\sin(c+dx))}{a^7d(a+b\tan(c+dx))^2}$$

```
[In] Integrate[Csc[c + d*x]^6/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -1/5*(Csc[c + d*x]^5*Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(a^2
*d*(a + b*Tan[c + d*x])^2) + ((-8*a^4*Cos[c + d*x] - 75*a^2*b^2*Cos[c + d*x
] - 75*b^4*Cos[c + d*x])*Csc[c + d*x]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Si
n[c + d*x])^2)/(15*a^6*d*(a + b*Tan[c + d*x])^2) + (b*(a^2 + 2*b^2)*Csc[c +
 d*x]^2*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(a^5*d*(a + b*T
an[c + d*x])^2) + ((-4*a^2*Cos[c + d*x] - 15*b^2*Cos[c + d*x])*Csc[c + d*x]
^3*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(15*a^4*d*(a + b*Tan
[c + d*x])^2) + (b*Csc[c + d*x]^4*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c
+ d*x])^2)/(2*a^3*d*(a + b*Tan[c + d*x])^2) - (2*(a^4*b + 4*a^2*b^3 + 3*b^5
)*Log[Sin[c + d*x]]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(a^
```

```
7*d*(a + b*Tan[c + d*x])^2) + (2*(a^4*b + 4*a^2*b^3 + 3*b^5)*Log[a*Cos[c +
d*x] + b*Sin[c + d*x]]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/
(a^7*d*(a + b*Tan[c + d*x])^2) + (Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c
+ d*x])*(a^4*b^2*Sin[c + d*x] + 2*a^2*b^4*Sin[c + d*x] + b^6*Sin[c + d*x]))
/(a^7*d*(a + b*Tan[c + d*x])^2)
```

Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{-\frac{1}{5a^2\tan(dx+c)^5}-\frac{2a^2+3b^2}{3a^4\tan(dx+c)^3}-\frac{a^4+6a^2b^2+5b^4}{a^6\tan(dx+c)}+\frac{b}{2a^3\tan(dx+c)^4}+\frac{2b\left(a^2+b^2\right)}{a^5\tan(dx+c)^2}-\frac{2b\left(a^4+4a^2b^2+3b^4\right)\ln(\tan(dx+c))}{a^7}-\frac{(}{a^6}}{d}$
default	$\frac{-\frac{1}{5a^2\tan(dx+c)^5}-\frac{2a^2+3b^2}{3a^4\tan(dx+c)^3}-\frac{a^4+6a^2b^2+5b^4}{a^6\tan(dx+c)}+\frac{b}{2a^3\tan(dx+c)^4}+\frac{2b\left(a^2+b^2\right)}{a^5\tan(dx+c)^2}-\frac{2b\left(a^4+4a^2b^2+3b^4\right)\ln(\tan(dx+c))}{a^7}-\frac{(}{a^6}}{d}$
risch	$-\frac{4i\left(-45a^2b^3+4ia^5-45b^5-4a^4b+45ia\,b^4e^{8i(dx+c)}-120ia^3b^2e^{6i(dx+c)}-180ia^3b^2e^{2i(dx+c)}-180ia\,b^4e^{2i(dx+c)}-180ia\,b\right.}{}$

```
[In] int(csc(d*x+c)^6/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/5/a^2/tan(d*x+c)^5-1/3*(2*a^2+3*b^2)/a^4/tan(d*x+c)^3-(a^4+6*a^2*b^
2+5*b^4)/a^6/tan(d*x+c)+1/2/a^3*b/tan(d*x+c)^4+2*b*(a^2+b^2)/a^5/tan(d*x+c)
^2-2*b*(a^4+4*a^2*b^2+3*b^4)/a^7*ln(tan(d*x+c))-(a^4+2*a^2*b^2+b^4)*b/a^6/(
a+b*tan(d*x+c))+2*b*(a^4+4*a^2*b^2+3*b^4)/a^7*ln(a+b*tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 787 vs. $2(213) = 426$.

Time = 0.32 (sec) , antiderivative size = 787, normalized size of antiderivative = 3.59

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{4\left(4a^6+45a^4b^2+45a^2b^4\right)\cos\left(dx+c\right)^6-75a^4b^2-90a^2b^4-10\left(4a^6+45a^4b^2+45a^2b^4\right)\cos\left(dx+c\right)^4}{}$$

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/30*(4*(4*a^6 + 45*a^4*b^2 + 45*a^2*b^4)*cos(d*x + c)^6 - 75*a^4*b^2 - 90*
a^2*b^4 - 10*(4*a^6 + 45*a^4*b^2 + 45*a^2*b^4)*cos(d*x + c)^4 + 15*(2*a^6 +
 23*a^4*b^2 + 24*a^2*b^4)*cos(d*x + c)^2 + 30*((a^4*b^2 + 4*a^2*b^4 + 3*b^6
)*cos(d*x + c)^6 - a^4*b^2 - 4*a^2*b^4 - 3*b^6 - 3*(a^4*b^2 + 4*a^2*b^4 + 3
*b^6)*cos(d*x + c)^4 + 3*(a^4*b^2 + 4*a^2*b^4 + 3*b^6)*cos(d*x + c)^2 - ((a
^5*b + 4*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^5 - 2*(a^5*b + 4*a^3*b^3 + 3*a*b^5
```

```
)*cos(d*x + c)^3 + (a^5*b + 4*a^3*b^3 + 3*a*b^5)*cos(d*x + c))*sin(d*x + c)
)*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) -
 30*((a^4*b^2 + 4*a^2*b^4 + 3*b^6)*cos(d*x + c)^6 - a^4*b^2 - 4*a^2*b^4 - 3
*b^6 - 3*(a^4*b^2 + 4*a^2*b^4 + 3*b^6)*cos(d*x + c)^4 + 3*(a^4*b^2 + 4*a^2*
b^4 + 3*b^6)*cos(d*x + c)^2 - ((a^5*b + 4*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^5
 - 2*(a^5*b + 4*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^3 + (a^5*b + 4*a^3*b^3 + 3*
a*b^5)*cos(d*x + c))*sin(d*x + c))*log(-1/4*cos(d*x + c)^2 + 1/4) + (4*(4*a
^5*b + 45*a^3*b^3 + 45*a*b^5)*cos(d*x + c)^5 - 10*(a^5*b + 33*a^3*b^3 + 36*
a*b^5)*cos(d*x + c)^3 - 15*(a^5*b - 10*a^3*b^3 - 12*a*b^5)*cos(d*x + c))*si
n(d*x + c))/(a^7*b*d*cos(d*x + c)^6 - 3*a^7*b*d*cos(d*x + c)^4 + 3*a^7*b*d*
cos(d*x + c)^2 - a^7*b*d - (a^8*d*cos(d*x + c)^5 - 2*a^8*d*cos(d*x + c)^3 +
 a^8*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\csc^6{(c+dx)}}{(a+b\tan{(c+dx)})^2}\,dx$$

```
[In] integrate(csc(d*x+c)**6/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)**6/(a + b*tan(c + d*x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.03

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{9\,a^4b\tan(dx+c)-60\,(a^4b+4\,a^2b^3+3\,b^5)\tan(dx+c)^5-6\,a^5-30\,(a^5+4\,a^3b^2+3\,ab^4)\tan(dx+c)^4+10\,(4\,a^4b+3\,a^2b^3)\tan(dx+c)^3-5\,(4\,a^5+3\,a^3b^2)\tan}{a^6b\tan(dx+c)^6+a^7\tan(dx+c)^5}}{30\,d}$$

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/30*((9*a^4*b*tan(d*x + c) - 60*(a^4*b + 4*a^2*b^3 + 3*b^5)*tan(d*x + c)^5
 - 6*a^5 - 30*(a^5 + 4*a^3*b^2 + 3*a*b^4)*tan(d*x + c)^4 + 10*(4*a^4*b + 3*
a^2*b^3)*tan(d*x + c)^3 - 5*(4*a^5 + 3*a^3*b^2)*tan(d*x + c)^2)/(a^6*b*tan(
d*x + c)^6 + a^7*tan(d*x + c)^5) + 60*(a^4*b + 4*a^2*b^3 + 3*b^5)*log(b*tan
(d*x + c) + a)/a^7 - 60*(a^4*b + 4*a^2*b^3 + 3*b^5)*log(tan(d*x + c))/a^7)/
d
```

Giac [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.52

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{60\left(a^4b+4a^2b^3+3b^5\right)\log(|\tan(dx+c)|)}{a^7}-\frac{60\left(a^4b^2+4a^2b^4+3b^6\right)\log(|b\tan(dx+c)+a|)}{a^7b}+\frac{30\left(2a^4b^2\tan(dx+c)+8a^2b^4\tan(dx+c)+6b^6\tan\cdots\right.}{(b\tan(dx+c)+a)\cdots}}{\cdots}$$

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/30*(60*(a^4*b + 4*a^2*b^3 + 3*b^5)*log(abs(tan(d*x + c)))/a^7 - 60*(a^4*
b^2 + 4*a^2*b^4 + 3*b^6)*log(abs(b*tan(d*x + c) + a))/(a^7*b) + 30*(2*a^4*b
^2*tan(d*x + c) + 8*a^2*b^4*tan(d*x + c) + 6*b^6*tan(d*x + c) + 3*a^5*b + 1
0*a^3*b^3 + 7*a*b^5)/((b*tan(d*x + c) + a)*a^7) - (137*a^4*b*tan(d*x + c)^5
 + 548*a^2*b^3*tan(d*x + c)^5 + 411*b^5*tan(d*x + c)^5 - 30*a^5*tan(d*x + c
)^4 - 180*a^3*b^2*tan(d*x + c)^4 - 150*a*b^4*tan(d*x + c)^4 + 60*a^4*b*tan(
d*x + c)^3 + 60*a^2*b^3*tan(d*x + c)^3 - 20*a^5*tan(d*x + c)^2 - 30*a^3*b^2
*tan(d*x + c)^2 + 15*a^4*b*tan(d*x + c) - 6*a^5)/(a^7*tan(d*x + c)^5))/d
```

Mupad [B] (verification not implemented)

Time = 5.99 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.08

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{4\,b\,\mathrm{atanh}\left(\frac{2\,b\,\left(a^2+3\,b^2\right)\,\left(a^2+b^2\right)\,\left(a+2\,b\tan(c+d\,x)\right)}{a\,\left(2\,a^4\,b+8\,a^2\,b^3+6\,b^5\right)}\right)\,\left(a^2+3\,b^2\right)\,\left(a^2+b^2\right)}{a^7\,d} - \frac{\frac{1}{5\,a}+\frac{\tan(c+d\,x)^4\,\left(a^4+4\,a^2\,b^2+3\,b^4\right)}{a^5}+\frac{\tan(c+d\,x)^2\,\left(4\,a^2+3\,b^2\right)}{6\,a^3}-\frac{3\,b\tan(c+d\,x)}{10\,a^2}+\frac{2\,b\tan(c+d\,x)^5\,\left(a^4+4\,a^2\,b^2+3\,b^4\right)}{a^6}-\frac{b\tan(c+d\,x\cdots}{3\cdots}}{d\,\left(b\tan\left(c+d\,x\right)^6+a\tan\left(c+d\,x\right)^5\right)}$$

```
[In] int(1/(sin(c + d*x)^6*(a + b*tan(c + d*x))^2),x)
```

```
[Out] (4*b*atanh((2*b*(a^2 + 3*b^2)*(a^2 + b^2)*(a + 2*b*tan(c + d*x)))/(a*(2*a^4
*b + 6*b^5 + 8*a^2*b^3)))*(a^2 + 3*b^2)*(a^2 + b^2))/(a^7*d) - (1/(5*a) + (
tan(c + d*x)^4*(a^4 + 3*b^4 + 4*a^2*b^2))/a^5 + (tan(c + d*x)^2*(4*a^2 + 3*
b^2))/(6*a^3) - (3*b*tan(c + d*x))/(10*a^2) + (2*b*tan(c + d*x)^5*(a^4 + 3*
b^4 + 4*a^2*b^2))/a^6 - (b*tan(c + d*x)^3*(4*a^2 + 3*b^2))/(3*a^4))/(d*(a*t
an(c + d*x)^5 + b*tan(c + d*x)^6))
```

3.67 $\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 382

$$\begin{aligned}
\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx
&= \frac{a(5a^8-180a^6b^2+390a^4b^4-68a^2b^6-3b^8)\,x}{16\,(a^2+b^2)^6} \\
&\quad+ \frac{a^4b(3a^4-22a^2b^2+15b^4)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^6\,d} \\
&\quad- \frac{a^6b}{2\,(a^2+b^2)^4\,d(a+b\tan(c+dx))^2} - \frac{2a^5b(a^2-3b^2)}{(a^2+b^2)^5\,d(a+b\tan(c+dx))} \\
&\quad- \frac{\cos^6(c+dx)\,(b(3a^2-b^2)+a(a^2-3b^2)\tan(c+dx))}{6\,(a^2+b^2)^3\,d} \\
&\quad+ \frac{\cos^4(c+dx)\,(6b(9a^4-4a^2b^2-b^4)+a(13a^4-62a^2b^2-3b^4)\tan(c+dx))}{24\,(a^2+b^2)^4\,d} \\
&\quad- \frac{a\cos^2(c+dx)\,(24a^3b(3a^2-5b^2)+(11a^6-119a^4b^2+65a^2b^4+3b^6)\tan(c+dx))}{16\,(a^2+b^2)^5\,d}
\end{aligned}$$

```
[Out] 1/16*a*(5*a^8-180*a^6*b^2+390*a^4*b^4-68*a^2*b^6-3*b^8)*x/(a^2+b^2)^6+a^4*b
*(3*a^4-22*a^2*b^2+15*b^4)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^6/d-1/2*
a^6*b/(a^2+b^2)^4/d/(a+b*tan(d*x+c))^2-2*a^5*b*(a^2-3*b^2)/(a^2+b^2)^5/d/(a
+b*tan(d*x+c))-1/6*cos(d*x+c)^6*(b*(3*a^2-b^2)+a*(a^2-3*b^2)*tan(d*x+c))/(a
^2+b^2)^3/d+1/24*cos(d*x+c)^4*(6*b*(9*a^4-4*a^2*b^2-b^4)+a*(13*a^4-62*a^2*b
^2-3*b^4)*tan(d*x+c))/(a^2+b^2)^4/d-1/16*a*cos(d*x+c)^2*(24*a^3*b*(3*a^2-5*
b^2)+(11*a^6-119*a^4*b^2+65*a^2*b^4+3*b^6)*tan(d*x+c))/(a^2+b^2)^5/d
```

Rubi [A] (verified)

Time = 2.12 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3597, 1661, 1643, 649, 209, 266}

$$\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= -\frac{\cos^6(c+dx)\left(a(a^2-3b^2)\tan(c+dx)+b(3a^2-b^2)\right)}{6d\left(a^2+b^2\right)^3} - \frac{a^6b}{2d\left(a^2+b^2\right)^4(a+b\tan(c+dx))^2} - \frac{2a^5b(a^2-3b^2)}{d\left(a^2+b^2\right)^5(a+b\tan(c+dx))} + \frac{\cos^4(c+dx)\left(a(13a^4-62a^2b^2-3b^4)\tan(c+dx)+6b(9a^4-4a^2b^2-b^4)\right)}{24d\left(a^2+b^2\right)^4} + \frac{a^4b(3a^4-22a^2b^2+15b^4)\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)^6} + \frac{ax(5a^8-180a^6b^2+390a^4b^4-68a^2b^6-3b^8)}{16\left(a^2+b^2\right)^6} - \frac{a\cos^2(c+dx)\left(24a^3b(3a^2-5b^2)+(11a^6-119a^4b^2+65a^2b^4+3b^6)\tan(c+dx)\right)}{16d\left(a^2+b^2\right)^5}$$

```
[In] Int[Sin[c + d*x]^6/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (a*(5*a^8 - 180*a^6*b^2 + 390*a^4*b^4 - 68*a^2*b^6 - 3*b^8)*x)/(16*(a^2 + b
^2)^6) + (a^4*b*(3*a^4 - 22*a^2*b^2 + 15*b^4)*Log[a*Cos[c + d*x] + b*Sin[c
+ d*x]])/((a^2 + b^2)^6*d) - (a^6*b)/(2*(a^2 + b^2)^4*d*(a + b*Tan[c + d*x]
)^2) - (2*a^5*b*(a^2 - 3*b^2))/((a^2 + b^2)^5*d*(a + b*Tan[c + d*x])) - (Co
s[c + d*x]^6*(b*(3*a^2 - b^2) + a*(a^2 - 3*b^2)*Tan[c + d*x]))/(6*(a^2 + b^
2)^3*d) + (Cos[c + d*x]^4*(6*b*(9*a^4 - 4*a^2*b^2 - b^4) + a*(13*a^4 - 62*a
^2*b^2 - 3*b^4)*Tan[c + d*x]))/(24*(a^2 + b^2)^4*d) - (a*Cos[c + d*x]^2*(24
*a^3*b*(3*a^2 - 5*b^2) + (11*a^6 - 119*a^4*b^2 + 65*a^2*b^4 + 3*b^6)*Tan[c
+ d*x]))/(16*(a^2 + b^2)^5*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{b\text{Subst}\left(\int \frac{x^6}{(a+x)^3(b^2+x^2)^4}\,dx, x, b\tan(c+dx)\right)}{d}\\ &= -\frac{\cos^6(c+dx)\left(b(3a^2-b^2)+a(a^2-3b^2)\tan(c+dx)\right)}{6\left(a^2+b^2\right)^3 d}\\ &\quad -\frac{\text{Subst}\left(\int \frac{-\frac{a^4b^6\left(a^2-3b^2\right)}{\left(a^2+b^2\right)^3}+\frac{3a^3b^6\left(5a^2+b^2\right)x}{\left(a^2+b^2\right)^3}+\frac{3a^2b^4\left(2a^4+11a^2b^2-3b^4\right)x^2}{\left(a^2+b^2\right)^3}+\frac{5ab^6\left(a^2-3b^2\right)x^3}{\left(a^2+b^2\right)^3}-6b^2x^4}{(a+x)^3(b^2+x^2)^3}\,dx, x, b\tan(c+dx)\right)}{6bd}\end{aligned}$$

$$= -\frac{\cos^6(c+dx)\left(b(3a^2-b^2)+a(a^2-3b^2)\tan(c+dx)\right)}{6\left(a^2+b^2\right)^3 d} + \frac{\cos^4(c+dx)\left(6b(9a^4-4a^2b^2-b^4)+a(13a^4-62a^2b^2-3b^4)\tan(c+dx)\right)}{24\left(a^2+b^2\right)^4 d} + \frac{\text{Subst}\left(\int \frac{-\frac{9a^4b^6\left(a^4-6a^2b^2+b^4\right)}{\left(a^2+b^2\right)^4}+\frac{9a^3b^6\left(13a^4+2a^2b^2-3b^4\right)x}{\left(a^2+b^2\right)^4}+\frac{3a^2b^4\left(8a^6+71a^4b^2-66a^2b^4-9b^6\right)x^2}{\left(a^2+b^2\right)^4}+\frac{3ab^6\left(13a^4-62a^2b^2-3b^4\right)x^3}{\left(a^2+b^2\right)^4}}{(a+x)^3(b^2+x^2)^2} \, d \cdots}{24b^3d}$$

$$= -\frac{\cos^6(c+dx)\left(b(3a^2-b^2)+a(a^2-3b^2)\tan(c+dx)\right)}{6\left(a^2+b^2\right)^3 d} + \frac{\cos^4(c+dx)\left(6b(9a^4-4a^2b^2-b^4)+a(13a^4-62a^2b^2-3b^4)\tan(c+dx)\right)}{24\left(a^2+b^2\right)^4 d} - \frac{a\cos^2(c+dx)\left(24a^3b(3a^2-5b^2)+(11a^6-119a^4b^2+65a^2b^4+3b^6)\tan(c+dx)\right)}{16\left(a^2+b^2\right)^5 d} - \frac{\text{Subst}\left(\int \frac{-\frac{3a^4b^6\left(5a^6-89a^4b^2+95a^2b^4-3b^6\right)}{\left(a^2+b^2\right)^5}+\frac{9a^3b^6\left(11a^6+9a^4b^2-63a^2b^4+3b^6\right)x}{\left(a^2+b^2\right)^5}+\frac{9a^2b^6\left(11a^6-71a^4b^2-15a^2b^4+3b^6\right)x^2}{\left(a^2+b^2\right)^5}+\frac{3ab^6\left(11a \cdots\right.}{\cdots}}{(a+x)^3(b^2+x^2)} \cdots}{48b^5d}$$

$$= -\frac{\cos^6(c+dx)\left(b(3a^2-b^2)+a(a^2-3b^2)\tan(c+dx)\right)}{6\left(a^2+b^2\right)^3 d} + \frac{\cos^4(c+dx)\left(6b(9a^4-4a^2b^2-b^4)+a(13a^4-62a^2b^2-3b^4)\tan(c+dx)\right)}{24\left(a^2+b^2\right)^4 d} - \frac{a\cos^2(c+dx)\left(24a^3b(3a^2-5b^2)+(11a^6-119a^4b^2+65a^2b^4+3b^6)\tan(c+dx)\right)}{16\left(a^2+b^2\right)^5 d} - \frac{\text{Subst}\left(\int \left(-\frac{48a^6b^6}{(a^2+b^2)^4(a+x)^3}-\frac{96a^5b^6(a^2-3b^2)}{(a^2+b^2)^5(a+x)^2}-\frac{48a^4b^6(3a^4-22a^2b^2+15b^4)}{(a^2+b^2)^6(a+x)}+\frac{3ab^6(-5a^8+180a^6b^2-390a^4b^4+68a^2 \cdots}{(a^2+b^2)^6 \cdots} \cdots\right.\right.}{48b^5d}$$

$$= \frac{a^4b(3a^4-22a^2b^2+15b^4)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^6 d} - \frac{a^6b}{2\left(a^2+b^2\right)^4 d(a+b\tan(c+dx))^2} - \frac{2a^5b(a^2-3b^2)}{\left(a^2+b^2\right)^5 d(a+b\tan(c+dx))} - \frac{\cos^6(c+dx)\left(b(3a^2-b^2)+a(a^2-3b^2)\tan(c+dx)\right)}{6\left(a^2+b^2\right)^3 d} + \frac{\cos^4(c+dx)\left(6b(9a^4-4a^2b^2-b^4)+a(13a^4-62a^2b^2-3b^4)\tan(c+dx)\right)}{24\left(a^2+b^2\right)^4 d} - \frac{a\cos^2(c+dx)\left(24a^3b(3a^2-5b^2)+(11a^6-119a^4b^2+65a^2b^4+3b^6)\tan(c+dx)\right)}{16\left(a^2+b^2\right)^5 d} - \frac{(ab)\text{Subst}\left(\int \frac{-5a^8+180a^6b^2-390a^4b^4+68a^2b^6+3b^8+16a^3(3a^4-22a^2b^2+15b^4)x}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{16\left(a^2+b^2\right)^6 d}$$

$$= \frac{a^4b(3a^4-22a^2b^2+15b^4)\log(a+b\tan(c+dx))}{(a^2+b^2)^6 d} - \frac{a^6b}{2(a^2+b^2)^4 d(a+b\tan(c+dx))^2} - \frac{2a^5b(a^2-3b^2)}{(a^2+b^2)^5 d(a+b\tan(c+dx))} - \frac{\cos^6(c+dx)\left(b(3a^2-b^2)+a(a^2-3b^2)\tan(c+dx)\right)}{6(a^2+b^2)^3 d} + \frac{\cos^4(c+dx)\left(6b(9a^4-4a^2b^2-b^4)+a(13a^4-62a^2b^2-3b^4)\tan(c+dx)\right)}{24(a^2+b^2)^4 d} - \frac{a\cos^2(c+dx)\left(24a^3b(3a^2-5b^2)+(11a^6-119a^4b^2+65a^2b^4+3b^6)\tan(c+dx)\right)}{16(a^2+b^2)^5 d} - \frac{(a^4b(3a^4-22a^2b^2+15b^4))\,\text{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{(a^2+b^2)^6 d} - \frac{(ab(-5a^8+180a^6b^2-390a^4b^4+68a^2b^6+3b^8))\,\text{Subst}\left(\int \frac{1}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{16(a^2+b^2)^6 d}$$

$$= \frac{a(5a^8-180a^6b^2+390a^4b^4-68a^2b^6-3b^8)x}{16(a^2+b^2)^6} + \frac{a^4b(3a^4-22a^2b^2+15b^4)\log(\cos(c+dx))}{(a^2+b^2)^6 d} + \frac{a^4b(3a^4-22a^2b^2+15b^4)\log(a+b\tan(c+dx))}{(a^2+b^2)^6 d} - \frac{a^6b}{2(a^2+b^2)^4 d(a+b\tan(c+dx))^2} - \frac{2a^5b(a^2-3b^2)}{(a^2+b^2)^5 d(a+b\tan(c+dx))} - \frac{\cos^6(c+dx)\left(b(3a^2-b^2)+a(a^2-3b^2)\tan(c+dx)\right)}{6(a^2+b^2)^3 d} + \frac{\cos^4(c+dx)\left(6b(9a^4-4a^2b^2-b^4)+a(13a^4-62a^2b^2-3b^4)\tan(c+dx)\right)}{24(a^2+b^2)^4 d} - \frac{a\cos^2(c+dx)\left(24a^3b(3a^2-5b^2)+(11a^6-119a^4b^2+65a^2b^4+3b^6)\tan(c+dx)\right)}{16(a^2+b^2)^5 d}$$

Mathematica [A] (verified)

Time = 6.72 (sec) , antiderivative size = 746, normalized size of antiderivative = 1.95

$$\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

$$= \frac{b\left(-\frac{3a^5(a^2-7b^2)\arctan(\tan(c+dx))}{2b(a^2+b^2)^5} - \frac{5a(a^2-3b^2)\arctan(\tan(c+dx))}{16b(a^2+b^2)^3} + \frac{9a(a^4-4a^2b^2-b^4)\arctan(\tan(c+dx))}{8b(a^2+b^2)^4} - \frac{3a^4(3a^2-5b^2)\cos^2(c+\ldots}{2(a^2+b^2)^5}\right.}{}$$

```
[In] Integrate[Sin[c + d*x]^6/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (b*((-3*a^5*(a^2 - 7*b^2)*ArcTan[Tan[c + d*x]])/(2*b*(a^2 + b^2)^5) - (5*a*
(a^2 - 3*b^2)*ArcTan[Tan[c + d*x]])/(16*b*(a^2 + b^2)^3) + (9*a*(a^4 - 4*a^
2*b^2 - b^4)*ArcTan[Tan[c + d*x]])/(8*b*(a^2 + b^2)^4) - (3*a^4*(3*a^2 - 5*
```

```
b^2)*Cos[c + d*x]^2)/(2*(a^2 + b^2)^5) + ((9*a^4 - 4*a^2*b^2 - b^4)*Cos[c +
 d*x]^4)/(4*(a^2 + b^2)^4) - ((3*a^2 - b^2)*Cos[c + d*x]^6)/(6*(a^2 + b^2)^
3) - (a^4*(3*a^4 - 22*a^2*b^2 + 15*b^4 - (a^5 - 18*a^3*b^2 + 21*a*b^4)/Sqrt
[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/(2*(a^2 + b^2)^6) + (a^4*(3*a^4 -
 22*a^2*b^2 + 15*b^4)*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^6 - (a^4*(3*a^4
- 22*a^2*b^2 + 15*b^4 + (a^5 - 18*a^3*b^2 + 21*a*b^4)/Sqrt[-b^2])*Log[Sqrt[
-b^2] + b*Tan[c + d*x]])/(2*(a^2 + b^2)^6) - (3*a^5*(a^2 - 7*b^2)*Cos[c + d
*x]*Sin[c + d*x])/(2*b*(a^2 + b^2)^5) - (5*a*(a^2 - 3*b^2)*Cos[c + d*x]*Sin
[c + d*x])/(16*b*(a^2 + b^2)^3) + (9*a*(a^4 - 4*a^2*b^2 - b^4)*Cos[c + d*x]
*Sin[c + d*x])/(8*b*(a^2 + b^2)^4) - (5*a*(a^2 - 3*b^2)*Cos[c + d*x]^3*Sin[
c + d*x])/(24*b*(a^2 + b^2)^3) + (3*a*(a^4 - 4*a^2*b^2 - b^4)*Cos[c + d*x]^
3*Sin[c + d*x])/(4*b*(a^2 + b^2)^4) - (a*(a^2 - 3*b^2)*Cos[c + d*x]^5*Sin[c
 + d*x])/(6*b*(a^2 + b^2)^3) - a^6/(2*(a^2 + b^2)^4*(a + b*Tan[c + d*x])^2)
 - (2*a^5*(a^2 - 3*b^2))/((a^2 + b^2)^5*(a + b*Tan[c + d*x]))))/d
```

Maple [A] (verified)

Time = 70.94 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.22

method	result
derivativedivides	$-\frac{b a^6}{2(a^2+b^2)^4(a+b\tan(dx+c))^2}+\frac{a^4 b(3a^4-22a^2b^2+15b^4)\ln(a+b\tan(dx+c))}{(a^2+b^2)^6}-\frac{2b a^5(a^2-3b^2)}{(a^2+b^2)^5(a+b\tan(dx+c))}+\frac{\left(-\frac{11}{16}a^9+\frac{27}{4}b^2a^7+\frac{2}{8}\right.}{}$
default	$-\frac{b a^6}{2(a^2+b^2)^4(a+b\tan(dx+c))^2}+\frac{a^4 b(3a^4-22a^2b^2+15b^4)\ln(a+b\tan(dx+c))}{(a^2+b^2)^6}-\frac{2b a^5(a^2-3b^2)}{(a^2+b^2)^5(a+b\tan(dx+c))}+\frac{\left(-\frac{11}{16}a^9+\frac{27}{4}b^2a^7+\frac{2}{8}\right.}{}$
risch	Expression too large to display

```
[In] int(sin(d*x+c)^6/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*b*a^6/(a^2+b^2)^4/(a+b*tan(d*x+c))^2+a^4*b*(3*a^4-22*a^2*b^2+15*b
^4)/(a^2+b^2)^6*ln(a+b*tan(d*x+c))-2*b*a^5*(a^2-3*b^2)/(a^2+b^2)^5/(a+b*tan
(d*x+c))+1/(a^2+b^2)^6*(((-11/16*a^9+27/4*b^2*a^7+27/8*b^4*a^5-17/4*b^6*a^3
-3/16*a*b^8)*tan(d*x+c)^5+(-9/2*a^8*b+3*a^6*b^3+15/2*a^4*b^5)*tan(d*x+c)^4+
(-5/6*a^9+12*b^2*a^7+2*b^4*a^5-34/3*b^6*a^3-1/2*a*b^8)*tan(d*x+c)^3+(-27/4*
a^8*b+19/2*a^6*b^3+15*a^4*b^5-3/2*a^2*b^7-1/4*b^9)*tan(d*x+c)^2+(-5/16*a^9+
21/4*b^2*a^7-3/8*b^4*a^5-23/4*b^6*a^3+3/16*a*b^8)*tan(d*x+c)-11/4*a^8*b+31/
6*a^6*b^3+13/2*a^4*b^5-3/2*a^2*b^7-1/12*b^9)/(1+tan(d*x+c)^2)^3+1/16*a*(1/2
*(-48*a^7*b+352*a^5*b^3-240*a^3*b^5)*ln(1+tan(d*x+c)^2)+(5*a^8-180*a^6*b^2+
390*a^4*b^4-68*a^2*b^6-3*b^8)*arctan(tan(d*x+c)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 932 vs. 2(372) = 744.

Time = 0.38 (sec) , antiderivative size = 932, normalized size of antiderivative = 2.44

$$\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{195\,a^8b^3 - 427\,a^6b^5 - 165\,a^4b^7 + 27\,a^2b^9 + 2\,b^{11} - 8\,(a^{10}b + 5\,a^8b^3 + 10\,a^6b^5 + 10\,a^4b^7 + 5\,a^2b^9 + b^{11})\cos(d}{}$$

```
[In] integrate(sin(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/48*(195*a^8*b^3 - 427*a^6*b^5 - 165*a^4*b^7 + 27*a^2*b^9 + 2*b^11 - 8*(a^
10*b + 5*a^8*b^3 + 10*a^6*b^5 + 10*a^4*b^7 + 5*a^2*b^9 + b^11)*cos(d*x + c)
^8 + 20*(2*a^10*b + 9*a^8*b^3 + 16*a^6*b^5 + 14*a^4*b^7 + 6*a^2*b^9 + b^11)
*cos(d*x + c)^6 - 2*(49*a^10*b + 162*a^8*b^3 + 198*a^6*b^5 + 112*a^4*b^7 +
33*a^2*b^9 + 6*b^11)*cos(d*x + c)^4 + 3*(5*a^9*b^2 - 180*a^7*b^4 + 390*a^5*
b^6 - 68*a^3*b^8 - 3*a*b^10)*d*x + (9*a^10*b - 46*a^8*b^3 + 994*a^6*b^5 + 1
44*a^4*b^7 - 43*a^2*b^9 - 2*b^11 + 3*(5*a^11 - 185*a^9*b^2 + 570*a^7*b^4 -
458*a^5*b^6 + 65*a^3*b^8 + 3*a*b^10)*d*x)*cos(d*x + c)^2 + 24*(3*a^8*b^3 -
22*a^6*b^5 + 15*a^4*b^7 + (3*a^10*b - 25*a^8*b^3 + 37*a^6*b^5 - 15*a^4*b^7)
*cos(d*x + c)^2 + 2*(3*a^9*b^2 - 22*a^7*b^4 + 15*a^5*b^6)*cos(d*x + c)*sin(
d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
+ b^2) - (8*(a^11 + 5*a^9*b^2 + 10*a^7*b^4 + 10*a^5*b^6 + 5*a^3*b^8 + a*b^1
0)*cos(d*x + c)^7 - 2*(13*a^11 + 55*a^9*b^2 + 90*a^7*b^4 + 70*a^5*b^6 + 25*
a^3*b^8 + 3*a*b^10)*cos(d*x + c)^5 + (33*a^11 + 49*a^9*b^2 - 54*a^7*b^4 - 1
26*a^5*b^6 - 59*a^3*b^8 - 3*a*b^10)*cos(d*x + c)^3 - (261*a^9*b^2 - 338*a^7
*b^4 + 120*a^5*b^6 - 150*a^3*b^8 - 5*a*b^10 + 6*(5*a^10*b - 180*a^8*b^3 + 3
90*a^6*b^5 - 68*a^4*b^7 - 3*a^2*b^9)*d*x)*cos(d*x + c))*sin(d*x + c))/((a^1
4 + 5*a^12*b^2 + 9*a^10*b^4 + 5*a^8*b^6 - 5*a^6*b^8 - 9*a^4*b^10 - 5*a^2*b^
12 - b^14)*d*cos(d*x + c)^2 + 2*(a^13*b + 6*a^11*b^3 + 15*a^9*b^5 + 20*a^7*
b^7 + 15*a^5*b^9 + 6*a^3*b^11 + a*b^13)*d*cos(d*x + c)*sin(d*x + c) + (a^12
*b^2 + 6*a^10*b^4 + 15*a^8*b^6 + 20*a^6*b^8 + 15*a^4*b^10 + 6*a^2*b^12 + b^
14)*d)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Exception raised: AttributeError}$$

```
[In] integrate(sin(d*x+c)**6/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1088 vs. $2(372) = 744$.

Time = 0.34 (sec) , antiderivative size = 1088, normalized size of antiderivative = 2.85

$$\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/48*(3*(5*a^9 - 180*a^7*b^2 + 390*a^5*b^4 - 68*a^3*b^6 - 3*a*b^8)*(d*x + c
)/(a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 +
b^12) + 48*(3*a^8*b - 22*a^6*b^3 + 15*a^4*b^5)*log(b*tan(d*x + c) + a)/(a^1
2 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)
- 24*(3*a^8*b - 22*a^6*b^3 + 15*a^4*b^5)*log(tan(d*x + c)^2 + 1)/(a^12 + 6*
a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12) - (252
*a^8*b - 644*a^6*b^3 + 68*a^4*b^5 + 4*a^2*b^7 + 3*(43*a^7*b^2 - 215*a^5*b^4
 + 65*a^3*b^6 + 3*a*b^8)*tan(d*x + c)^7 + 6*(31*a^8*b - 127*a^6*b^3 + 5*a^4
*b^5 + 3*a^2*b^7)*tan(d*x + c)^6 + (33*a^9 + 403*a^7*b^2 - 2005*a^5*b^4 + 5
29*a^3*b^6 + 24*a*b^8)*tan(d*x + c)^5 + 4*(164*a^8*b - 515*a^6*b^3 + 65*a^4
*b^5 + 27*a^2*b^7 + 3*b^9)*tan(d*x + c)^4 + (40*a^9 + 335*a^7*b^2 - 2171*a^
5*b^4 + 429*a^3*b^6 + 15*a*b^8)*tan(d*x + c)^3 + 2*(357*a^8*b - 987*a^6*b^3
 + 125*a^4*b^5 + 31*a^2*b^7 + 2*b^9)*tan(d*x + c)^2 + (15*a^9 + 93*a^7*b^2
- 763*a^5*b^4 + 127*a^3*b^6 + 8*a*b^8)*tan(d*x + c))/(a^12 + 5*a^10*b^2 + 1
0*a^8*b^4 + 10*a^6*b^6 + 5*a^4*b^8 + a^2*b^10 + (a^10*b^2 + 5*a^8*b^4 + 10*
a^6*b^6 + 10*a^4*b^8 + 5*a^2*b^10 + b^12)*tan(d*x + c)^8 + 2*(a^11*b + 5*a^
9*b^3 + 10*a^7*b^5 + 10*a^5*b^7 + 5*a^3*b^9 + a*b^11)*tan(d*x + c)^7 + (a^1
2 + 8*a^10*b^2 + 25*a^8*b^4 + 40*a^6*b^6 + 35*a^4*b^8 + 16*a^2*b^10 + 3*b^1
2)*tan(d*x + c)^6 + 6*(a^11*b + 5*a^9*b^3 + 10*a^7*b^5 + 10*a^5*b^7 + 5*a^3
*b^9 + a*b^11)*tan(d*x + c)^5 + 3*(a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*
b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*tan(d*x + c)^4 + 6*(a^11*b + 5*a^9*b^
3 + 10*a^7*b^5 + 10*a^5*b^7 + 5*a^3*b^9 + a*b^11)*tan(d*x + c)^3 + (3*a^12
+ 16*a^10*b^2 + 35*a^8*b^4 + 40*a^6*b^6 + 25*a^4*b^8 + 8*a^2*b^10 + b^12)*t
an(d*x + c)^2 + 2*(a^11*b + 5*a^9*b^3 + 10*a^7*b^5 + 10*a^5*b^7 + 5*a^3*b^9
 + a*b^11)*tan(d*x + c)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 923 vs. $2(372) = 744$.

Time = 0.69 (sec) , antiderivative size = 923, normalized size of antiderivative = 2.42

$$\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

$$= \frac{\frac{3\left(5a^9-180a^7b^2+390a^5b^4-68a^3b^6-3ab^8\right)(dx+c)}{a^{12}+6a^{10}b^2+15a^8b^4+20a^6b^6+15a^4b^8+6a^2b^{10}+b^{12}} - \frac{24\left(3a^8b-22a^6b^3+15a^4b^5\right)\log\left(\tan(dx+c)^2+1\right)}{a^{12}+6a^{10}b^2+15a^8b^4+20a^6b^6+15a^4b^8+6a^2b^{10}+b^{12}} + \frac{48\left(3a^8b^2-22a^6b^4+15a^4 \cdots\right.}{a^{12}b+6a^{10}b^3+15a^8b^5+20 \cdots}}{\cdots}$$

```
[In] integrate(sin(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/48*(3*(5*a^9 - 180*a^7*b^2 + 390*a^5*b^4 - 68*a^3*b^6 - 3*a*b^8)*(d*x + c
)/(a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 +
b^12) - 24*(3*a^8*b - 22*a^6*b^3 + 15*a^4*b^5)*log(tan(d*x + c)^2 + 1)/(a^1
2 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)
+ 48*(3*a^8*b^2 - 22*a^6*b^4 + 15*a^4*b^6)*log(abs(b*tan(d*x + c) + a))/(a^
12*b + 6*a^10*b^3 + 15*a^8*b^5 + 20*a^6*b^7 + 15*a^4*b^9 + 6*a^2*b^11 + b^1
3) - 24*(9*a^8*b^3*tan(d*x + c)^2 - 66*a^6*b^5*tan(d*x + c)^2 + 45*a^4*b^7*
tan(d*x + c)^2 + 22*a^9*b^2*tan(d*x + c) - 140*a^7*b^4*tan(d*x + c) + 78*a^
5*b^6*tan(d*x + c) + 14*a^10*b - 72*a^8*b^3 + 34*a^6*b^5)/((a^12 + 6*a^10*b
^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*(b*tan(d*x +
 c) + a)^2) + (132*a^8*b*tan(d*x + c)^6 - 968*a^6*b^3*tan(d*x + c)^6 + 660*
a^4*b^5*tan(d*x + c)^6 - 33*a^9*tan(d*x + c)^5 + 324*a^7*b^2*tan(d*x + c)^5
 + 162*a^5*b^4*tan(d*x + c)^5 - 204*a^3*b^6*tan(d*x + c)^5 - 9*a*b^8*tan(d*
x + c)^5 + 180*a^8*b*tan(d*x + c)^4 - 2760*a^6*b^3*tan(d*x + c)^4 + 2340*a^
4*b^5*tan(d*x + c)^4 - 40*a^9*tan(d*x + c)^3 + 576*a^7*b^2*tan(d*x + c)^3 +
 96*a^5*b^4*tan(d*x + c)^3 - 544*a^3*b^6*tan(d*x + c)^3 - 24*a*b^8*tan(d*x
+ c)^3 + 72*a^8*b*tan(d*x + c)^2 - 2448*a^6*b^3*tan(d*x + c)^2 + 2700*a^4*b
^5*tan(d*x + c)^2 - 72*a^2*b^7*tan(d*x + c)^2 - 12*b^9*tan(d*x + c)^2 - 15*
a^9*tan(d*x + c) + 252*a^7*b^2*tan(d*x + c) - 18*a^5*b^4*tan(d*x + c) - 276
*a^3*b^6*tan(d*x + c) + 9*a*b^8*tan(d*x + c) - 720*a^6*b^3 + 972*a^4*b^5 -
72*a^2*b^7 - 4*b^9)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*
b^8 + 6*a^2*b^10 + b^12)*(tan(d*x + c)^2 + 1)^3))/d
```

Mupad [B] (verification not implemented)

Time = 6.86 (sec) , antiderivative size = 1068, normalized size of antiderivative = 2.80

$$\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(\frac{3\,b}{{\left(a^2+b^2\right)}^2}-\frac{34\,b^3}{{\left(a^2+b^2\right)}^3}+\frac{99\,b^5}{{\left(a^2+b^2\right)}^4}-\frac{108\,b^7}{{\left(a^2+b^2\right)}^5}+\frac{40\,b^9}{{\left(a^2+b^2\right)}^6}\right)}{d}$$
$$-\frac{\frac{{\mathrm{tan}\left(c+d\,x\right)}^6\,\left(31\,a^8\,b-127\,a^6\,b^3+5\,a^4\,b^5+3\,a^2\,b^7\right)}{8\,\left(a^{10}+5\,a^8\,b^2+10\,a^6\,b^4+10\,a^4\,b^6+5\,a^2\,b^8+b^{10}\right)}+\frac{{\mathrm{tan}\left(c+d\,x\right)}^7\,\left(43\,a^7\,b^2-215\,a^5\,b^4+65\,a^3\,b^6+3\,a\,b^8\right)}{16\,\left(a^{10}+5\,a^8\,b^2+10\,a^6\,b^4+10\,a^4\,b^6+5\,a^2\,b^8+b^{10}\right)}+\frac{{\mathrm{tan}\left(c+d\,x\right)}^5\,\left(33\,a^9+403\,a^7\,b^2-2005\ldots\right)}{48\,\left(a^{10}+5\,a^8\,b^2+10\,a^6\,b^4+10\ldots\right)}}{d\,\left({\mathrm{tan}\left(c+d\,x\right)}^2\,\left(3\,a^2+b^2\right)\ldots\right)}$$
$$-\frac{\ln\left(\mathrm{tan}\left(c+d\,x\right)+1{}\mathrm{i}\right)\,\left(a^3\,5{}\mathrm{i}-18\,a^2\,b+a\,b^2\,3{}\mathrm{i}\right)}{32\,d\,\left(-a^6+a^5\,b\,6{}\mathrm{i}+15\,a^4\,b^2-a^3\,b^3\,20{}\mathrm{i}-15\,a^2\,b^4+a\,b^5\,6{}\mathrm{i}+b^6\right)}$$
$$-\frac{\ln\left(\mathrm{tan}\left(c+d\,x\right)-\mathrm{i}\right)\,\left(5\,a^3-a^2\,b\,18{}\mathrm{i}+3\,a\,b^2\right)}{32\,d\,\left(-a^6\,1{}\mathrm{i}+6\,a^5\,b+a^4\,b^2\,15{}\mathrm{i}-20\,a^3\,b^3-a^2\,b^4\,15{}\mathrm{i}+6\,a\,b^5+b^6\,1{}\mathrm{i}\right)}$$

```
[In] int(sin(c + d*x)^6/(a + b*tan(c + d*x))^3,x)
```

```
[Out] (log(a + b*tan(c + d*x))*((3*b)/(a^2 + b^2)^2 - (34*b^3)/(a^2 + b^2)^3 + (9
9*b^5)/(a^2 + b^2)^4 - (108*b^7)/(a^2 + b^2)^5 + (40*b^9)/(a^2 + b^2)^6))/d
```

```
 - ((tan(c + d*x)^6*(31*a^8*b + 3*a^2*b^7 + 5*a^4*b^5 - 127*a^6*b^3))/(8*(a
^10 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2)) + (tan(c + d
*x)^7*(3*a*b^8 + 65*a^3*b^6 - 215*a^5*b^4 + 43*a^7*b^2))/(16*(a^10 + b^10 +
 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2)) + (tan(c + d*x)^5*(24*a*
b^8 + 33*a^9 + 529*a^3*b^6 - 2005*a^5*b^4 + 403*a^7*b^2))/(48*(a^10 + b^10
+ 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2)) + (tan(c + d*x)^4*(164*
a^8*b + 3*b^9 + 27*a^2*b^7 + 65*a^4*b^5 - 515*a^6*b^3))/(12*(a^10 + b^10 +
5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2)) + (a^2*(63*a^6*b + b^7 +
17*a^2*b^5 - 161*a^4*b^3))/(12*(a^2 + b^2)*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b
^4 + 4*a^6*b^2)) + (tan(c + d*x)^3*(15*a*b^8 + 40*a^9 + 429*a^3*b^6 - 2171*
a^5*b^4 + 335*a^7*b^2))/(48*(a^2 + b^2)*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4
+ 4*a^6*b^2)) + (tan(c + d*x)^2*(357*a^8*b + 2*b^9 + 31*a^2*b^7 + 125*a^4*b
^5 - 987*a^6*b^3))/(24*(a^2 + b^2)*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a
^6*b^2)) + (a*tan(c + d*x)*(15*a^8 + 8*b^8 + 127*a^2*b^6 - 763*a^4*b^4 + 93
*a^6*b^2))/(48*(a^2 + b^2)*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2))
)/(d*(tan(c + d*x)^2*(3*a^2 + b^2) + tan(c + d*x)^6*(a^2 + 3*b^2) + a^2 + t
an(c + d*x)^4*(3*a^2 + 3*b^2) + b^2*tan(c + d*x)^8 + 2*a*b*tan(c + d*x) + 6
*a*b*tan(c + d*x)^3 + 6*a*b*tan(c + d*x)^5 + 2*a*b*tan(c + d*x)^7)) - (log(
tan(c + d*x) + 1i)*(a*b^2*3i - 18*a^2*b + a^3*5i))/(32*d*(a*b^5*6i + a^5*b*
6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)) - (log(tan(c + d*x
) - 1i)*(3*a*b^2 - a^2*b*18i + 5*a^3))/(32*d*(6*a*b^5 + 6*a^5*b - a^6*1i +
b^6*1i - a^2*b^4*15i - 20*a^3*b^3 + a^4*b^2*15i))
```

3.68 $\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 285

$$\begin{aligned}
&\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx\\
&=\frac{3a(a^6-25a^4b^2+35a^2b^4-3b^6)\,x}{8\,(a^2+b^2)^5}\\
&\quad+\frac{3a^2b(a^4-5a^2b^2+2b^4)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^5\,d}\\
&\quad-\frac{a^4b}{2\,(a^2+b^2)^3\,d(a+b\tan(c+dx))^2}-\frac{2a^3b(a^2-2b^2)}{(a^2+b^2)^4\,d(a+b\tan(c+dx))}\\
&\quad+\frac{\cos^4(c+dx)\,(b(3a^2-b^2)+a(a^2-3b^2)\tan(c+dx))}{4\,(a^2+b^2)^3\,d}\\
&\quad-\frac{a\cos^2(c+dx)\,(24ab(a^2-b^2)+(5a^4-34a^2b^2+9b^4)\tan(c+dx))}{8\,(a^2+b^2)^4\,d}
\end{aligned}$$

```
[Out] 3/8*a*(a^6-25*a^4*b^2+35*a^2*b^4-3*b^6)*x/(a^2+b^2)^5+3*a^2*b*(a^4-5*a^2*b^
2+2*b^4)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^5/d-1/2*a^4*b/(a^2+b^2)^3/
d/(a+b*tan(d*x+c))^2-2*a^3*b*(a^2-2*b^2)/(a^2+b^2)^4/d/(a+b*tan(d*x+c))+1/4
*cos(d*x+c)^4*(b*(3*a^2-b^2)+a*(a^2-3*b^2)*tan(d*x+c))/(a^2+b^2)^3/d-1/8*a*
cos(d*x+c)^2*(24*a*b*(a^2-b^2)+(5*a^4-34*a^2*b^2+9*b^4)*tan(d*x+c))/(a^2+b^
2)^4/d
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3597, 1661, 1643, 649, 209, 266}

$$\begin{aligned}&\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx\\&=\frac{\cos^4(c+dx)\,(a(a^2-3b^2)\tan(c+dx)+b(3a^2-b^2))}{4d\,(a^2+b^2)^3}-\frac{a^4b}{2d\,(a^2+b^2)^3\,(a+b\tan(c+dx))^2}\\&\quad-\frac{a\cos^2(c+dx)\,(24ab(a^2-b^2)+(5a^4-34a^2b^2+9b^4)\tan(c+dx))}{8d\,(a^2+b^2)^4}\\&\quad+\frac{3a^2b(a^4-5a^2b^2+2b^4)\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^5}\\&\quad-\frac{2a^3b(a^2-2b^2)}{d\,(a^2+b^2)^4\,(a+b\tan(c+dx))}+\frac{3ax(a^6-25a^4b^2+35a^2b^4-3b^6)}{8\,(a^2+b^2)^5}\end{aligned}$$

```
[In] Int[Sin[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a*(a^6 - 25*a^4*b^2 + 35*a^2*b^4 - 3*b^6)*x)/(8*(a^2 + b^2)^5) + (3*a^2*
b*(a^4 - 5*a^2*b^2 + 2*b^4)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b
^2)^5*d) - (a^4*b)/(2*(a^2 + b^2)^3*d*(a + b*Tan[c + d*x])^2) - (2*a^3*b*(a
^2 - 2*b^2))/((a^2 + b^2)^4*d*(a + b*Tan[c + d*x])) + (Cos[c + d*x]^4*(b*(3
*a^2 - b^2) + a*(a^2 - 3*b^2)*Tan[c + d*x]))/(4*(a^2 + b^2)^3*d) - (a*Cos[c
 + d*x]^2*(24*a*b*(a^2 - b^2) + (5*a^4 - 34*a^2*b^2 + 9*b^4)*Tan[c + d*x]))
/(8*(a^2 + b^2)^4*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{b\text{Subst}\left(\int \frac{x^4}{(a+x)^3(b^2+x^2)^3}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{\cos^4(c+dx)\left(b(3a^2-b^2)+a(a^2-3b^2)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^3 d} - \frac{\text{Subst}\left(\int \frac{\frac{a^4b^4\left(a^2-3b^2\right)}{\left(a^2+b^2\right)^3}-\frac{a^3b^4\left(9a^2+5b^2\right)x}{\left(a^2+b^2\right)^3}-\frac{a^2b^2\left(4a^4+21a^2b^2-3b^4\right)x^2}{\left(a^2+b^2\right)^3}-\frac{3ab^4\left(a^2-3b^2\right)x^3}{\left(a^2+b^2\right)^3}}{(a+x)^3(b^2+x^2)^2}\,dx, x, b\tan(c+dx)\right)}{4bd}$$

$$= \frac{\cos^4(c+dx)\left(b(3a^2-b^2)+a(a^2-3b^2)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^3 d} - \frac{a\cos^2(c+dx)\left(24ab(a^2-b^2)+(5a^4-34a^2b^2+9b^4)\tan(c+dx)\right)}{8\left(a^2+b^2\right)^4 d} + \frac{\text{Subst}\left(\int \frac{\frac{3a^4b^4\left(a^4-10a^2b^2+5b^4\right)}{\left(a^2+b^2\right)^4}-\frac{a^3b^4\left(15a^4+26a^2b^2-37b^4\right)x}{\left(a^2+b^2\right)^4}-\frac{3a^2b^4\left(5a^4-18a^2b^2-7b^4\right)x^2}{\left(a^2+b^2\right)^4}-\frac{ab^4\left(5a^4-34a^2b^2+9b^4\right)x^3}{\left(a^2+b^2\right)^4}}{(a+x)^3(b^2+x^2)}\,dx, x, b\,\mathrm{ta}\right.}{8b^3d}$$

$$= \frac{\cos^4(c+dx)\left(b(3a^2-b^2)+a(a^2-3b^2)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^3 d} - \frac{a\cos^2(c+dx)\left(24ab(a^2-b^2)+(5a^4-34a^2b^2+9b^4)\tan(c+dx)\right)}{8\left(a^2+b^2\right)^4 d} + \frac{\text{Subst}\left(\int\left(\frac{8a^4b^4}{(a^2+b^2)^3(a+x)^3}+\frac{16a^3b^4(a^2-2b^2)}{(a^2+b^2)^4(a+x)^2}+\frac{24a^2b^4(a^4-5a^2b^2+2b^4)}{(a^2+b^2)^5(a+x)}+\frac{3ab^4(a^6-25a^4b^2+35a^2b^4-3b^6-8a(a^4-5a^2}{(a^2+b^2)^5(b^2+x^2)}\right.\right.}{8b^3d}$$

$$= \frac{3a^2b(a^4-5a^2b^2+2b^4)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^5 d} - \frac{a^4b}{2\left(a^2+b^2\right)^3 d(a+b\tan(c+dx))^2} - \frac{2a^3b(a^2-2b^2)}{\left(a^2+b^2\right)^4 d(a+b\tan(c+dx))} + \frac{\cos^4(c+dx)\left(b(3a^2-b^2)+a(a^2-3b^2)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^3 d} - \frac{a\cos^2(c+dx)\left(24ab(a^2-b^2)+(5a^4-34a^2b^2+9b^4)\tan(c+dx)\right)}{8\left(a^2+b^2\right)^4 d} + \frac{(3ab)\text{Subst}\left(\int\frac{a^6-25a^4b^2+35a^2b^4-3b^6-8a(a^4-5a^2b^2+2b^4)x}{b^2+x^2}\,dx,x,b\tan(c+dx)\right)}{8\left(a^2+b^2\right)^5 d}$$

$$= \frac{3a^2b(a^4-5a^2b^2+2b^4)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^5 d} - \frac{a^4b}{2\left(a^2+b^2\right)^3 d(a+b\tan(c+dx))^2} - \frac{2a^3b(a^2-2b^2)}{\left(a^2+b^2\right)^4 d(a+b\tan(c+dx))} + \frac{\cos^4(c+dx)\left(b(3a^2-b^2)+a(a^2-3b^2)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^3 d} - \frac{a\cos^2(c+dx)\left(24ab(a^2-b^2)+(5a^4-34a^2b^2+9b^4)\tan(c+dx)\right)}{8\left(a^2+b^2\right)^4 d} - \frac{(3a^2b(a^4-5a^2b^2+2b^4))\,\text{Subst}\left(\int\frac{x}{b^2+x^2}\,dx,x,b\tan(c+dx)\right)}{\left(a^2+b^2\right)^5 d} + \frac{(3ab(a^6-25a^4b^2+35a^2b^4-3b^6))\,\text{Subst}\left(\int\frac{1}{b^2+x^2}\,dx,x,b\tan(c+dx)\right)}{8\left(a^2+b^2\right)^5 d}$$

$$= \frac{3a(a^6-25a^4b^2+35a^2b^4-3b^6)\,x}{8\,(a^2+b^2)^5} + \frac{3a^2b(a^4-5a^2b^2+2b^4)\log(\cos(c+dx))}{(a^2+b^2)^5\,d}$$
$$+ \frac{3a^2b(a^4-5a^2b^2+2b^4)\log(a+b\tan(c+dx))}{(a^2+b^2)^5\,d}$$
$$- \frac{a^4b}{2\,(a^2+b^2)^3\,d(a+b\tan(c+dx))^2} - \frac{2a^3b(a^2-2b^2)}{(a^2+b^2)^4\,d(a+b\tan(c+dx))}$$
$$+ \frac{\cos^4(c+dx)\,(b(3a^2-b^2)+a(a^2-3b^2)\tan(c+dx))}{4\,(a^2+b^2)^3\,d}$$
$$- \frac{a\cos^2(c+dx)\,(24ab(a^2-b^2)+(5a^4-34a^2b^2+9b^4)\tan(c+dx))}{8\,(a^2+b^2)^4\,d}$$

Mathematica [A] (verified)

Time = 6.50 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.83

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{b\left(-\frac{a^3(a^2-5b^2)\arctan(\tan(c+dx))}{b(a^2+b^2)^4} + \frac{3a(a^2-3b^2)\arctan(\tan(c+dx))}{8b(a^2+b^2)^3} - \frac{3a^2(a-b)(a+b)\cos^2(c+dx)}{(a^2+b^2)^4} + \frac{(3a^2-b^2)\cos^4(c+dx)}{4(a^2+b^2)^3} - \frac{a^2\left(3a^4-\right.}{}\right.}{}$$

```
[In] Integrate[Sin[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (b*(-((a^3*(a^2 - 5*b^2)*ArcTan[Tan[c + d*x]])/(b*(a^2 + b^2)^4)) + (3*a*(a
^2 - 3*b^2)*ArcTan[Tan[c + d*x]])/(8*b*(a^2 + b^2)^3) - (3*a^2*(a - b)*(a +
 b)*Cos[c + d*x]^2)/(a^2 + b^2)^4 + ((3*a^2 - b^2)*Cos[c + d*x]^4)/(4*(a^2
+ b^2)^3) - (a^2*(3*a^4 - 15*a^2*b^2 + 6*b^4 - (a^5 - 13*a^3*b^2 + 10*a*b^4
)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/(2*(a^2 + b^2)^5) + (3*a^2*
(a^4 - 5*a^2*b^2 + 2*b^4)*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^5 - (a^2*(3*
a^4 - 15*a^2*b^2 + 6*b^4 + (a^5 - 13*a^3*b^2 + 10*a*b^4)/Sqrt[-b^2])*Log[Sq
rt[-b^2] + b*Tan[c + d*x]])/(2*(a^2 + b^2)^5) - (a^3*(a^2 - 5*b^2)*Cos[c +
d*x]*Sin[c + d*x])/(b*(a^2 + b^2)^4) + (3*a*(a^2 - 3*b^2)*Cos[c + d*x]*Sin[
c + d*x])/(8*b*(a^2 + b^2)^3) + (a*(a^2 - 3*b^2)*Cos[c + d*x]^3*Sin[c + d*x
])/(4*b*(a^2 + b^2)^3) - a^4/(2*(a^2 + b^2)^3*(a + b*Tan[c + d*x])^2) - (2*
a^3*(a^2 - 2*b^2))/((a^2 + b^2)^4*(a + b*Tan[c + d*x]))))/d
```

Maple [A] (verified)

Time = 20.58 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.16

method	result
derivativedivides	$\frac{\frac{\left(-\frac{5}{8}a^{7}+\frac{29}{8}a^{5}b^{2}+\frac{25}{8}a^{3}b^{4}-\frac{9}{8}a\,b^{6}\right)\left(\tan^{3}(dx+c)\right)+\left(-3a^{6}b+3b^{5}a^{2}\right)\left(\tan^{2}(dx+c)\right)+\left(-\frac{3}{8}a^{7}+\frac{27}{8}a^{5}b^{2}+\frac{15}{8}a^{3}b^{4}-\frac{15}{8}a\,b^{6}\right)\tan(dx+c)}{\left(1+\tan^{2}(dx+c)\right)^{2}}}{(a^{2}+}$
default	$\frac{\frac{\left(-\frac{5}{8}a^{7}+\frac{29}{8}a^{5}b^{2}+\frac{25}{8}a^{3}b^{4}-\frac{9}{8}a\,b^{6}\right)\left(\tan^{3}(dx+c)\right)+\left(-3a^{6}b+3b^{5}a^{2}\right)\left(\tan^{2}(dx+c)\right)+\left(-\frac{3}{8}a^{7}+\frac{27}{8}a^{5}b^{2}+\frac{15}{8}a^{3}b^{4}-\frac{15}{8}a\,b^{6}\right)\tan(dx+c)}{\left(1+\tan^{2}(dx+c)\right)^{2}}}{(a^{2}+}$
risch	$-\frac{9iaxb}{8(5ia^{4}b-10ia^{2}b^{3}+ib^{5}-a^{5}+10a^{3}b^{2}-5a\,b^{4})}-\frac{3a^{2}x}{8(5ia^{4}b-10ia^{2}b^{3}+ib^{5}-a^{5}+10a^{3}b^{2}-5a\,b^{4})}-\frac{12ia^{2}}{a^{10}+5a^{8}b^{2}+10a^{6}b^{4}+}$

```
[In] int(sin(d*x+c)^4/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a^2+b^2)^5*(((-5/8*a^7+29/8*a^5*b^2+25/8*a^3*b^4-9/8*a*b^6)*tan(d*x
+c)^3+(-3*a^6*b+3*a^2*b^5)*tan(d*x+c)^2+(-3/8*a^7+27/8*a^5*b^2+15/8*a^3*b^4
-15/8*a*b^6)*tan(d*x+c)-9/4*a^6*b+5/4*a^4*b^3+13/4*b^5*a^2-1/4*b^7)/(1+tan(
d*x+c)^2)^2+3/8*a*(1/2*(-8*a^5*b+40*a^3*b^3-16*a*b^5)*ln(1+tan(d*x+c)^2)+(a
^6-25*a^4*b^2+35*a^2*b^4-3*b^6)*arctan(tan(d*x+c))))-1/2*b/(a^2+b^2)^3*a^4/
(a+b*tan(d*x+c))^2+3*a^2*b*(a^4-5*a^2*b^2+2*b^4)/(a^2+b^2)^5*ln(a+b*tan(d*x
+c))-2*a^3*b*(a^2-2*b^2)/(a^2+b^2)^4/(a+b*tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 705 vs. $2(277) = 554$.

Time = 0.34 (sec) , antiderivative size = 705, normalized size of antiderivative = 2.47

$$\int \frac{\sin^{4}(c+dx)}{(a+b\tan(c+dx))^{3}}\,dx = \frac{119\,a^{6}b^{3}-159\,a^{4}b^{5}-51\,a^{2}b^{7}+3\,b^{9}+8\left(a^{8}b+4\,a^{6}b^{3}+6\,a^{4}b^{5}+4\,a^{2}b^{7}+b^{9}\right)\cos\left(dx+c\right)^{6}-8\left(5\,a^{8}b+16\right.}{}$$

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/32*(119*a^6*b^3 - 159*a^4*b^5 - 51*a^2*b^7 + 3*b^9 + 8*(a^8*b + 4*a^6*b^3
 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cos(d*x + c)^6 - 8*(5*a^8*b + 16*a^6*b^3 +
18*a^4*b^5 + 8*a^2*b^7 + b^9)*cos(d*x + c)^4 + 12*(a^7*b^2 - 25*a^5*b^4 + 3
5*a^3*b^6 - 3*a*b^8)*d*x - (a^8*b + 110*a^6*b^3 - 420*a^4*b^5 - 78*a^2*b^7
+ 3*b^9 - 12*(a^9 - 26*a^7*b^2 + 60*a^5*b^4 - 38*a^3*b^6 + 3*a*b^8)*d*x)*co
s(d*x + c)^2 + 48*(a^6*b^3 - 5*a^4*b^5 + 2*a^2*b^7 + (a^8*b - 6*a^6*b^3 + 7
```

```
*a^4*b^5 - 2*a^2*b^7)*cos(d*x + c)^2 + 2*(a^7*b^2 - 5*a^5*b^4 + 2*a^3*b^6)*
cos(d*x + c)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2
)*cos(d*x + c)^2 + b^2) + 2*(4*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a
*b^8)*cos(d*x + c)^5 - 2*(5*a^9 + 12*a^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 - 3*a*
b^8)*cos(d*x + c)^3 + (77*a^7*b^2 - 69*a^5*b^4 + 63*a^3*b^6 - 15*a*b^8 + 12
*(a^8*b - 25*a^6*b^3 + 35*a^4*b^5 - 3*a^2*b^7)*d*x)*cos(d*x + c))*sin(d*x +
 c))/((a^12 + 4*a^10*b^2 + 5*a^8*b^4 - 5*a^4*b^8 - 4*a^2*b^10 - b^12)*d*cos
(d*x + c)^2 + 2*(a^11*b + 5*a^9*b^3 + 10*a^7*b^5 + 10*a^5*b^7 + 5*a^3*b^9 +
 a*b^11)*d*cos(d*x + c)*sin(d*x + c) + (a^10*b^2 + 5*a^8*b^4 + 10*a^6*b^6 +
 10*a^4*b^8 + 5*a^2*b^10 + b^12)*d)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Exception raised: AttributeError}$$

```
[In] integrate(sin(d*x+c)**4/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 744 vs. 2(277) = 554.

Time = 0.55 (sec) , antiderivative size = 744, normalized size of antiderivative = 2.61

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{3\,(a^7-25\,a^5b^2+35\,a^3b^4-3\,ab^6)(dx+c)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}} + \frac{24\,(a^6b-5\,a^4b^3+2\,a^2b^5)\log(b\tan(dx+c)+a)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}} - \frac{12\,(a^6b-5\,a^4b^3+2\,a^2b^5)\log\left(\tan(dx+c)^2+1\right)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}} - }{}$$

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(a^7 - 25*a^5*b^2 + 35*a^3*b^4 - 3*a*b^6)*(d*x + c)/(a^10 + 5*a^8*b^
2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) + 24*(a^6*b - 5*a^4*b^3 + 2
*a^2*b^5)*log(b*tan(d*x + c) + a)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b
^6 + 5*a^2*b^8 + b^10) - 12*(a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*log(tan(d*x + c
)^2 + 1)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) -
(38*a^6*b - 56*a^4*b^3 + 2*a^2*b^5 + 3*(7*a^5*b^2 - 22*a^3*b^4 + 3*a*b^6)*t
an(d*x + c)^5 + 6*(5*a^6*b - 12*a^4*b^3 - a^2*b^5)*tan(d*x + c)^4 + (5*a^7
+ 49*a^5*b^2 - 133*a^3*b^4 + 15*a*b^6)*tan(d*x + c)^3 + 2*(35*a^6*b - 61*a^
4*b^3 + a^2*b^5 + b^7)*tan(d*x + c)^2 + (3*a^7 + 22*a^5*b^2 - 73*a^3*b^4 +
```

```
4*a*b^6)*tan(d*x + c))/(a^10 + 4*a^8*b^2 + 6*a^6*b^4 + 4*a^4*b^6 + a^2*b^8
+ (a^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)*tan(d*x + c)^6 + 2*(
a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*tan(d*x + c)^5 + (a^10 +
 6*a^8*b^2 + 14*a^6*b^4 + 16*a^4*b^6 + 9*a^2*b^8 + 2*b^10)*tan(d*x + c)^4 +
 4*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*tan(d*x + c)^3 + (2*
a^10 + 9*a^8*b^2 + 16*a^6*b^4 + 14*a^4*b^6 + 6*a^2*b^8 + b^10)*tan(d*x + c)
^2 + 2*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*tan(d*x + c)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 588 vs. $2(277) = 554$.

Time = 0.68 (sec) , antiderivative size = 588, normalized size of antiderivative = 2.06

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{3\,(a^7-25\,a^5b^2+35\,a^3b^4-3\,ab^6)(dx+c)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}} - \frac{12\,(a^6b-5\,a^4b^3+2\,a^2b^5)\log\left(\tan(dx+c)^2+1\right)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}} + \frac{24\,(a^6b^2-5\,a^4b^4+2\,a^2b^6)\log(|b\tan(dx+c)+a|)}{a^{10}b+5\,a^8b^3+10\,a^6b^5+10\,a^4b^7+5\,a^2b^9+b^{11}}}{}$$

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a^7 - 25*a^5*b^2 + 35*a^3*b^4 - 3*a*b^6)*(d*x + c)/(a^10 + 5*a^8*b^
2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) - 12*(a^6*b - 5*a^4*b^3 + 2
*a^2*b^5)*log(tan(d*x + c)^2 + 1)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b
^6 + 5*a^2*b^8 + b^10) + 24*(a^6*b^2 - 5*a^4*b^4 + 2*a^2*b^6)*log(abs(b*tan
(d*x + c) + a))/(a^10*b + 5*a^8*b^3 + 10*a^6*b^5 + 10*a^4*b^7 + 5*a^2*b^9 +
 b^11) - (21*a^5*b^2*tan(d*x + c)^5 - 66*a^3*b^4*tan(d*x + c)^5 + 9*a*b^6*t
an(d*x + c)^5 + 30*a^6*b*tan(d*x + c)^4 - 72*a^4*b^3*tan(d*x + c)^4 - 6*a^2
*b^5*tan(d*x + c)^4 + 5*a^7*tan(d*x + c)^3 + 49*a^5*b^2*tan(d*x + c)^3 - 13
3*a^3*b^4*tan(d*x + c)^3 + 15*a*b^6*tan(d*x + c)^3 + 70*a^6*b*tan(d*x + c)^
2 - 122*a^4*b^3*tan(d*x + c)^2 + 2*a^2*b^5*tan(d*x + c)^2 + 2*b^7*tan(d*x +
 c)^2 + 3*a^7*tan(d*x + c) + 22*a^5*b^2*tan(d*x + c) - 73*a^3*b^4*tan(d*x +
 c) + 4*a*b^6*tan(d*x + c) + 38*a^6*b - 56*a^4*b^3 + 2*a^2*b^5)/((a^8 + 4*a
^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(b*tan(d*x + c)^3 + a*tan(d*x + c)^2
+ b*tan(d*x + c) + a)^2))/d
```

Mupad [B] (verification not implemented)

Time = 6.09 (sec) , antiderivative size = 717, normalized size of antiderivative = 2.52

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\ln\left(a+b\tan(c+d\,x)\right)\left(\frac{3\,b}{(a^2+b^2)^2}-\frac{24\,b^3}{(a^2+b^2)^3}+\frac{45\,b^5}{(a^2+b^2)^4}-\frac{24\,b^7}{(a^2+b^2)^5}\right)}{d}$$
$$-\frac{\frac{19\,a^6\,b-28\,a^4\,b^3+a^2\,b^5}{4\,(a^8+4\,a^6\,b^2+6\,a^4\,b^4+4\,a^2\,b^6+b^8)}+\frac{\tan(c+d\,x)^2\,(35\,a^6\,b-61\,a^4\,b^3+a^2\,b^5+b^7)}{4\,(a^8+4\,a^6\,b^2+6\,a^4\,b^4+4\,a^2\,b^6+b^8)}-\frac{3\tan(c+d\,x)^4\,(-5\,a^6\,b+12\,a^4\,b^3+a^2\,b^5)}{4\,(a^8+4\,a^6\,b^2+6\,a^4\,b^4+4\,a^2\,b^6+b^8)}+\frac{3\tan(c+d\,x)^5}{8\,(a^8+4\,a^6}}{d\,\left(\tan\left(c+d\,x\right)^2\,(2\,a^2+b^2)+\tan\left(c+d\,x\right)^4\,(a^2+2\,b^2)+a^2+b^2\tan\left(c+d\,x\right.\right.}$$
$$-\frac{3\,\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,(a^2\,1\mathrm{i}+3\,b\,a)}{16\,d\,\left(a^5+a^4\,b\,5\mathrm{i}-10\,a^3\,b^2-a^2\,b^3\,10\mathrm{i}+5\,a\,b^4+b^5\,1\mathrm{i}\right)}$$
$$-\frac{3\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,(3\,a\,b-a^2\,1\mathrm{i})}{16\,d\,\left(a^5-a^4\,b\,5\mathrm{i}-10\,a^3\,b^2+a^2\,b^3\,10\mathrm{i}+5\,a\,b^4-b^5\,1\mathrm{i}\right)}$$

```
[In] int(sin(c + d*x)^4/(a + b*tan(c + d*x))^3,x)
```

```
[Out] (log(a + b*tan(c + d*x))*((3*b)/(a^2 + b^2)^2 - (24*b^3)/(a^2 + b^2)^3 + (4
5*b^5)/(a^2 + b^2)^4 - (24*b^7)/(a^2 + b^2)^5))/d - ((19*a^6*b + a^2*b^5 -
28*a^4*b^3)/(4*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (tan(c +
d*x)^2*(35*a^6*b + b^7 + a^2*b^5 - 61*a^4*b^3))/(4*(a^8 + b^8 + 4*a^2*b^6 +
 6*a^4*b^4 + 4*a^6*b^2)) - (3*tan(c + d*x)^4*(a^2*b^5 - 5*a^6*b + 12*a^4*b^
3))/(4*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (3*tan(c + d*x)^5
*(3*a*b^6 - 22*a^3*b^4 + 7*a^5*b^2))/(8*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4
+ 4*a^6*b^2)) + (tan(c + d*x)^3*(15*a*b^6 + 5*a^7 - 133*a^3*b^4 + 49*a^5*b^
2))/(8*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (a*tan(c + d*x)*(
3*a^6 + 4*b^6 - 73*a^2*b^4 + 22*a^4*b^2))/(8*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4
*b^4 + 4*a^6*b^2)))/(d*(tan(c + d*x)^2*(2*a^2 + b^2) + tan(c + d*x)^4*(a^2
+ 2*b^2) + a^2 + b^2*tan(c + d*x)^6 + 2*a*b*tan(c + d*x) + 4*a*b*tan(c + d*
x)^3 + 2*a*b*tan(c + d*x)^5)) - (3*log(tan(c + d*x) - 1i)*(3*a*b + a^2*1i))
/(16*d*(5*a*b^4 + a^4*b*5i + a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2)) - (3
*log(tan(c + d*x) + 1i)*(3*a*b - a^2*1i))/(16*d*(5*a*b^4 - a^4*b*5i + a^5 -
 b^5*1i + a^2*b^3*10i - 10*a^3*b^2))
```

3.69 $\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 206

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{a(a^4-14a^2b^2+9b^4)\,x}{2\,(a^2+b^2)^4} + \frac{b(3a^4-8a^2b^2+b^4)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^4\,d} - \frac{a^2b}{2\,(a^2+b^2)^2\,d(a+b\tan(c+dx))^2} - \frac{2ab(a^2-b^2)}{(a^2+b^2)^3\,d(a+b\tan(c+dx))} - \frac{\cos^2(c+dx)\,(b(3a^2-b^2)+a(a^2-3b^2)\tan(c+dx))}{2\,(a^2+b^2)^3\,d}$$

```
[Out] 1/2*a*(a^4-14*a^2*b^2+9*b^4)*x/(a^2+b^2)^4+b*(3*a^4-8*a^2*b^2+b^4)*ln(a*cos
(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^4/d-1/2*a^2*b/(a^2+b^2)^2/d/(a+b*tan(d*x+c)
)^2-2*a*b*(a^2-b^2)/(a^2+b^2)^3/d/(a+b*tan(d*x+c))-1/2*cos(d*x+c)^2*(b*(3*a
^2-b^2)+a*(a^2-3*b^2)*tan(d*x+c))/(a^2+b^2)^3/d
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used

$= \{3597, 1661, 1643, 649, 209, 266\}$

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{a^2b}{2d\,(a^2+b^2)^2\,(a+b\tan(c+dx))^2} - \frac{2ab(a^2-b^2)}{d\,(a^2+b^2)^3\,(a+b\tan(c+dx))} - \frac{\cos^2(c+dx)\,(a(a^2-3b^2)\tan(c+dx)+b(3a^2-b^2))}{2d\,(a^2+b^2)^3} + \frac{b(3a^4-8a^2b^2+b^4)\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^4} + \frac{ax(a^4-14a^2b^2+9b^4)}{2\,(a^2+b^2)^4}$$

```
[In] Int[Sin[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (a*(a^4 - 14*a^2*b^2 + 9*b^4)*x)/(2*(a^2 + b^2)^4) + (b*(3*a^4 - 8*a^2*b^2
+ b^4)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^4*d) - (a^2*b)/(2
*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x])^2) - (2*a*b*(a^2 - b^2))/((a^2 + b^2)
^3*d*(a + b*Tan[c + d*x])) - (Cos[c + d*x]^2*(b*(3*a^2 - b^2) + a*(a^2 - 3*
b^2)*Tan[c + d*x]))/(2*(a^2 + b^2)^3*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{b\text{Subst}\left(\int \frac{x^2}{(a+x)^3(b^2+x^2)^2}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{\cos^2(c+dx)\left(b(3a^2-b^2)+a(a^2-3b^2)\tan(c+dx)\right)}{2\left(a^2+b^2\right)^3 d} - \frac{\text{Subst}\left(\int \frac{-\frac{a^4b^2\left(a^2-3b^2\right)}{\left(a^2+b^2\right)^3}+\frac{a^3b^2\left(3a^2+7b^2\right)x}{\left(a^2+b^2\right)^3}+\frac{b^2\left(3a^4-3a^2b^2-2b^4\right)x^2}{\left(a^2+b^2\right)^3}+\frac{ab^2\left(a^2-3b^2\right)x^3}{\left(a^2+b^2\right)^3}}{(a+x)^3(b^2+x^2)}\,dx, x, b\tan(c+dx)\right)}{2bd}$$

$$= -\frac{\cos^2(c+dx)\left(b(3a^2-b^2)+a(a^2-3b^2)\tan(c+dx)\right)}{2\left(a^2+b^2\right)^3 d} - \frac{\text{Subst}\left(\int \left(-\frac{2a^2b^2}{(a^2+b^2)^2(a+x)^3}+\frac{4ab^2\left(-a^2+b^2\right)}{(a^2+b^2)^3(a+x)^2}-\frac{2\left(3a^4b^2-8a^2b^4+b^6\right)}{(a^2+b^2)^4(a+x)}+\frac{b^2\left(-a\left(a^4-14a^2b^2+9b^4\right)+2\left(3a^4-8a^2b^2+b^4\right)\right.}{(a^2+b^2)^4(b^2+x^2)}\right.\right.}{2bd}$$

$$= \frac{b(3a^4-8a^2b^2+b^4)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^4 d} - \frac{a^2b}{2\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^2} - \frac{2ab(a^2-b^2)}{\left(a^2+b^2\right)^3 d(a+b\tan(c+dx))} - \frac{\cos^2(c+dx)\left(b(3a^2-b^2)+a(a^2-3b^2)\tan(c+dx)\right)}{2\left(a^2+b^2\right)^3 d} - \frac{b\text{Subst}\left(\int \frac{-a\left(a^4-14a^2b^2+9b^4\right)+2\left(3a^4-8a^2b^2+b^4\right)x}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{2\left(a^2+b^2\right)^4 d}$$

$$= \frac{b(3a^4-8a^2b^2+b^4)\log(a+b\tan(c+dx))}{(a^2+b^2)^4 d} - \frac{a^2b}{2(a^2+b^2)^2 d(a+b\tan(c+dx))^2} - \frac{2ab(a^2-b^2)}{(a^2+b^2)^3 d(a+b\tan(c+dx))} - \frac{\cos^2(c+dx)\left(b(3a^2-b^2)+a(a^2-3b^2)\tan(c+dx)\right)}{2(a^2+b^2)^3 d} - \frac{(b(3a^4-8a^2b^2+b^4))\operatorname{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{(a^2+b^2)^4 d} + \frac{(ab(a^4-14a^2b^2+9b^4))\operatorname{Subst}\left(\int \frac{1}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{2(a^2+b^2)^4 d}$$

$$= \frac{a(a^4-14a^2b^2+9b^4)x}{2(a^2+b^2)^4} + \frac{b(3a^4-8a^2b^2+b^4)\log(\cos(c+dx))}{(a^2+b^2)^4 d} + \frac{b(3a^4-8a^2b^2+b^4)\log(a+b\tan(c+dx))}{(a^2+b^2)^4 d} - \frac{a^2b}{2(a^2+b^2)^2 d(a+b\tan(c+dx))^2} - \frac{2ab(a^2-b^2)}{(a^2+b^2)^3 d(a+b\tan(c+dx))} - \frac{\cos^2(c+dx)\left(b(3a^2-b^2)+a(a^2-3b^2)\tan(c+dx)\right)}{2(a^2+b^2)^3 d}$$

Mathematica [A] (verified)

Time = 4.21 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.53

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{b\left(\frac{a(a^2-3b^2)(a^2+b^2)\arctan(\tan(c+dx))}{b} + (3a^2-b^2)(a^2+b^2)\cos^2(c+dx) + \left(3a^4-8a^2b^2+b^4-\frac{a^5-8a^3b^2+3ab^4}{\sqrt{-b^2}}\right)\right.}{}$$

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -1/2*(b*((a*(a^2 - 3*b^2)*(a^2 + b^2)*ArcTan[Tan[c + d*x]])/b + (3*a^2 - b^
2)*(a^2 + b^2)*Cos[c + d*x]^2 + (3*a^4 - 8*a^2*b^2 + b^4 - (a^5 - 8*a^3*b^2
 + 3*a*b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]] - 2*(3*a^4 - 8*a^2
*b^2 + b^4)*Log[a + b*Tan[c + d*x]] + (3*a^4 - 8*a^2*b^2 + b^4 + (a^5 - 8*a
^3*b^2 + 3*a*b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + (a*(a^2 -
3*b^2)*(a^2 + b^2)*Sin[2*(c + d*x)])/(2*b) + (a^2*(a^2 + b^2)^2)/(a + b*Tan
[c + d*x])^2 + (4*(a^5 - a*b^4))/(a + b*Tan[c + d*x])))/((a^2 + b^2)^4*d)
```

Maple [A] (verified)

Time = 5.73 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{-\frac{a^2 b}{2(a^2+b^2)^2(a+b\tan(dx+c))^2}+\frac{b(3a^4-8a^2b^2+b^4)\ln(a+b\tan(dx+c))}{(a^2+b^2)^4}-\frac{2ab(a^2-b^2)}{(a^2+b^2)^3(a+b\tan(dx+c))}+\frac{(-\frac{1}{2}a^5+a^3b^2+\frac{3}{2}ab^4)\tan(}{1+\tan^2(}}{d}$
default	$\frac{-\frac{a^2 b}{2(a^2+b^2)^2(a+b\tan(dx+c))^2}+\frac{b(3a^4-8a^2b^2+b^4)\ln(a+b\tan(dx+c))}{(a^2+b^2)^4}-\frac{2ab(a^2-b^2)}{(a^2+b^2)^3(a+b\tan(dx+c))}+\frac{(-\frac{1}{2}a^5+a^3b^2+\frac{3}{2}ab^4)\tan(}{1+\tan^2(}}{d}$
risch	$-\frac{ixb}{4ia^3b-4ia\,b^3-a^4+6a^2b^2-b^4}-\frac{xa}{2(4ia^3b-4ia\,b^3-a^4+6a^2b^2-b^4)}+\frac{ie^{2i(dx+c)}}{8(-3ib\,a^2+ib^3+a^3-3a\,b^2)d}-\frac{ie^{-2i(}}{8(3ib\,a^2-ib^3+}$

```
[In] int(sin(d*x+c)^2/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*a^2*b/(a^2+b^2)^2/(a+b*tan(d*x+c))^2+b*(3*a^4-8*a^2*b^2+b^4)/(a^2
+b^2)^4*ln(a+b*tan(d*x+c))-2*a*b*(a^2-b^2)/(a^2+b^2)^3/(a+b*tan(d*x+c))+1/(
a^2+b^2)^4*(((-1/2*a^5+a^3*b^2+3/2*a*b^4)*tan(d*x+c)-3/2*a^4*b-a^2*b^3+1/2*
b^5)/(1+tan(d*x+c)^2)+1/4*(-6*a^4*b+16*a^2*b^3-2*b^5)*ln(1+tan(d*x+c)^2)+1/
2*(a^5-14*a^3*b^2+9*a*b^4)*arctan(tan(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 526 vs. 2(200) = 400.

Time = 0.30 (sec) , antiderivative size = 526, normalized size of antiderivative = 2.55

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{13\,a^4b^3-8\,a^2b^5-b^7-2\,(a^6b+3\,a^4b^3+3\,a^2b^5+b^7)\cos{(dx+c)}^4+2\,(a^5b^2-14\,a^3b^4+9\,ab^6)dx-(a^6b+}{}$$

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(13*a^4*b^3 - 8*a^2*b^5 - b^7 - 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)
*cos(d*x + c)^4 + 2*(a^5*b^2 - 14*a^3*b^4 + 9*a*b^6)*d*x - (a^6*b + 23*a^4*
b^3 - 21*a^2*b^5 - 3*b^7 - 2*(a^7 - 15*a^5*b^2 + 23*a^3*b^4 - 9*a*b^6)*d*x)
*cos(d*x + c)^2 + 2*(3*a^4*b^3 - 8*a^2*b^5 + b^7 + (3*a^6*b - 11*a^4*b^3 +
9*a^2*b^5 - b^7)*cos(d*x + c)^2 + 2*(3*a^5*b^2 - 8*a^3*b^4 + a*b^6)*cos(d*x
 + c)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d
*x + c)^2 + b^2) - 2*((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(d*x + c)^3
- 2*(4*a^5*b^2 - 3*a^3*b^4 + 3*a*b^6 + (a^6*b - 14*a^4*b^3 + 9*a^2*b^5)*d*x
)*cos(d*x + c))*sin(d*x + c))/((a^10 + 3*a^8*b^2 + 2*a^6*b^4 - 2*a^4*b^6 -
3*a^2*b^8 - b^10)*d*cos(d*x + c)^2 + 2*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a
```

```
^3*b^7 + a*b^9)*d*cos(d*x + c)*sin(d*x + c) + (a^8*b^2 + 4*a^6*b^4 + 6*a^4*
b^6 + 4*a^2*b^8 + b^10)*d)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Exception raised: AttributeError}$$

```
[In] integrate(sin(d*x+c)**2/(a+b*tan(d*x+c))**3,x)
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 463 vs. $2(200) = 400$.

Time = 0.45 (sec) , antiderivative size = 463, normalized size of antiderivative = 2.25

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{(a^5-14\,a^3b^2+9\,ab^4)(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} + \frac{2\,(3\,a^4b-8\,a^2b^3+b^5)\log(b\tan(dx+c)+a)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} - \frac{(3\,a^4b-8\,a^2b^3+b^5)\log\left(\tan(dx+c)^2+1\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} - \frac{}{a^8+3\,a^6b^2+3\,a^4b^4+}}{}$$

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
[Out] 1/2*((a^5 - 14*a^3*b^2 + 9*a*b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
4*a^2*b^6 + b^8) + 2*(3*a^4*b - 8*a^2*b^3 + b^5)*log(b*tan(d*x + c) + a)/(a
^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - (3*a^4*b - 8*a^2*b^3 + b^5)
*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) -
(8*a^4*b - 4*a^2*b^3 + (5*a^3*b^2 - 7*a*b^4)*tan(d*x + c)^3 + (7*a^4*b - 6*
a^2*b^3 - b^5)*tan(d*x + c)^2 + (a^5 + 7*a^3*b^2 - 6*a*b^4)*tan(d*x + c))/(
a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 + (a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 +
b^8)*tan(d*x + c)^4 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*tan(d*x + c
)^3 + (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*tan(d*x + c)^2 + 2*(a
^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*tan(d*x + c)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 482 vs. $2(200) = 400$.

Time = 0.64 (sec) , antiderivative size = 482, normalized size of antiderivative = 2.34

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{(a^5-14a^3b^2+9ab^4)(dx+c)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8} - \frac{(3a^4b-8a^2b^3+b^5)\log\left(\tan(dx+c)^2+1\right)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8} + \frac{2(3a^4b^2-8a^2b^4+b^6)\log(|b\tan(dx+c)+a|)}{a^8b+4a^6b^3+6a^4b^5+4a^2b^7+b^9} + \frac{3a^4b\tan(dx+c)^2}{}}{}$$

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*((a^5 - 14*a^3*b^2 + 9*a*b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
4*a^2*b^6 + b^8) - (3*a^4*b - 8*a^2*b^3 + b^5)*log(tan(d*x + c)^2 + 1)/(a^8
 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 2*(3*a^4*b^2 - 8*a^2*b^4 + b^
6)*log(abs(b*tan(d*x + c) + a))/(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7
+ b^9) + (3*a^4*b*tan(d*x + c)^2 - 8*a^2*b^3*tan(d*x + c)^2 + b^5*tan(d*x +
 c)^2 - a^5*tan(d*x + c) + 2*a^3*b^2*tan(d*x + c) + 3*a*b^4*tan(d*x + c) -
10*a^2*b^3 + 2*b^5)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(tan(d
*x + c)^2 + 1)) - (9*a^4*b^3*tan(d*x + c)^2 - 24*a^2*b^5*tan(d*x + c)^2 + 3
*b^7*tan(d*x + c)^2 + 22*a^5*b^2*tan(d*x + c) - 48*a^3*b^4*tan(d*x + c) + 2
*a*b^6*tan(d*x + c) + 14*a^6*b - 22*a^4*b^3)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4
+ 4*a^2*b^6 + b^8)*(b*tan(d*x + c) + a)^2))/d
```

Mupad [B] (verification not implemented)

Time = 5.52 (sec) , antiderivative size = 433, normalized size of antiderivative = 2.10

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{\tan(c+d\,x)^2\,\left(-7\,a^4\,b+6\,a^2\,b^3+b^5\right)}{2\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{\tan(c+d\,x)^3\,\left(7\,a\,b^4-5\,a^3\,b^2\right)}{2\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}-\frac{2\,a^2\,\left(2\,a^2\,b-b^3\right)}{\left(a^2+b^2\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}-\frac{a\tan(c+d\,x)\,\left(a^4+7\,a^2\,b^2-6\,b^4\right)}{2\,\left(a^2+b^2\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}}{d\,\left({\tan\left(c+d\,x\right)}^2\,\left(a^2+b^2\right)+a^2+b^2\,{\tan\left(c+d\,x\right)}^4+2\,a\,b\,\tan\left(c+d\,x\right)+2\,a\,b\,{\tan\left(c+d\,x\right)}^3\right)} + \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(\frac{3\,b}{{\left(a^2+b^2\right)}^2}-\frac{14\,b^3}{{\left(a^2+b^2\right)}^3}+\frac{12\,b^5}{{\left(a^2+b^2\right)}^4}\right)}{d} + \frac{\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)\,\left(-2\,b+a\,1{}\mathrm{i}\right)}{4\,d\,\left(a^4-a^3\,b\,4{}\mathrm{i}-6\,a^2\,b^2+a\,b^3\,4{}\mathrm{i}+b^4\right)} + \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(a-b\,2{}\mathrm{i}\right)}{4\,d\,\left(a^4\,1{}\mathrm{i}-4\,a^3\,b-a^2\,b^2\,6{}\mathrm{i}+4\,a\,b^3+b^4\,1{}\mathrm{i}\right)}$$

```
[In] int(sin(c + d*x)^2/(a + b*tan(c + d*x))^3,x)
```

```
[Out] ((tan(c + d*x)^2*(b^5 - 7*a^4*b + 6*a^2*b^3))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3
*a^4*b^2)) + (tan(c + d*x)^3*(7*a*b^4 - 5*a^3*b^2))/(2*(a^6 + b^6 + 3*a^2*b
^4 + 3*a^4*b^2)) - (2*a^2*(2*a^2*b - b^3))/((a^2 + b^2)*(a^4 + b^4 + 2*a^2*
```

```
b^2)) - (a*tan(c + d*x)*(a^4 - 6*b^4 + 7*a^2*b^2))/(2*(a^2 + b^2)*(a^4 + b^
4 + 2*a^2*b^2)))/(d*(tan(c + d*x)^2*(a^2 + b^2) + a^2 + b^2*tan(c + d*x)^4
+ 2*a*b*tan(c + d*x) + 2*a*b*tan(c + d*x)^3)) + (log(a + b*tan(c + d*x))*((
3*b)/(a^2 + b^2)^2 - (14*b^3)/(a^2 + b^2)^3 + (12*b^5)/(a^2 + b^2)^4))/d +
(log(tan(c + d*x) + 1i)*(a*1i - 2*b))/(4*d*(a*b^3*4i - a^3*b*4i + a^4 + b^4
 - 6*a^2*b^2)) + (log(tan(c + d*x) - 1i)*(a - b*2i))/(4*d*(4*a*b^3 - 4*a^3*
b + a^4*1i + b^4*1i - a^2*b^2*6i))
```

3.70 $\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^3} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 95

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^3} \, dx = -\frac{\cot(c+dx)}{a^3d} - \frac{3b\log(\tan(c+dx))}{a^4d} + \frac{3b\log(a+b\tan(c+dx))}{a^4d} - \frac{b}{2a^2d(a+b\tan(c+dx))^2} - \frac{2b}{a^3d(a+b\tan(c+dx))}$$

```
[Out] -cot(d*x+c)/a^3/d-3*b*ln(tan(d*x+c))/a^4/d+3*b*ln(a+b*tan(d*x+c))/a^4/d-1/2
*b/a^2/d/(a+b*tan(d*x+c))^2-2*b/a^3/d/(a+b*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3597, 46}

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^3} \, dx = -\frac{3b\log(\tan(c+dx))}{a^4d} + \frac{3b\log(a+b\tan(c+dx))}{a^4d} - \frac{2b}{a^3d(a+b\tan(c+dx))} - \frac{\cot(c+dx)}{a^3d} - \frac{b}{2a^2d(a+b\tan(c+dx))^2}$$

```
[In] Int[Csc[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -(Cot[c + d*x]/(a^3*d)) - (3*b*Log[Tan[c + d*x]])/(a^4*d) + (3*b*Log[a + b*
Tan[c + d*x]])/(a^4*d) - b/(2*a^2*d*(a + b*Tan[c + d*x])^2) - (2*b)/(a^3*d*
(a + b*Tan[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{b\text{Subst}\left(\int \frac{1}{x^2(a+x)^3}\,dx, x, b\tan(c+dx)\right)}{d}\\ &= \frac{b\text{Subst}\left(\int \left(\frac{1}{a^3x^2}-\frac{3}{a^4x}+\frac{1}{a^2(a+x)^3}+\frac{2}{a^3(a+x)^2}+\frac{3}{a^4(a+x)}\right)dx, x, b\tan(c+dx)\right)}{d}\\ &= -\frac{\cot(c+dx)}{a^3d}-\frac{3b\log(\tan(c+dx))}{a^4d}+\frac{3b\log(a+b\tan(c+dx))}{a^4d}\\ &\quad -\frac{b}{2a^2d(a+b\tan(c+dx))^2}-\frac{2b}{a^3d(a+b\tan(c+dx))}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 241 vs. $2(95) = 190$.

Time = 4.19 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.54

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{-2a^3(a^2+b^2)\cot(c+dx)+b(-2a^2(a^2+b^2)(2+3\log(\sin(c+dx))-3\log(a\cos(c+dx)+b\sin(c+dx))}{}$$

```
[In] Integrate[Csc[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-2*a^3*(a^2 + b^2)*Cot[c + d*x] + b*(-2*a^2*(a^2 + b^2)*(2 + 3*Log[Sin[c +
 d*x]] - 3*Log[a*Cos[c + d*x] + b*Sin[c + d*x]]) - a^2*b^2*Sec[c + d*x]^2 +
 2*a*b*(2*a^2 + b^2 - 6*(a^2 + b^2)*Log[Sin[c + d*x]] + 6*(a^2 + b^2)*Log[a
*Cos[c + d*x] + b*Sin[c + d*x]])*Tan[c + d*x] - 2*b^2*(-3*a^2 - 2*b^2 + 3*(
a^2 + b^2)*Log[Sin[c + d*x]] - 3*(a^2 + b^2)*Log[a*Cos[c + d*x] + b*Sin[c +
 d*x]])*Tan[c + d*x]^2))/(2*a^4*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2)
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{-\frac{1}{a^3\tan(dx+c)}-\frac{3b\ln(\tan(dx+c))}{a^4}-\frac{b}{2a^2(a+b\tan(dx+c))^2}+\frac{3b\ln(a+b\tan(dx+c))}{a^4}-\frac{2b}{a^3(a+b\tan(dx+c))}}{d}$
default	$\frac{-\frac{1}{a^3\tan(dx+c)}-\frac{3b\ln(\tan(dx+c))}{a^4}-\frac{b}{2a^2(a+b\tan(dx+c))^2}+\frac{3b\ln(a+b\tan(dx+c))}{a^4}-\frac{2b}{a^3(a+b\tan(dx+c))}}{d}$
risch	$-\frac{2i\left(a^4e^{4i(dx+c)}-9a^2b^2e^{4i(dx+c)}+3b^4e^{4i(dx+c)}-4ia^3b\,e^{4i(dx+c)}+9ia\,b^3e^{4i(dx+c)}+2a^4e^{2i(dx+c)}-6b^4e^{2i(dx+c)}-4ia^3b\right.}{\left(e^{2i(dx+c)}-1\right)\left(-ib\,e^{2i(dx+c)}+a\,e^{2i(dx+c)}+ib+a\right)^2(-ib+a)^2a^3d}$

[In] int(csc(d*x+c)^2/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(-1/a^3/tan(d*x+c)-3/a^4*b*ln(tan(d*x+c))-1/2/a^2*b/(a+b*tan(d*x+c))^2+3/a^4*b*ln(a+b*tan(d*x+c))-2/a^3*b/(a+b*tan(d*x+c)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 565 vs. 2(93) = 186.

Time = 0.28 (sec) , antiderivative size = 565, normalized size of antiderivative = 5.95

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{2\left(a^7+4\,a^5b^2-2\,a^3b^4-3\,ab^6\right)\cos\left(dx+c\right)^3-2\left(2\,a^5b^2-3\,a^3b^4-3\,ab^6\right)\cos\left(dx+c\right)+3\left(2\left(a^5b^2+2\,a^3b^4\right.\right.}{}$$

[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="fricas")

[Out] 1/2*(2*(a^7 + 4*a^5*b^2 - 2*a^3*b^4 - 3*a*b^6)*cos(d*x + c)^3 - 2*(2*a^5*b^2 - 3*a^3*b^4 - 3*a*b^6)*cos(d*x + c) + 3*(2*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cos(d*x + c)^3 - 2*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cos(d*x + c) - (a^4*b^3 + 2*a^2*b^5 + b^7 + (a^6*b + a^4*b^3 - a^2*b^5 - b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 3*(2*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cos(d*x + c)^3 - 2*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cos(d*x + c) - (a^4*b^3 + 2*a^2*b^5 + b^7 + (a^6*b + a^4*b^3 - a^2*b^5 - b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/4*cos(d*x + c)^2 + 1/4) - (5*a^4*b^3 + 3*a^2*b^5 - 4*(a^6*b + 5*a^4*b^3 + 3*a^2*b^5)*cos(d*x + c)^2)*sin(d*x + c))/(2*(a^9*b + 2*a^7*b^3 + a^5*b^5)*d*cos(d*x + c)^3 - 2*(a^9*b + 2*a^7*b^3 + a^5*b^5)*d*cos(d*x + c) - ((a^10 + a^8*b^2 - a^6*b^4 - a^4*b^6)*d*cos(d*x + c)^2 + (a^8*b^2 + 2*a^6*b^4 + a^4*b^6)*d)*sin(d*x + c))

Sympy [F]

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

```
[In] integrate(csc(d*x+c)**2/(a+b*tan(d*x+c))**3,x)
[Out] Integral(csc(c + d*x)**2/(a + b*tan(c + d*x))**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.57 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.14

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{\frac{6\,b^2\tan(dx+c)^2+9\,ab\tan(dx+c)+2\,a^2}{a^3b^2\tan(dx+c)^3+2\,a^4b\tan(dx+c)^2+a^5\tan(dx+c)} - \frac{6\,b\log(b\tan(dx+c)+a)}{a^4} + \frac{6\,b\log(\tan(dx+c))}{a^4}}{2\,d}$$

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
[Out] -1/2*((6*b^2*tan(d*x + c)^2 + 9*a*b*tan(d*x + c) + 2*a^2)/(a^3*b^2*tan(d*x
 + c)^3 + 2*a^4*b*tan(d*x + c)^2 + a^5*tan(d*x + c)) - 6*b*log(b*tan(d*x + c
) + a)/a^4 + 6*b*log(tan(d*x + c))/a^4)/d
```

Giac [A] (verification not implemented)

none

Time = 0.57 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.19

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{6\,b\log(|b\tan(dx+c)+a|)}{a^4} - \frac{6\,b\log(|\tan(dx+c)|)}{a^4} + \frac{2\,(3\,b\tan(dx+c)-a)}{a^4\tan(dx+c)} - \frac{9\,b^3\tan(dx+c)^2+22\,ab^2\tan(dx+c)+14\,a^2b}{(b\tan(dx+c)+a)^2a^4}}{2\,d}$$

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="giac")
[Out] 1/2*(6*b*log(abs(b*tan(d*x + c) + a))/a^4 - 6*b*log(abs(tan(d*x + c)))/a^4
+ 2*(3*b*tan(d*x + c) - a)/(a^4*tan(d*x + c)) - (9*b^3*tan(d*x + c)^2 + 22*
a*b^2*tan(d*x + c) + 14*a^2*b)/((b*tan(d*x + c) + a)^2*a^4))/d
```

Mupad [B] (verification not implemented)

Time = 4.87 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{6\,b\,\mathrm{atanh}\left(\frac{2\,b\tan(c+d\,x)}{a}+1\right)}{a^4\,d} - \frac{\frac{1}{a}+\frac{3\,b^2\tan(c+d\,x)^2}{a^3}+\frac{9\,b\tan(c+d\,x)}{2\,a^2}}{d\,\left(a^2\tan\left(c+d\,x\right)+2\,a\,b\tan\left(c+d\,x\right)^2+b^2\tan\left(c+d\,x\right)^3\right)}$$

```
[In] int(1/(sin(c + d*x)^2*(a + b*tan(c + d*x))^3),x)
```

```
[Out] (6*b*atanh((2*b*tan(c + d*x))/a + 1))/(a^4*d) - (1/a + (3*b^2*tan(c + d*x)^
2)/a^3 + (9*b*tan(c + d*x))/(2*a^2))/(d*(a^2*tan(c + d*x) + b^2*tan(c + d*x
)^3 + 2*a*b*tan(c + d*x)^2))
```

3.71 $\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 178

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{(a^2+6b^2)\cot(c+dx)}{a^5d}+\frac{3b\cot^2(c+dx)}{2a^4d}-\frac{\cot^3(c+dx)}{3a^3d}-\frac{b(3a^2+10b^2)\log(\tan(c+dx))}{a^6d}+\frac{b(3a^2+10b^2)\log(a+b\tan(c+dx))}{a^6d}-\frac{b(a^2+b^2)}{2a^4d(a+b\tan(c+dx))^2}-\frac{2b(a^2+2b^2)}{a^5d(a+b\tan(c+dx))}$$

```
[Out] -(a^2+6*b^2)*cot(d*x+c)/a^5/d+3/2*b*cot(d*x+c)^2/a^4/d-1/3*cot(d*x+c)^3/a^3
/d-b*(3*a^2+10*b^2)*ln(tan(d*x+c))/a^6/d+b*(3*a^2+10*b^2)*ln(a+b*tan(d*x+c)
)/a^6/d-1/2*b*(a^2+b^2)/a^4/d/(a+b*tan(d*x+c))^2-2*b*(a^2+2*b^2)/a^5/d/(a+b
*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used

$= \{3597, 908\}$

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{3b\cot^2(c+dx)}{2a^4d} - \frac{\cot^3(c+dx)}{3a^3d} - \frac{b(3a^2+10b^2)\log(\tan(c+dx))}{a^6d} + \frac{b(3a^2+10b^2)\log(a+b\tan(c+dx))}{a^6d} - \frac{2b(a^2+2b^2)}{a^5d(a+b\tan(c+dx))} - \frac{(a^2+6b^2)\cot(c+dx)}{a^5d} - \frac{b(a^2+b^2)}{2a^4d(a+b\tan(c+dx))^2}$$

```
[In] Int[Csc[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -(((a^2 + 6*b^2)*Cot[c + d*x])/(a^5*d)) + (3*b*Cot[c + d*x]^2)/(2*a^4*d) -
Cot[c + d*x]^3/(3*a^3*d) - (b*(3*a^2 + 10*b^2)*Log[Tan[c + d*x]])/(a^6*d) +
 (b*(3*a^2 + 10*b^2)*Log[a + b*Tan[c + d*x]])/(a^6*d) - (b*(a^2 + b^2))/(2*
a^4*d*(a + b*Tan[c + d*x])^2) - (2*b*(a^2 + 2*b^2))/(a^5*d*(a + b*Tan[c + d
*x]))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{b\text{Subst}\left(\int \frac{b^2+x^2}{x^4(a+x)^3}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{b\text{Subst}\left(\int \left(\frac{b^2}{a^3x^4} - \frac{3b^2}{a^4x^3} + \frac{a^2+6b^2}{a^5x^2} + \frac{-3a^2-10b^2}{a^6x} + \frac{a^2+b^2}{a^4(a+x)^3} + \frac{2(a^2+2b^2)}{a^5(a+x)^2} + \frac{3a^2+10b^2}{a^6(a+x)}\right)\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{(a^2+6b^2)\cot(c+dx)}{a^5d} + \frac{3b\cot^2(c+dx)}{2a^4d} - \frac{\cot^3(c+dx)}{3a^3d} - \frac{b(3a^2+10b^2)\log(\tan(c+dx))}{a^6d} + \frac{b(3a^2+10b^2)\log(a+b\tan(c+dx))}{a^6d} - \frac{b(a^2+b^2)}{2a^4d(a+b\tan(c+dx))^2} - \frac{2b(a^2+2b^2)}{a^5d(a+b\tan(c+dx))}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 456 vs. $2(178) = 356$.

Time = 6.67 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.56

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{b^3\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))}{2a^4d(a+b\tan(c+dx))^3}$$
$$-\frac{\csc^3(c+dx)\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3}{3a^3d(a+b\tan(c+dx))^3}$$
$$-\frac{2(a^2\cos(c+dx)+9b^2\cos(c+dx))\csc(c+dx)\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3}{3a^5d(a+b\tan(c+dx))^3}$$
$$+\frac{3b\csc^2(c+dx)\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3}{2a^4d(a+b\tan(c+dx))^3}$$
$$+\frac{(-3a^2b-10b^3)\log(\sin(c+dx))\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3}{a^6d(a+b\tan(c+dx))^3}$$
$$+\frac{(3a^2b+10b^3)\log(a\cos(c+dx)+b\sin(c+dx))\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3}{a^6d(a+b\tan(c+dx))^3}$$
$$+\frac{\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\left(3a^2b^2\sin(c+dx)+4b^4\sin(c+dx)\right)}{a^6d(a+b\tan(c+dx))^3}$$

```
[In] Integrate[Csc[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -1/2*(b^3*Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x]))/(a^4*d*(a + b*T
an[c + d*x])^3) - (Csc[c + d*x]^3*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c
+ d*x])^3)/(3*a^3*d*(a + b*Tan[c + d*x])^3) - (2*(a^2*Cos[c + d*x] + 9*b^2*
Cos[c + d*x])*Csc[c + d*x]*Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])
^3)/(3*a^5*d*(a + b*Tan[c + d*x])^3) + (3*b*Csc[c + d*x]^2*Sec[c + d*x]^3*(
a*Cos[c + d*x] + b*Sin[c + d*x])^3)/(2*a^4*d*(a + b*Tan[c + d*x])^3) + ((-3
*a^2*b - 10*b^3)*Log[Sin[c + d*x]]*Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c
 + d*x])^3)/(a^6*d*(a + b*Tan[c + d*x])^3) + ((3*a^2*b + 10*b^3)*Log[a*Cos[
c + d*x] + b*Sin[c + d*x]]*Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])
^3)/(a^6*d*(a + b*Tan[c + d*x])^3) + (Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Si
n[c + d*x])^2*(3*a^2*b^2*Sin[c + d*x] + 4*b^4*Sin[c + d*x]))/(a^6*d*(a + b*
Tan[c + d*x])^3)
```

Maple [A] (verified)

Time = 2.47 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{-\frac{1}{3a^3\tan(dx+c)^3}-\frac{a^2+6b^2}{a^5\tan(dx+c)}+\frac{3b}{2a^4\tan(dx+c)^2}-\frac{b\left(3a^2+10b^2\right)\ln(\tan(dx+c))}{a^6}+\frac{b\left(3a^2+10b^2\right)\ln(a+b\tan(dx+c))}{a^6}-\frac{\left(a^2+\right.}{2a^4(a+b\tan}}{d}$
default	$\frac{-\frac{1}{3a^3\tan(dx+c)^3}-\frac{a^2+6b^2}{a^5\tan(dx+c)}+\frac{3b}{2a^4\tan(dx+c)^2}-\frac{b\left(3a^2+10b^2\right)\ln(\tan(dx+c))}{a^6}+\frac{b\left(3a^2+10b^2\right)\ln(a+b\tan(dx+c))}{a^6}-\frac{\left(a^2+\right.}{2a^4(a+b\tan}}{d}$
risch	$-\frac{2i\left(29a^2b^3-2ia^5+30b^5+90ia\,b^4e^{4i(dx+c)}+18ia^3b^2e^{8i(dx+c)}-150ia\,b^4e^{6i(dx+c)}-9a^4b\,e^{8i(dx+c)}-a^4b\,e^{2i(dx+c)}+15a^4\right.}{}$

```
[In] int(csc(d*x+c)^4/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3/a^3/tan(d*x+c)^3-(a^2+6*b^2)/a^5/tan(d*x+c)+3/2/a^4*b/tan(d*x+c)^
2-b*(3*a^2+10*b^2)/a^6*ln(tan(d*x+c))+b*(3*a^2+10*b^2)/a^6*ln(a+b*tan(d*x+c
))-1/2*(a^2+b^2)*b/a^4/(a+b*tan(d*x+c))^2-2*b*(a^2+2*b^2)/a^5/(a+b*tan(d*x+
c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 811 vs. $2(172) = 344$.

Time = 0.30 (sec) , antiderivative size = 811, normalized size of antiderivative = 4.56

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$=\frac{2\left(2\,a^7+27\,a^5b^2+a^3b^4-30\,ab^6\right)\cos\left(dx+c\right)^5-2\left(3\,a^7+43\,a^5b^2-8\,a^3b^4-60\,ab^6\right)\cos\left(dx+c\right)^3+6\left(5\,a\right.}{}$$

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(2*a^7 + 27*a^5*b^2 + a^3*b^4 - 30*a*b^6)*cos(d*x + c)^5 - 2*(3*a^7
+ 43*a^5*b^2 - 8*a^3*b^4 - 60*a*b^6)*cos(d*x + c)^3 + 6*(5*a^5*b^2 - 3*a^3*
b^4 - 10*a*b^6)*cos(d*x + c) + 3*(2*(3*a^5*b^2 + 13*a^3*b^4 + 10*a*b^6)*cos
(d*x + c)^5 - 4*(3*a^5*b^2 + 13*a^3*b^4 + 10*a*b^6)*cos(d*x + c)^3 + 2*(3*a
^5*b^2 + 13*a^3*b^4 + 10*a*b^6)*cos(d*x + c) + (3*a^4*b^3 + 13*a^2*b^5 + 10
*b^7 - (3*a^6*b + 10*a^4*b^3 - 3*a^2*b^5 - 10*b^7)*cos(d*x + c)^4 + (3*a^6*
b + 7*a^4*b^3 - 16*a^2*b^5 - 20*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(2*a*
b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 3*(2*(3*a
^5*b^2 + 13*a^3*b^4 + 10*a*b^6)*cos(d*x + c)^5 - 4*(3*a^5*b^2 + 13*a^3*b^4
+ 10*a*b^6)*cos(d*x + c)^3 + 2*(3*a^5*b^2 + 13*a^3*b^4 + 10*a*b^6)*cos(d*x
+ c) + (3*a^4*b^3 + 13*a^2*b^5 + 10*b^7 - (3*a^6*b + 10*a^4*b^3 - 3*a^2*b^5
 - 10*b^7)*cos(d*x + c)^4 + (3*a^6*b + 7*a^4*b^3 - 16*a^2*b^5 - 20*b^7)*cos
(d*x + c)^2)*sin(d*x + c))*log(-1/4*cos(d*x + c)^2 + 1/4) + (24*a^4*b^3 + 3
```

```
0*a^2*b^5 + 4*(2*a^6*b + 29*a^4*b^3 + 30*a^2*b^5)*cos(d*x + c)^4 - 3*(a^6*b
 + 45*a^4*b^3 + 50*a^2*b^5)*cos(d*x + c)^2)*sin(d*x + c))/(2*(a^9*b + a^7*b
^3)*d*cos(d*x + c)^5 - 4*(a^9*b + a^7*b^3)*d*cos(d*x + c)^3 + 2*(a^9*b + a^
7*b^3)*d*cos(d*x + c) - ((a^10 - a^6*b^4)*d*cos(d*x + c)^4 - (a^10 - a^8*b^
2 - 2*a^6*b^4)*d*cos(d*x + c)^2 - (a^8*b^2 + a^6*b^4)*d)*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

```
[In] integrate(csc(d*x+c)**4/(a+b*tan(d*x+c))**3,x)
[Out] Integral(csc(c + d*x)**4/(a + b*tan(c + d*x))**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.08

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{5a^3b\tan(dx+c)-6(3a^2b^2+10b^4)\tan(dx+c)^4-2a^4-9(3a^3b+10ab^3)\tan(dx+c)^3-2(3a^4+10a^2b^2)\tan(dx+c)^2}{a^5b^2\tan(dx+c)^5+2a^6b\tan(dx+c)^4+a^7\tan(dx+c)^3} + \frac{6(3a^2b+10b^3)\log(b\tan(dx}{a^6}}{6d}$$

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
[Out] 1/6*((5*a^3*b*tan(d*x + c) - 6*(3*a^2*b^2 + 10*b^4)*tan(d*x + c)^4 - 2*a^4
- 9*(3*a^3*b + 10*a*b^3)*tan(d*x + c)^3 - 2*(3*a^4 + 10*a^2*b^2)*tan(d*x +
c)^2)/(a^5*b^2*tan(d*x + c)^5 + 2*a^6*b*tan(d*x + c)^4 + a^7*tan(d*x + c)^3
) + 6*(3*a^2*b + 10*b^3)*log(b*tan(d*x + c) + a)/a^6 - 6*(3*a^2*b + 10*b^3)
*log(tan(d*x + c))/a^6)/d
```

Giac [A] (verification not implemented)

none

Time = 0.60 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.33

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{\frac{6(3a^2b+10b^3)\log(|\tan(dx+c)|)}{a^6} - \frac{6(3a^2b^2+10b^4)\log(|b\tan(dx+c)+a|)}{a^6b} + \frac{3\left(9a^2b^3\tan(dx+c)^2+30b^5\tan(dx+c)^2+22a^3b^2\tan(dx+c)+6\right.}{(b\tan(dx+c)+a)^2a^6}}{6d}$$

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(6*(3*a^2*b + 10*b^3)*log(abs(tan(d*x + c)))/a^6 - 6*(3*a^2*b^2 + 10*b
^4)*log(abs(b*tan(d*x + c) + a))/(a^6*b) + 3*(9*a^2*b^3*tan(d*x + c)^2 + 30
*b^5*tan(d*x + c)^2 + 22*a^3*b^2*tan(d*x + c) + 68*a*b^4*tan(d*x + c) + 14*
a^4*b + 39*a^2*b^3)/((b*tan(d*x + c) + a)^2*a^6) - (33*a^2*b*tan(d*x + c)^3
 + 110*b^3*tan(d*x + c)^3 - 6*a^3*tan(d*x + c)^2 - 36*a*b^2*tan(d*x + c)^2
+ 9*a^2*b*tan(d*x + c) - 2*a^3)/(a^6*tan(d*x + c)^3))/d
```

Mupad [B] (verification not implemented)

Time = 5.05 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.12

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{2\,b\,\mathrm{atanh}\left(\frac{b\,(3\,a^2+10\,b^2)\,(a+2\,b\tan(c+d\,x))}{a\,(3\,a^2\,b+10\,b^3)}\right)\,(3\,a^2+10\,b^2)}{a^6\,d} - \frac{\frac{1}{3\,a}+\frac{\tan(c+d\,x)^2\,(3\,a^2+10\,b^2)}{3\,a^3}-\frac{5\,b\tan(c+d\,x)}{6\,a^2}+\frac{b^2\tan(c+d\,x)^4\,(3\,a^2+10\,b^2)}{a^5}+\frac{3\,b\tan(c+d\,x)^3\,(3\,a^2+10\,b^2)}{2\,a^4}}{d\,\left(a^2\tan\left(c+d\,x\right)^3+2\,a\,b\tan\left(c+d\,x\right)^4+b^2\tan\left(c+d\,x\right)^5\right)}$$

```
[In] int(1/(sin(c + d*x)^4*(a + b*tan(c + d*x))^3),x)
```

```
[Out] (2*b*atanh((b*(3*a^2 + 10*b^2)*(a + 2*b*tan(c + d*x)))/(a*(3*a^2*b + 10*b^3
)))*(3*a^2 + 10*b^2))/(a^6*d) - (1/(3*a) + (tan(c + d*x)^2*(3*a^2 + 10*b^2)
)/(3*a^3) - (5*b*tan(c + d*x))/(6*a^2) + (b^2*tan(c + d*x)^4*(3*a^2 + 10*b^
2))/a^5 + (3*b*tan(c + d*x)^3*(3*a^2 + 10*b^2))/(2*a^4))/(d*(a^2*tan(c + d*
x)^3 + b^2*tan(c + d*x)^5 + 2*a*b*tan(c + d*x)^4))
```

3.72 $\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 265

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{(a^4+12a^2b^2+15b^4)\cot(c+dx)}{a^7d} + \frac{b(3a^2+5b^2)\cot^2(c+dx)}{a^6d} - \frac{2(a^2+3b^2)\cot^3(c+dx)}{3a^5d} + \frac{3b\cot^4(c+dx)}{4a^4d} - \frac{\cot^5(c+dx)}{5a^3d} - \frac{b(3a^4+20a^2b^2+21b^4)\log(\tan(c+dx))}{a^8d} + \frac{b(3a^4+20a^2b^2+21b^4)\log(a+b\tan(c+dx))}{a^8d} - \frac{b(a^2+b^2)^2}{2a^6d(a+b\tan(c+dx))^2} - \frac{2b(a^2+b^2)(a^2+3b^2)}{a^7d(a+b\tan(c+dx))}$$

```
[Out] -(a^4+12*a^2*b^2+15*b^4)*cot(d*x+c)/a^7/d+b*(3*a^2+5*b^2)*cot(d*x+c)^2/a^6/
d-2/3*(a^2+3*b^2)*cot(d*x+c)^3/a^5/d+3/4*b*cot(d*x+c)^4/a^4/d-1/5*cot(d*x+c
)^5/a^3/d-b*(3*a^4+20*a^2*b^2+21*b^4)*ln(tan(d*x+c))/a^8/d+b*(3*a^4+20*a^2*
b^2+21*b^4)*ln(a+b*tan(d*x+c))/a^8/d-1/2*b*(a^2+b^2)^2/a^6/d/(a+b*tan(d*x+c
))^2-2*b*(a^2+b^2)*(a^2+3*b^2)/a^7/d/(a+b*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used

$= \{3597, 908\}$

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{3b\cot^4(c+dx)}{4a^4d} - \frac{\cot^5(c+dx)}{5a^3d} - \frac{2b(a^2+b^2)\,(a^2+3b^2)}{a^7d(a+b\tan(c+dx))} - \frac{b(a^2+b^2)^2}{2a^6d(a+b\tan(c+dx))^2} + \frac{b(3a^2+5b^2)\cot^2(c+dx)}{a^6d} - \frac{2(a^2+3b^2)\cot^3(c+dx)}{3a^5d} - \frac{b(3a^4+20a^2b^2+21b^4)\log(\tan(c+dx))}{a^8d} + \frac{b(3a^4+20a^2b^2+21b^4)\log(a+b\tan(c+dx))}{a^8d} - \frac{(a^4+12a^2b^2+15b^4)\cot(c+dx)}{a^7d}$$

```
[In] Int[Csc[c + d*x]^6/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -(((a^4 + 12*a^2*b^2 + 15*b^4)*Cot[c + d*x])/(a^7*d)) + (b*(3*a^2 + 5*b^2)*
Cot[c + d*x]^2)/(a^6*d) - (2*(a^2 + 3*b^2)*Cot[c + d*x]^3)/(3*a^5*d) + (3*b
*Cot[c + d*x]^4)/(4*a^4*d) - Cot[c + d*x]^5/(5*a^3*d) - (b*(3*a^4 + 20*a^2*
b^2 + 21*b^4)*Log[Tan[c + d*x]])/(a^8*d) + (b*(3*a^4 + 20*a^2*b^2 + 21*b^4)
*Log[a + b*Tan[c + d*x]])/(a^8*d) - (b*(a^2 + b^2)^2)/(2*a^6*d*(a + b*Tan[c
 + d*x])^2) - (2*b*(a^2 + b^2)*(a^2 + 3*b^2))/(a^7*d*(a + b*Tan[c + d*x]))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{b\text{Subst}\left(\int \frac{(b^2+x^2)^2}{x^6(a+x)^3}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{b\text{Subst}\left(\int \left(\frac{b^4}{a^3x^6} - \frac{3b^4}{a^4x^5} + \frac{2b^2(a^2+3b^2)}{a^5x^4} - \frac{2(3a^2b^2+5b^4)}{a^6x^3} + \frac{a^4+12a^2b^2+15b^4}{a^7x^2} + \frac{-3a^4-20a^2b^2-21b^4}{a^8x} + \frac{(a^2+b^2)^2}{a^6(a+x)^3} + \right.\right.}{d}$$

$$= -\frac{(a^4+12a^2b^2+15b^4)\cot(c+dx)}{a^7d} + \frac{b(3a^2+5b^2)\cot^2(c+dx)}{a^6d} - \frac{2(a^2+3b^2)\cot^3(c+dx)}{3a^5d} + \frac{3b\cot^4(c+dx)}{4a^4d} - \frac{\cot^5(c+dx)}{5a^3d} - \frac{b(3a^4+20a^2b^2+21b^4)\log(\tan(c+dx))}{a^8d} + \frac{b(3a^4+20a^2b^2+21b^4)\log(a+b\tan(c+dx))}{a^8d} - \frac{b(a^2+b^2)^2}{2a^6d(a+b\tan(c+dx))^2} - \frac{2b(a^2+b^2)(a^2+3b^2)}{a^7d(a+b\tan(c+dx))}$$

Mathematica [A] (verified)

Time = 6.95 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.86

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{\csc^5(c+dx)\left(\sec^2(c+dx)\left((8a^7+567a^5b^2+630a^3b^4-1215ab^6)\cos(3(c+dx))-(24a^7+619a^5b^2+63\right.\right.}{}$$

```
[In] Integrate[Csc[c + d*x]^6/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -1/960*(Csc[c + d*x]^5*(Sec[c + d*x]^2*((8*a^7 + 567*a^5*b^2 + 630*a^3*b^4
- 1215*a*b^6)*Cos[3*(c + d*x)] - (24*a^7 + 619*a^5*b^2 + 630*a^3*b^4 - 675*
a*b^6)*Cos[5*(c + d*x)] + 8*a^7*Cos[7*(c + d*x)] + 187*a^5*b^2*Cos[7*(c + d
*x)] + 210*a^3*b^4*Cos[7*(c + d*x)] - 135*a*b^6*Cos[7*(c + d*x)] - 126*a^6*
b*Sin[3*(c + d*x)] + 1665*a^4*b^3*Sin[3*(c + d*x)] + 4635*a^2*b^5*Sin[3*(c
+ d*x)] + 1890*b^7*Sin[3*(c + d*x)] + 10*a^6*b*Sin[5*(c + d*x)] - 1215*a^4*
b^3*Sin[5*(c + d*x)] - 2565*a^2*b^5*Sin[5*(c + d*x)] - 630*b^7*Sin[5*(c + d
*x)] + 16*a^6*b*Sin[7*(c + d*x)] + 345*a^4*b^3*Sin[7*(c + d*x)] + 585*a^2*b
^5*Sin[7*(c + d*x)] + 90*b^7*Sin[7*(c + d*x)]) + 960*b*(3*a^4 + 20*a^2*b^2
+ 21*b^4)*(Log[Sin[c + d*x]] - Log[a*Cos[c + d*x] + b*Sin[c + d*x]])*Sin[c
+ d*x]^5*(a + b*Tan[c + d*x])^2 + 5*Sec[c + d*x]*(40*a^7 - 27*a^5*b^2 - 42*
a^3*b^4 + 135*a*b^6 - 3*b*(8*a^6 + 89*a^4*b^2 + 345*a^2*b^4 + 210*b^6)*Tan[
c + d*x])))/(a^8*d*(a + b*Tan[c + d*x])^2)
```

Maple [A] (verified)

Time = 4.39 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{\frac{b(3a^4+20a^2b^2+21b^4)\ln(a+b\tan(dx+c))}{a^8}-\frac{(a^4+2a^2b^2+b^4)b}{2a^6(a+b\tan(dx+c))^2}-\frac{2b(a^4+4a^2b^2+3b^4)}{a^7(a+b\tan(dx+c))}-\frac{1}{5a^3\tan(dx+c)^5}-\frac{2a^2+6b^2}{3a^5\tan(dx+c)^3}-\frac{a^4+\cdots}{a^7\cdots}}{d}$
default	$\frac{\frac{b(3a^4+20a^2b^2+21b^4)\ln(a+b\tan(dx+c))}{a^8}-\frac{(a^4+2a^2b^2+b^4)b}{2a^6(a+b\tan(dx+c))^2}-\frac{2b(a^4+4a^2b^2+3b^4)}{a^7(a+b\tan(dx+c))}-\frac{1}{5a^3\tan(dx+c)^5}-\frac{2a^2+6b^2}{3a^5\tan(dx+c)^3}-\frac{a^4+\cdots}{a^7\cdots}}{d}$
risch	$-\frac{2i(-315b^6+187a^4b^2+120a^2b^4+8a^6+120a^6e^{6i(dx+c)}+1890b^6e^{2i(dx+c)}-4725b^6e^{8i(dx+c)}-24a^6e^{2i(dx+c)}-315b^6e^{12i(\cdots}}{\cdots}$

```
[In] int(csc(d*x+c)^6/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*(3*a^4+20*a^2*b^2+21*b^4)/a^8*ln(a+b*tan(d*x+c))-1/2*(a^4+2*a^2*b^2+
b^4)*b/a^6/(a+b*tan(d*x+c))^2-2*b*(a^4+4*a^2*b^2+3*b^4)/a^7/(a+b*tan(d*x+c)
)-1/5/a^3/tan(d*x+c)^5-1/3*(2*a^2+6*b^2)/a^5/tan(d*x+c)^3-(a^4+12*a^2*b^2+1
5*b^4)/a^7/tan(d*x+c)+3/4/a^4*b/tan(d*x+c)^4+b*(3*a^2+5*b^2)/a^6/tan(d*x+c)
^2-b*(3*a^4+20*a^2*b^2+21*b^4)/a^8*ln(tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1018 vs. $2(257) = 514$.

Time = 0.31 (sec) , antiderivative size = 1018, normalized size of antiderivative = 3.84

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/60*(4*(8*a^7 + 187*a^5*b^2 + 120*a^3*b^4 - 315*a*b^6)*cos(d*x + c)^7 - 4*
(20*a^7 + 482*a^5*b^2 + 255*a^3*b^4 - 945*a*b^6)*cos(d*x + c)^5 + 10*(6*a^7
 + 157*a^5*b^2 + 60*a^3*b^4 - 378*a*b^6)*cos(d*x + c)^3 - 30*(13*a^5*b^2 +
2*a^3*b^4 - 42*a*b^6)*cos(d*x + c) + 30*(2*(3*a^5*b^2 + 20*a^3*b^4 + 21*a*b
^6)*cos(d*x + c)^7 - 6*(3*a^5*b^2 + 20*a^3*b^4 + 21*a*b^6)*cos(d*x + c)^5 +
 6*(3*a^5*b^2 + 20*a^3*b^4 + 21*a*b^6)*cos(d*x + c)^3 - 2*(3*a^5*b^2 + 20*a
^3*b^4 + 21*a*b^6)*cos(d*x + c) - (3*a^4*b^3 + 20*a^2*b^5 + 21*b^7 + (3*a^6
*b + 17*a^4*b^3 + a^2*b^5 - 21*b^7)*cos(d*x + c)^6 - (6*a^6*b + 31*a^4*b^3
- 18*a^2*b^5 - 63*b^7)*cos(d*x + c)^4 + (3*a^6*b + 11*a^4*b^3 - 39*a^2*b^5
- 63*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c)
 + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 30*(2*(3*a^5*b^2 + 20*a^3*b^4 + 21*a
*b^6)*cos(d*x + c)^7 - 6*(3*a^5*b^2 + 20*a^3*b^4 + 21*a*b^6)*cos(d*x + c)^5
 + 6*(3*a^5*b^2 + 20*a^3*b^4 + 21*a*b^6)*cos(d*x + c)^3 - 2*(3*a^5*b^2 + 20
*a^3*b^4 + 21*a*b^6)*cos(d*x + c) - (3*a^4*b^3 + 20*a^2*b^5 + 21*b^7 + (3*a
```

```
^6*b + 17*a^4*b^3 + a^2*b^5 - 21*b^7)*cos(d*x + c)^6 - (6*a^6*b + 31*a^4*b^
3 - 18*a^2*b^5 - 63*b^7)*cos(d*x + c)^4 + (3*a^6*b + 11*a^4*b^3 - 39*a^2*b^
5 - 63*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/4*cos(d*x + c)^2 + 1/4) -
(285*a^4*b^3 + 630*a^2*b^5 - 8*(8*a^6*b + 195*a^4*b^3 + 315*a^2*b^5)*cos(d*
x + c)^6 + 10*(7*a^6*b + 330*a^4*b^3 + 567*a^2*b^5)*cos(d*x + c)^4 + 15*(a^
6*b - 135*a^4*b^3 - 252*a^2*b^5)*cos(d*x + c)^2)*sin(d*x + c))/(2*a^9*b*d*c
os(d*x + c)^7 - 6*a^9*b*d*cos(d*x + c)^5 + 6*a^9*b*d*cos(d*x + c)^3 - 2*a^9
*b*d*cos(d*x + c) - (a^8*b^2*d + (a^10 - a^8*b^2)*d*cos(d*x + c)^6 - (2*a^1
0 - 3*a^8*b^2)*d*cos(d*x + c)^4 + (a^10 - 3*a^8*b^2)*d*cos(d*x + c)^2)*sin(
d*x + c))
```

Sympy [F]

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\csc^6{(c+dx)}}{(a+b\tan{(c+dx)})^3}\,dx$$

```
[In] integrate(csc(d*x+c)**6/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)**6/(a + b*tan(c + d*x))**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.06

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{21\,a^5b\tan(dx+c)-60\,(3\,a^4b^2+20\,a^2b^4+21\,b^6)\tan(dx+c)^6-12\,a^6-90\,(3\,a^5b+20\,a^3b^3+21\,ab^5)\tan(dx+c)^5-20\,(3\,a^6+20\,a^4b^2+21\,a^2b^4)\tan(dx+c)}{a^7b^2\tan(dx+c)^7+2\,a^8b\tan(dx+c)^6+a^9\tan(dx+c)^5}}{}$$

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*((21*a^5*b*tan(d*x + c) - 60*(3*a^4*b^2 + 20*a^2*b^4 + 21*b^6)*tan(d*x
 + c)^6 - 12*a^6 - 90*(3*a^5*b + 20*a^3*b^3 + 21*a*b^5)*tan(d*x + c)^5 - 20
*(3*a^6 + 20*a^4*b^2 + 21*a^2*b^4)*tan(d*x + c)^4 + 5*(20*a^5*b + 21*a^3*b^
3)*tan(d*x + c)^3 - 2*(20*a^6 + 21*a^4*b^2)*tan(d*x + c)^2)/(a^7*b^2*tan(d*
x + c)^7 + 2*a^8*b*tan(d*x + c)^6 + a^9*tan(d*x + c)^5) + 60*(3*a^4*b + 20*
a^2*b^3 + 21*b^5)*log(b*tan(d*x + c) + a)/a^8 - 60*(3*a^4*b + 20*a^2*b^3 +
21*b^5)*log(tan(d*x + c))/a^8)/d
```

Giac [A] (verification not implemented)

none

Time = 0.69 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.44

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx =$$

$$-\frac{\frac{60\,(3\,a^4b+20\,a^2b^3+21\,b^5)\log(|\tan(dx+c)|)}{a^8}-\frac{60\,(3\,a^4b^2+20\,a^2b^4+21\,b^6)\log(|b\tan(dx+c)+a|)}{a^8b}+\frac{30\left(9\,a^4b^3\tan(dx+c)^2+60\,a^2b^5\tan(dx+\right.}{}}{}$$

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/60*(60*(3*a^4*b + 20*a^2*b^3 + 21*b^5)*log(abs(tan(d*x + c)))/a^8 - 60*(
3*a^4*b^2 + 20*a^2*b^4 + 21*b^6)*log(abs(b*tan(d*x + c) + a))/(a^8*b) + 30*
(9*a^4*b^3*tan(d*x + c)^2 + 60*a^2*b^5*tan(d*x + c)^2 + 63*b^7*tan(d*x + c)
^2 + 22*a^5*b^2*tan(d*x + c) + 136*a^3*b^4*tan(d*x + c) + 138*a*b^6*tan(d*x
 + c) + 14*a^6*b + 78*a^4*b^3 + 76*a^2*b^5)/((b*tan(d*x + c) + a)^2*a^8) -
(411*a^4*b*tan(d*x + c)^5 + 2740*a^2*b^3*tan(d*x + c)^5 + 2877*b^5*tan(d*x
+ c)^5 - 60*a^5*tan(d*x + c)^4 - 720*a^3*b^2*tan(d*x + c)^4 - 900*a*b^4*tan
(d*x + c)^4 + 180*a^4*b*tan(d*x + c)^3 + 300*a^2*b^3*tan(d*x + c)^3 - 40*a^
5*tan(d*x + c)^2 - 120*a^3*b^2*tan(d*x + c)^2 + 45*a^4*b*tan(d*x + c) - 12*
a^5)/(a^8*tan(d*x + c)^5))/d
```

Mupad [B] (verification not implemented)

Time = 5.84 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.12

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

$$= \frac{2\,b\,\mathrm{atanh}\left(\frac{b\,(a+2\,b\tan(c+d\,x))\,(3\,a^4+20\,a^2\,b^2+21\,b^4)}{a\,(3\,a^4\,b+20\,a^2\,b^3+21\,b^5)}\right)\,(3\,a^4+20\,a^2\,b^2+21\,b^4)}{a^8\,d}$$

$$-\frac{\frac{1}{5\,a}+\frac{\tan(c+d\,x)^4\,(3\,a^4+20\,a^2\,b^2+21\,b^4)}{3\,a^5}+\frac{\tan(c+d\,x)^2\,(20\,a^2+21\,b^2)}{30\,a^3}-\frac{7\,b\tan(c+d\,x)}{20\,a^2}+\frac{b^2\tan(c+d\,x)^6\,(3\,a^4+20\,a^2\,b^2+21\,b^4)}{a^7}+\frac{3\,}{}}{d\,\left(a^2\tan\left(c+d\,x\right)^5+2\,a\,b\tan\left(c+d\,x\right)^6+b^2\tan\left(c+d\right.\right.}$$

```
[In] int(1/(sin(c + d*x)^6*(a + b*tan(c + d*x))^3),x)
```

```
[Out] (2*b*atanh((b*(a + 2*b*tan(c + d*x))*(3*a^4 + 21*b^4 + 20*a^2*b^2))/(a*(3*a
^4*b + 21*b^5 + 20*a^2*b^3)))*(3*a^4 + 21*b^4 + 20*a^2*b^2))/(a^8*d) - (1/(
5*a) + (tan(c + d*x)^4*(3*a^4 + 21*b^4 + 20*a^2*b^2))/(3*a^5) + (tan(c + d*
x)^2*(20*a^2 + 21*b^2))/(30*a^3) - (7*b*tan(c + d*x))/(20*a^2) + (b^2*tan(c
 + d*x)^6*(3*a^4 + 21*b^4 + 20*a^2*b^2))/a^7 + (3*b*tan(c + d*x)^5*(3*a^4 +
 21*b^4 + 20*a^2*b^2))/(2*a^6) - (b*tan(c + d*x)^3*(20*a^2 + 21*b^2))/(12*a
^4))/(d*(a^2*tan(c + d*x)^5 + b^2*tan(c + d*x)^7 + 2*a*b*tan(c + d*x)^6))
```

3.73 $\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^4} dx$

Optimal result

Integrand size = 21, antiderivative size = 366

$$\begin{aligned}\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^4} dx = & \frac{(3a^8-132a^6b^2+370a^4b^4-132a^2b^6+3b^8)\,x}{8\,(a^2+b^2)^6} \\ & + \frac{4ab(a^2-b^2)\,(a^4-8a^2b^2+b^4)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^6\,d} \\ & - \frac{a^4b}{3\,(a^2+b^2)^3\,d(a+b\tan(c+dx))^3} \\ & - \frac{a^3b(a^2-2b^2)}{(a^2+b^2)^4\,d(a+b\tan(c+dx))^2} - \frac{3a^2b(a^4-5a^2b^2+2b^4)}{(a^2+b^2)^5\,d(a+b\tan(c+dx))} \\ & + \frac{\cos^4(c+dx)\,(4ab(a^2-b^2)+(a^4-6a^2b^2+b^4)\tan(c+dx))}{4\,(a^2+b^2)^4\,d} \\ & - \frac{\cos^2(c+dx)\,(16ab(2a^4-5a^2b^2+b^4)+(5a^6-65a^4b^2+55a^2b^4-3b^6)\tan(c+dx))}{8\,(a^2+b^2)^5\,d}\end{aligned}$$

```
[Out] 1/8*(3*a^8-132*a^6*b^2+370*a^4*b^4-132*a^2*b^6+3*b^8)*x/(a^2+b^2)^6+4*a*b*(
a^2-b^2)*(a^4-8*a^2*b^2+b^4)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^6/d-1/
3*a^4*b/(a^2+b^2)^3/d/(a+b*tan(d*x+c))^3-a^3*b*(a^2-2*b^2)/(a^2+b^2)^4/d/(a
+b*tan(d*x+c))^2-3*a^2*b*(a^4-5*a^2*b^2+2*b^4)/(a^2+b^2)^5/d/(a+b*tan(d*x+c
))+1/4*cos(d*x+c)^4*(4*a*b*(a^2-b^2)+(a^4-6*a^2*b^2+b^4)*tan(d*x+c))/(a^2+b
^2)^4/d-1/8*cos(d*x+c)^2*(16*a*b*(2*a^4-5*a^2*b^2+b^4)+(5*a^6-65*a^4*b^2+55
*a^2*b^4-3*b^6)*tan(d*x+c))/(a^2+b^2)^5/d
```

Rubi [A] (verified)

Time = 2.43 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3597, 1661, 1643, 649, 209, 266}

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx$$
$$= -\frac{a^4b}{3d\,(a^2+b^2)^3\,(a+b\tan(c+dx))^3} - \frac{3a^2b(a^4-5a^2b^2+2b^4)}{d\,(a^2+b^2)^5\,(a+b\tan(c+dx))}$$
$$+ \frac{\cos^4(c+dx)\,(4ab(a^2-b^2)+(a^4-6a^2b^2+b^4)\tan(c+dx))}{4d\,(a^2+b^2)^4}$$
$$+ \frac{4ab(a^2-b^2)\,(a^4-8a^2b^2+b^4)\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^6}$$
$$- \frac{a^3b(a^2-2b^2)}{d\,(a^2+b^2)^4\,(a+b\tan(c+dx))^2}$$
$$- \frac{\cos^2(c+dx)\,(16ab(2a^4-5a^2b^2+b^4)+(5a^6-65a^4b^2+55a^2b^4-3b^6)\tan(c+dx))}{8d\,(a^2+b^2)^5}$$
$$+ \frac{x(3a^8-132a^6b^2+370a^4b^4-132a^2b^6+3b^8)}{8\,(a^2+b^2)^6}$$

```
[In] Int[Sin[c + d*x]^4/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] ((3*a^8 - 132*a^6*b^2 + 370*a^4*b^4 - 132*a^2*b^6 + 3*b^8)*x)/(8*(a^2 + b^2
)^6) + (4*a*b*(a^2 - b^2)*(a^4 - 8*a^2*b^2 + b^4)*Log[a*Cos[c + d*x] + b*Si
n[c + d*x]])/((a^2 + b^2)^6*d) - (a^4*b)/(3*(a^2 + b^2)^3*d*(a + b*Tan[c +
d*x])^3) - (a^3*b*(a^2 - 2*b^2))/((a^2 + b^2)^4*d*(a + b*Tan[c + d*x])^2) -
 (3*a^2*b*(a^4 - 5*a^2*b^2 + 2*b^4))/((a^2 + b^2)^5*d*(a + b*Tan[c + d*x]))
 + (Cos[c + d*x]^4*(4*a*b*(a^2 - b^2) + (a^4 - 6*a^2*b^2 + b^4)*Tan[c + d*x
]))/(4*(a^2 + b^2)^4*d) - (Cos[c + d*x]^2*(16*a*b*(2*a^4 - 5*a^2*b^2 + b^4)
 + (5*a^6 - 65*a^4*b^2 + 55*a^2*b^4 - 3*b^6)*Tan[c + d*x]))/(8*(a^2 + b^2)^
5*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{b\text{Subst}\left(\int \frac{x^4}{(a+x)^4(b^2+x^2)^3}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{\cos^4(c+dx)\left(4ab(a^2-b^2)+(a^4-6a^2b^2+b^4)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^4 d}$$

$$- \frac{\text{Subst}\left(\int \frac{\frac{a^4b^4\left(a^4-6a^2b^2+b^4\right)}{\left(a^2+b^2\right)^4}-\frac{4a^3b^4\left(3a^2-b^2\right)x}{\left(a^2+b^2\right)^3}-\frac{2a^2b^2\left(2a^6+17a^4b^2-12a^2b^4-3b^6\right)x^2}{\left(a^2+b^2\right)^4}-\frac{4ab^4\left(3a^4-14a^2b^2-b^4\right)x^3}{\left(a^2+b^2\right)^4}-\frac{3b^4\left(a^4-6a^2b^2+ …

$$= \frac{\cos^4(c+dx)\left(4ab(a^2-b^2)+(a^4-6a^2b^2+b^4)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^4 d}$$
$$-\frac{\cos^2(c+dx)\left(16ab(2a^4-5a^2b^2+b^4)+(5a^6-65a^4b^2+55a^2b^4-3b^6)\tan(c+dx)\right)}{8\left(a^2+b^2\right)^5 d}$$
$$+\frac{\text{Subst}\left(\int \frac{\frac{a^4b^4\left(3a^6-55a^4b^2+65a^2b^4-5b^6\right)}{\left(a^2+b^2\right)^5}-\frac{4a^3b^4\left(a^2+5b^2\right)\left(5a^4-10a^2b^2+b^4\right)x}{\left(a^2+b^2\right)^5}-\frac{30a^2b^4\left(a^4-6a^2b^2+b^4\right)x^2}{\left(a^2+b^2\right)^4}-\frac{4ab^4\left(5a^2+b^2\right)\left(a^4-10a\right.}{\left(a^2+b^2\right)^5}}{(a+x)^4(b^2+x^2)}\right.}{8b^3d}$$

$$= \frac{\cos^4(c+dx)\left(4ab(a^2-b^2)+(a^4-6a^2b^2+b^4)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^4 d}$$
$$-\frac{\cos^2(c+dx)\left(16ab(2a^4-5a^2b^2+b^4)+(5a^6-65a^4b^2+55a^2b^4-3b^6)\tan(c+dx)\right)}{8\left(a^2+b^2\right)^5 d}$$
$$+\frac{\text{Subst}\left(\int \left(\frac{8a^4b^4}{(a^2+b^2)^3(a+x)^4}+\frac{16a^3b^4(a^2-2b^2)}{(a^2+b^2)^4(a+x)^3}+\frac{24a^2b^4(a^4-5a^2b^2+2b^4)}{(a^2+b^2)^5(a+x)^2}+\frac{32ab^4(a^2-b^2)(a^4-8a^2b^2+b^4)}{(a^2+b^2)^6(a+x)}+\frac{b^4(3a^8-}{}\right.\right.}{8b^3d}$$

$$= \frac{4ab(a^2-b^2)\left(a^4-8a^2b^2+b^4\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^6 d}$$
$$-\frac{a^4b}{3\left(a^2+b^2\right)^3 d(a+b\tan(c+dx))^3}$$
$$-\frac{a^3b(a^2-2b^2)}{\left(a^2+b^2\right)^4 d(a+b\tan(c+dx))^2}-\frac{3a^2b(a^4-5a^2b^2+2b^4)}{\left(a^2+b^2\right)^5 d(a+b\tan(c+dx))}$$
$$+\frac{\cos^4(c+dx)\left(4ab(a^2-b^2)+(a^4-6a^2b^2+b^4)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^4 d}$$
$$-\frac{\cos^2(c+dx)\left(16ab(2a^4-5a^2b^2+b^4)+(5a^6-65a^4b^2+55a^2b^4-3b^6)\tan(c+dx)\right)}{8\left(a^2+b^2\right)^5 d}$$
$$+\frac{b\,\text{Subst}\left(\int \frac{3a^8-132a^6b^2+370a^4b^4-132a^2b^6+3b^8-32a(a^2-b^2)(a^4-8a^2b^2+b^4)x}{b^2+x^2}\,dx,x,b\tan(c+dx)\right)}{8\left(a^2+b^2\right)^6 d}$$

$$= \frac{4ab(a^2-b^2)(a^4-8a^2b^2+b^4)\log(a+b\tan(c+dx))}{(a^2+b^2)^6 d}$$
$$- \frac{a^4b}{3(a^2+b^2)^3 d(a+b\tan(c+dx))^3}$$
$$- \frac{a^3b(a^2-2b^2)}{(a^2+b^2)^4 d(a+b\tan(c+dx))^2} - \frac{3a^2b(a^4-5a^2b^2+2b^4)}{(a^2+b^2)^5 d(a+b\tan(c+dx))}$$
$$+ \frac{\cos^4(c+dx)\left(4ab(a^2-b^2)+(a^4-6a^2b^2+b^4)\tan(c+dx)\right)}{4(a^2+b^2)^4 d}$$
$$- \frac{\cos^2(c+dx)\left(16ab(2a^4-5a^2b^2+b^4)+(5a^6-65a^4b^2+55a^2b^4-3b^6)\tan(c+dx)\right)}{8(a^2+b^2)^5 d}$$
$$- \frac{(4ab(a^2-b^2)(a^4-8a^2b^2+b^4))\,\mathrm{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{(a^2+b^2)^6 d}$$
$$+ \frac{(b(3a^8-132a^6b^2+370a^4b^4-132a^2b^6+3b^8))\,\mathrm{Subst}\left(\int \frac{1}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{8(a^2+b^2)^6 d}$$

$$= \frac{(3a^8-132a^6b^2+370a^4b^4-132a^2b^6+3b^8)\,x}{8(a^2+b^2)^6}$$
$$+ \frac{4ab(a^2-b^2)(a^4-8a^2b^2+b^4)\log(\cos(c+dx))}{(a^2+b^2)^6 d}$$
$$+ \frac{4ab(a^2-b^2)(a^4-8a^2b^2+b^4)\log(a+b\tan(c+dx))}{(a^2+b^2)^6 d}$$
$$- \frac{a^4b}{3(a^2+b^2)^3 d(a+b\tan(c+dx))^3}$$
$$- \frac{a^3b(a^2-2b^2)}{(a^2+b^2)^4 d(a+b\tan(c+dx))^2} - \frac{3a^2b(a^4-5a^2b^2+2b^4)}{(a^2+b^2)^5 d(a+b\tan(c+dx))}$$
$$+ \frac{\cos^4(c+dx)\left(4ab(a^2-b^2)+(a^4-6a^2b^2+b^4)\tan(c+dx)\right)}{4(a^2+b^2)^4 d}$$
$$- \frac{\cos^2(c+dx)\left(16ab(2a^4-5a^2b^2+b^4)+(5a^6-65a^4b^2+55a^2b^4-3b^6)\tan(c+dx)\right)}{8(a^2+b^2)^5 d}$$

Mathematica [A] (verified)

Time = 5.76 (sec) , antiderivative size = 590, normalized size of antiderivative = 1.61

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx =$$
$$-\frac{b\left(-\frac{9(a^2+b^2)^2(a^4-6a^2b^2+b^4)\arctan(\tan(c+dx))}{b} + \frac{24a^2(a^2+b^2)(a^4-10a^2b^2+5b^4)\arctan(\tan(c+dx))}{b} + 48a(a^2+b^2)(2a^4-5a\right.}{}$$

```
[In] Integrate[Sin[c + d*x]^4/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -1/24*(b*((-9*(a^2 + b^2)^2*(a^4 - 6*a^2*b^2 + b^4)*ArcTan[Tan[c + d*x]])/b
 + (24*a^2*(a^2 + b^2)*(a^4 - 10*a^2*b^2 + 5*b^4)*ArcTan[Tan[c + d*x]])/b +
 48*a*(a^2 + b^2)*(2*a^4 - 5*a^2*b^2 + b^4)*Cos[c + d*x]^2 - 24*a*(a - b)*(
a + b)*(a^2 + b^2)^2*Cos[c + d*x]^4 + 12*a*(4*a^6 - 36*a^4*b^2 + 36*a^2*b^4
 - 4*b^6 + (-a^7 + 24*a^5*b^2 - 45*a^3*b^4 + 10*a*b^6)/Sqrt[-b^2])*Log[Sqrt
[-b^2] - b*Tan[c + d*x]] - 96*a*(a - b)*(a + b)*(a^4 - 8*a^2*b^2 + b^4)*Log
[a + b*Tan[c + d*x]] + 12*a*(4*a^6 - 36*a^4*b^2 + 36*a^2*b^4 - 4*b^6 + (a^7
 - 24*a^5*b^2 + 45*a^3*b^4 - 10*a*b^6)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c
 + d*x]] - (6*(a^2 + b^2)^2*(a^4 - 6*a^2*b^2 + b^4)*Cos[c + d*x]^3*Sin[c +
d*x])/b - (9*(a^2 + b^2)^2*(a^4 - 6*a^2*b^2 + b^4)*Sin[2*(c + d*x)])/(2*b)
+ (12*a^2*(a^2 + b^2)*(a^4 - 10*a^2*b^2 + 5*b^4)*Sin[2*(c + d*x)])/b + (8*a
^4*(a^2 + b^2)^3)/(a + b*Tan[c + d*x])^3 + (24*a^3*(a^2 - 2*b^2)*(a^2 + b^2
)^2)/(a + b*Tan[c + d*x])^2 + (72*a^2*(a^2 + b^2)*(a^4 - 5*a^2*b^2 + 2*b^4)
)/(a + b*Tan[c + d*x])))/((a^2 + b^2)^6*d)
```

Maple [A] (verified)

Time = 55.09 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.16

method	result
derivativedivides	$\frac{\frac{\left(-\frac{5}{8}a^{8}+\frac{15}{2}a^{6}b^{2}+\frac{5}{4}a^{4}b^{4}-\frac{13}{2}b^{6}a^{2}+\frac{3}{8}b^{8}\right)\left(\tan^{3}(dx+c)\right)+\left(-4b\,a^{7}+6b^{3}a^{5}+8a^{3}b^{5}-2a\,b^{7}\right)\left(\tan^{2}(dx+c)\right)+\left(-\frac{3}{8}a^{8}+\frac{13}{2}a^{6}b^{2}-\frac{15}{2}b^{6}a^{2}}{\left(1+\tan^{2}(dx+c)\right)^{2}}}{}$
default	$\frac{\frac{\left(-\frac{5}{8}a^{8}+\frac{15}{2}a^{6}b^{2}+\frac{5}{4}a^{4}b^{4}-\frac{13}{2}b^{6}a^{2}+\frac{3}{8}b^{8}\right)\left(\tan^{3}(dx+c)\right)+\left(-4b\,a^{7}+6b^{3}a^{5}+8a^{3}b^{5}-2a\,b^{7}\right)\left(\tan^{2}(dx+c)\right)+\left(-\frac{3}{8}a^{8}+\frac{13}{2}a^{6}b^{2}-\frac{15}{2}b^{6}a^{2}}{\left(1+\tan^{2}(dx+c)\right)^{2}}}{}$
risch	Expression too large to display

```
[In] int(sin(d*x+c)^4/(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a^2+b^2)^6*(((-5/8*a^8+15/2*a^6*b^2+5/4*a^4*b^4-13/2*b^6*a^2+3/8*b^
8)*tan(d*x+c)^3+(-4*a^7*b+6*a^5*b^3+8*a^3*b^5-2*a*b^7)*tan(d*x+c)^2+(-3/8*a
^8+13/2*a^6*b^2-15/2*b^6*a^2+5/8*b^8-5/4*a^4*b^4)*tan(d*x+c)-3*b*a^7+7*b^3*
a^5+7*a^3*b^5-3*a*b^7)/(1+tan(d*x+c)^2)^2+1/16*(-32*a^7*b+288*a^5*b^3-288*a
^3*b^5+32*a*b^7)*ln(1+tan(d*x+c)^2)+1/8*(3*a^8-132*a^6*b^2+370*a^4*b^4-132*
a^2*b^6+3*b^8)*arctan(tan(d*x+c)))-1/3*a^4*b/(a^2+b^2)^3/(a+b*tan(d*x+c))^3
-3*a^2*b*(a^4-5*a^2*b^2+2*b^4)/(a^2+b^2)^5/(a+b*tan(d*x+c))-a^3*b*(a^2-2*b^
2)/(a^2+b^2)^4/(a+b*tan(d*x+c))^2+4*b*a*(a^6-9*a^4*b^2+9*a^2*b^4-b^6)/(a^2+
b^2)^6*ln(a+b*tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1053 vs. $2(358) = 716$.

Time = 0.37 (sec) , antiderivative size = 1053, normalized size of antiderivative = 2.88

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/24*(6*(a^10*b + 5*a^8*b^3 + 10*a^6*b^5 + 10*a^4*b^7 + 5*a^2*b^9 + b^11)*c
os(d*x + c)^7 - 3*(11*a^10*b + 45*a^8*b^3 + 70*a^6*b^5 + 50*a^4*b^7 + 15*a^
2*b^9 + b^11)*cos(d*x + c)^5 - (6*a^10*b + 342*a^8*b^3 - 1830*a^6*b^5 + 614
*a^4*b^7 - 216*a^2*b^9 + 12*b^11 - 3*(3*a^11 - 141*a^9*b^2 + 766*a^7*b^4 -
1242*a^5*b^6 + 399*a^3*b^8 - 9*a*b^10)*d*x)*cos(d*x + c)^3 + 3*(114*a^8*b^3
 - 381*a^6*b^5 + 187*a^4*b^7 - 67*a^2*b^9 + 3*b^11 + 3*(3*a^9*b^2 - 132*a^7
*b^4 + 370*a^5*b^6 - 132*a^3*b^8 + 3*a*b^10)*d*x)*cos(d*x + c) + 48*((a^10*
b - 12*a^8*b^3 + 36*a^6*b^5 - 28*a^4*b^7 + 3*a^2*b^9)*cos(d*x + c)^3 + 3*(a
^8*b^3 - 9*a^6*b^5 + 9*a^4*b^7 - a^2*b^9)*cos(d*x + c) + (a^7*b^4 - 9*a^5*b
^6 + 9*a^3*b^8 - a*b^10 + (3*a^9*b^2 - 28*a^7*b^4 + 36*a^5*b^6 - 12*a^3*b^8
 + a*b^10)*cos(d*x + c)^2)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c
) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) + (143*a^7*b^4 - 537*a^5*b^6 + 105*a^
3*b^8 + 33*a*b^10 + 6*(a^11 + 5*a^9*b^2 + 10*a^7*b^4 + 10*a^5*b^6 + 5*a^3*b
^8 + a*b^10)*cos(d*x + c)^6 - 15*(a^11 + 3*a^9*b^2 + 2*a^7*b^4 - 2*a^5*b^6
- 3*a^3*b^8 - a*b^10)*cos(d*x + c)^4 + 3*(3*a^8*b^3 - 132*a^6*b^5 + 370*a^4
*b^7 - 132*a^2*b^9 + 3*b^11)*d*x + (216*a^9*b^2 - 734*a^7*b^4 + 1590*a^5*b^
6 - 522*a^3*b^8 - 54*a*b^10 + 3*(9*a^10*b - 399*a^8*b^3 + 1242*a^6*b^5 - 76
6*a^4*b^7 + 141*a^2*b^9 - 3*b^11)*d*x)*cos(d*x + c)^2)*sin(d*x + c))/((a^15
 + 3*a^13*b^2 - 3*a^11*b^4 - 25*a^9*b^6 - 45*a^7*b^8 - 39*a^5*b^10 - 17*a^3
*b^12 - 3*a*b^14)*d*cos(d*x + c)^3 + 3*(a^13*b^2 + 6*a^11*b^4 + 15*a^9*b^6
+ 20*a^7*b^8 + 15*a^5*b^10 + 6*a^3*b^12 + a*b^14)*d*cos(d*x + c) + ((3*a^14
*b + 17*a^12*b^3 + 39*a^10*b^5 + 45*a^8*b^7 + 25*a^6*b^9 + 3*a^4*b^11 - 3*a
^2*b^13 - b^15)*d*cos(d*x + c)^2 + (a^12*b^3 + 6*a^10*b^5 + 15*a^8*b^7 + 20
*a^6*b^9 + 15*a^4*b^11 + 6*a^2*b^13 + b^15)*d)*sin(d*x + c))
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \text{Exception raised: AttributeError}$$

```
[In] integrate(sin(d*x+c)**4/(a+b*tan(d*x+c))**4,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 997 vs. 2(358) = 716.

Time = 0.42 (sec) , antiderivative size = 997, normalized size of antiderivative = 2.72

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx$$
$$= \frac{3\,(3\,a^8-132\,a^6b^2+370\,a^4b^4-132\,a^2b^6+3\,b^8)\,(dx+c)}{a^{12}+6\,a^{10}b^2+15\,a^8b^4+20\,a^6b^6+15\,a^4b^8+6\,a^2b^{10}+b^{12}}+\frac{96\,(a^7b-9\,a^5b^3+9\,a^3b^5-ab^7)\log(b\tan(dx+c)+a)}{a^{12}+6\,a^{10}b^2+15\,a^8b^4+20\,a^6b^6+15\,a^4b^8+6\,a^2b^{10}+b^{12}}-\frac{48\,(a^7b-9\,a^5b^3+9\,a^3b^5-a}{a^{12}+6\,a^{10}b^2+15\,a^8b^4+20\,a}$$

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/24*(3*(3*a^8 - 132*a^6*b^2 + 370*a^4*b^4 - 132*a^2*b^6 + 3*b^8)*(d*x + c)
/(a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b
^12) + 96*(a^7*b - 9*a^5*b^3 + 9*a^3*b^5 - a*b^7)*log(b*tan(d*x + c) + a)/(
a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^1
2) - 48*(a^7*b - 9*a^5*b^3 + 9*a^3*b^5 - a*b^7)*log(tan(d*x + c)^2 + 1)/(a^
12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)
 - (176*a^8*b - 608*a^6*b^3 + 176*a^4*b^5 + 3*(29*a^6*b^3 - 185*a^4*b^5 + 1
03*a^2*b^7 - 3*b^9)*tan(d*x + c)^6 + 3*(71*a^7*b^2 - 411*a^5*b^4 + 165*a^3*
b^6 + 7*a*b^8)*tan(d*x + c)^5 + (149*a^8*b - 512*a^6*b^3 - 1006*a^4*b^5 + 6
00*a^2*b^7 - 15*b^9)*tan(d*x + c)^4 + 3*(5*a^9 + 152*a^7*b^2 - 822*a^5*b^4
+ 320*a^3*b^6 + 9*a*b^8)*tan(d*x + c)^3 + (331*a^8*b - 1183*a^6*b^3 - 239*a
^4*b^5 + 315*a^2*b^7)*tan(d*x + c)^2 + 3*(3*a^9 + 73*a^7*b^2 - 423*a^5*b^4
+ 147*a^3*b^6)*tan(d*x + c))/(a^13 + 5*a^11*b^2 + 10*a^9*b^4 + 10*a^7*b^6 +
 5*a^5*b^8 + a^3*b^10 + (a^10*b^3 + 5*a^8*b^5 + 10*a^6*b^7 + 10*a^4*b^9 + 5
*a^2*b^11 + b^13)*tan(d*x + c)^7 + 3*(a^11*b^2 + 5*a^9*b^4 + 10*a^7*b^6 + 1
0*a^5*b^8 + 5*a^3*b^10 + a*b^12)*tan(d*x + c)^6 + (3*a^12*b + 17*a^10*b^3 +
 40*a^8*b^5 + 50*a^6*b^7 + 35*a^4*b^9 + 13*a^2*b^11 + 2*b^13)*tan(d*x + c)^
5 + (a^13 + 11*a^11*b^2 + 40*a^9*b^4 + 70*a^7*b^6 + 65*a^5*b^8 + 31*a^3*b^1
0 + 6*a*b^12)*tan(d*x + c)^4 + (6*a^12*b + 31*a^10*b^3 + 65*a^8*b^5 + 70*a^
6*b^7 + 40*a^4*b^9 + 11*a^2*b^11 + b^13)*tan(d*x + c)^3 + (2*a^13 + 13*a^11
*b^2 + 35*a^9*b^4 + 50*a^7*b^6 + 40*a^5*b^8 + 17*a^3*b^10 + 3*a*b^12)*tan(d
*x + c)^2 + 3*(a^12*b + 5*a^10*b^3 + 10*a^8*b^5 + 10*a^6*b^7 + 5*a^4*b^9 +
a^2*b^11)*tan(d*x + c)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 902 vs. $2(358) = 716$.

Time = 0.91 (sec) , antiderivative size = 902, normalized size of antiderivative = 2.46

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx$$

$$= \frac{\frac{3\,(3\,a^8-132\,a^6b^2+370\,a^4b^4-132\,a^2b^6+3\,b^8)(dx+c)}{a^{12}+6\,a^{10}b^2+15\,a^8b^4+20\,a^6b^6+15\,a^4b^8+6\,a^2b^{10}+b^{12}} - \frac{48\,(a^7b-9\,a^5b^3+9\,a^3b^5-ab^7)\log\left(\tan(dx+c)^2+1\right)}{a^{12}+6\,a^{10}b^2+15\,a^8b^4+20\,a^6b^6+15\,a^4b^8+6\,a^2b^{10}+b^{12}} + \frac{96\,(a^7b^2-9\,a^5b^4+9\,a^3b^6-ab}{a^{12}b+6\,a^{10}b^3+15\,a^8b^5+20\,a^6}}{}$$

```
[In] integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(3*(3*a^8 - 132*a^6*b^2 + 370*a^4*b^4 - 132*a^2*b^6 + 3*b^8)*(d*x + c)
/(a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b
^12) - 48*(a^7*b - 9*a^5*b^3 + 9*a^3*b^5 - a*b^7)*log(tan(d*x + c)^2 + 1)/(
a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^1
2) + 96*(a^7*b^2 - 9*a^5*b^4 + 9*a^3*b^6 - a*b^8)*log(abs(b*tan(d*x + c) +
a))/(a^12*b + 6*a^10*b^3 + 15*a^8*b^5 + 20*a^6*b^7 + 15*a^4*b^9 + 6*a^2*b^1
1 + b^13) + 3*(24*a^7*b*tan(d*x + c)^4 - 216*a^5*b^3*tan(d*x + c)^4 + 216*a
^3*b^5*tan(d*x + c)^4 - 24*a*b^7*tan(d*x + c)^4 - 5*a^8*tan(d*x + c)^3 + 60
*a^6*b^2*tan(d*x + c)^3 + 10*a^4*b^4*tan(d*x + c)^3 - 52*a^2*b^6*tan(d*x +
c)^3 + 3*b^8*tan(d*x + c)^3 + 16*a^7*b*tan(d*x + c)^2 - 384*a^5*b^3*tan(d*x
 + c)^2 + 496*a^3*b^5*tan(d*x + c)^2 - 64*a*b^7*tan(d*x + c)^2 - 3*a^8*tan(
d*x + c) + 52*a^6*b^2*tan(d*x + c) - 10*a^4*b^4*tan(d*x + c) - 60*a^2*b^6*t
an(d*x + c) + 5*b^8*tan(d*x + c) - 160*a^5*b^3 + 272*a^3*b^5 - 48*a*b^7)/((
a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^1
2)*(tan(d*x + c)^2 + 1)^2) - 8*(22*a^7*b^4*tan(d*x + c)^3 - 198*a^5*b^6*tan
(d*x + c)^3 + 198*a^3*b^8*tan(d*x + c)^3 - 22*a*b^10*tan(d*x + c)^3 + 75*a^
8*b^3*tan(d*x + c)^2 - 630*a^6*b^5*tan(d*x + c)^2 + 567*a^4*b^7*tan(d*x + c
)^2 - 48*a^2*b^9*tan(d*x + c)^2 + 87*a^9*b^2*tan(d*x + c) - 666*a^7*b^4*tan
(d*x + c) + 531*a^5*b^6*tan(d*x + c) - 36*a^3*b^8*tan(d*x + c) + 35*a^10*b
- 231*a^8*b^3 + 165*a^6*b^5 - 9*a^4*b^7)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 +
 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*(b*tan(d*x + c) + a)^3))/d
```

Mupad [B] (verification not implemented)

Time = 7.61 (sec) , antiderivative size = 962, normalized size of antiderivative = 2.63

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx$$

$$= \frac{\ln(a+b\tan(c+d\,x))\,\left(\frac{4\,a\,b}{(a^2+b^2)^3}-\frac{48\,a\,b^3}{(a^2+b^2)^4}+\frac{120\,a\,b^5}{(a^2+b^2)^5}-\frac{80\,a\,b^7}{(a^2+b^2)^6}\right)}{d}$$

$$-\frac{\frac{2\,(11\,a^8\,b-38\,a^6\,b^3+11\,a^4\,b^5)}{3\,(a^{10}+5\,a^8\,b^2+10\,a^6\,b^4+10\,a^4\,b^6+5\,a^2\,b^8+b^{10})}-\frac{\tan(c+d\,x)^6\,(-29\,a^6\,b^3+185\,a^4\,b^5-103\,a^2\,b^7+3\,b^9)}{8\,(a^{10}+5\,a^8\,b^2+10\,a^6\,b^4+10\,a^4\,b^6+5\,a^2\,b^8+b^{10})}+\frac{\tan(c+d\,x)^5\,(71\,a^7\,b^2-411\,a^5\,b^4+}{8\,(a^{10}+5\,a^8\,b^2+10\,a^6\,b^4+10\,a^4}}{d\,\left(\tan(c+d\,x)^2\,(2\,a^3+3\,a\,b^2)+\tan(c+d\,x)^5\,(}$$

$$+\frac{\ln(\tan(c+d\,x)-\mathrm{i})\,(-3\,a^2+a\,b\,14\mathrm{i}+3\,b^2)}{16\,d\,(-a^6\,1\mathrm{i}+6\,a^5\,b+a^4\,b^2\,15\mathrm{i}-20\,a^3\,b^3-a^2\,b^4\,15\mathrm{i}+6\,a\,b^5+b^6\,1\mathrm{i})}$$

$$-\frac{\ln(\tan(c+d\,x)+1\mathrm{i})\,(3\,a^2+a\,b\,14\mathrm{i}-3\,b^2)}{16\,d\,(a^6\,1\mathrm{i}+6\,a^5\,b-a^4\,b^2\,15\mathrm{i}-20\,a^3\,b^3+a^2\,b^4\,15\mathrm{i}+6\,a\,b^5-b^6\,1\mathrm{i})}$$

```
[In] int(sin(c + d*x)^4/(a + b*tan(c + d*x))^4,x)
```

```
[Out] (log(a + b*tan(c + d*x))*((4*a*b)/(a^2 + b^2)^3 - (48*a*b^3)/(a^2 + b^2)^4
+ (120*a*b^5)/(a^2 + b^2)^5 - (80*a*b^7)/(a^2 + b^2)^6))/d - ((2*(11*a^8*b
+ 11*a^4*b^5 - 38*a^6*b^3))/(3*(a^10 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a
^6*b^4 + 5*a^8*b^2)) - (tan(c + d*x)^6*(3*b^9 - 103*a^2*b^7 + 185*a^4*b^5 -
 29*a^6*b^3))/(8*(a^10 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8
*b^2)) + (tan(c + d*x)^5*(7*a*b^8 + 165*a^3*b^6 - 411*a^5*b^4 + 71*a^7*b^2)
)/(8*(a^10 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2)) + (ta
n(c + d*x)^2*(331*a^8*b + 315*a^2*b^7 - 239*a^4*b^5 - 1183*a^6*b^3))/(24*(a
^10 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2)) + (tan(c + d
*x)^3*(9*a*b^8 + 5*a^9 + 320*a^3*b^6 - 822*a^5*b^4 + 152*a^7*b^2))/(8*(a^10
 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2)) - (tan(c + d*x)
^4*(15*b^9 - 149*a^8*b - 600*a^2*b^7 + 1006*a^4*b^5 + 512*a^6*b^3))/(24*(a^
10 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2)) + (a*tan(c +
d*x)*(3*a^8 + 147*a^2*b^6 - 423*a^4*b^4 + 73*a^6*b^2))/(8*(a^10 + b^10 + 5*
a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2)))/(d*(tan(c + d*x)^2*(3*a*b^
2 + 2*a^3) + tan(c + d*x)^5*(3*a^2*b + 2*b^3) + a^3 + tan(c + d*x)^4*(6*a*b
^2 + a^3) + tan(c + d*x)^3*(6*a^2*b + b^3) + b^3*tan(c + d*x)^7 + 3*a*b^2*t
an(c + d*x)^6 + 3*a^2*b*tan(c + d*x))) + (log(tan(c + d*x) - 1i)*(a*b*14i -
 3*a^2 + 3*b^2))/(16*d*(6*a*b^5 + 6*a^5*b - a^6*1i + b^6*1i - a^2*b^4*15i -
 20*a^3*b^3 + a^4*b^2*15i)) - (log(tan(c + d*x) + 1i)*(a*b*14i + 3*a^2 - 3*
b^2))/(16*d*(6*a*b^5 + 6*a^5*b + a^6*1i - b^6*1i + a^2*b^4*15i - 20*a^3*b^3
 - a^4*b^2*15i))
```

3.74 $\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 264

$$\begin{aligned}\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx =& \frac{(a^6-25a^4b^2+35a^2b^4-3b^6)\,x}{2\,(a^2+b^2)^5} \\ &+ \frac{4ab(a^4-5a^2b^2+2b^4)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^5\,d} \\ &- \frac{a^2b}{3\,(a^2+b^2)^2\,d(a+b\tan(c+dx))^3} \\ &- \frac{ab(a^2-b^2)}{(a^2+b^2)^3\,d(a+b\tan(c+dx))^2} \\ &- \frac{b(3a^4-8a^2b^2+b^4)}{(a^2+b^2)^4\,d(a+b\tan(c+dx))} \\ &- \frac{\cos^2(c+dx)\,(4ab(a^2-b^2)+(a^4-6a^2b^2+b^4)\tan(c+dx))}{2\,(a^2+b^2)^4\,d}\end{aligned}$$

```
[Out] 1/2*(a^6-25*a^4*b^2+35*a^2*b^4-3*b^6)*x/(a^2+b^2)^5+4*a*b*(a^4-5*a^2*b^2+2*
b^4)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^5/d-1/3*a^2*b/(a^2+b^2)^2/d/(a
+b*tan(d*x+c))^3-a*b*(a^2-b^2)/(a^2+b^2)^3/d/(a+b*tan(d*x+c))^2-b*(3*a^4-8*
a^2*b^2+b^4)/(a^2+b^2)^4/d/(a+b*tan(d*x+c))-1/2*cos(d*x+c)^2*(4*a*b*(a^2-b^
2)+(a^4-6*a^2*b^2+b^4)*tan(d*x+c))/(a^2+b^2)^4/d
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3597, 1661, 1643, 649, 209, 266}

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx = -\frac{a^2b}{3d\left(a^2+b^2\right)^2(a+b\tan(c+dx))^3} - \frac{ab(a^2-b^2)}{d\left(a^2+b^2\right)^3(a+b\tan(c+dx))^2} - \frac{b(3a^4-8a^2b^2+b^4)}{d\left(a^2+b^2\right)^4(a+b\tan(c+dx))} - \frac{\cos^2(c+dx)\left(4ab(a^2-b^2)+(a^4-6a^2b^2+b^4)\tan(c+dx)\right)}{2d\left(a^2+b^2\right)^4} + \frac{4ab(a^4-5a^2b^2+2b^4)\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)^5} + \frac{x(a^6-25a^4b^2+35a^2b^4-3b^6)}{2\left(a^2+b^2\right)^5}$$

```
[In] Int[Sin[c + d*x]^2/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] ((a^6 - 25*a^4*b^2 + 35*a^2*b^4 - 3*b^6)*x)/(2*(a^2 + b^2)^5) + (4*a*b*(a^4
 - 5*a^2*b^2 + 2*b^4)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^5*
d) - (a^2*b)/(3*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x])^3) - (a*b*(a^2 - b^2))
/((a^2 + b^2)^3*d*(a + b*Tan[c + d*x])^2) - (b*(3*a^4 - 8*a^2*b^2 + b^4))/(
(a^2 + b^2)^4*d*(a + b*Tan[c + d*x])) - (Cos[c + d*x]^2*(4*a*b*(a^2 - b^2)
+ (a^4 - 6*a^2*b^2 + b^4)*Tan[c + d*x]))/(2*(a^2 + b^2)^4*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{b\text{Subst}\left(\int \frac{x^2}{(a+x)^4(b^2+x^2)^2}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{\cos^2(c+dx)\left(4ab(a^2-b^2)+(a^4-6a^2b^2+b^4)\tan(c+dx)\right)}{2\left(a^2+b^2\right)^4 d} - \frac{\text{Subst}\left(\int \frac{-\frac{a^4b^2\left(a^4-6a^2b^2+b^4\right)}{\left(a^2+b^2\right)^4}+\frac{4a^3b^2\left(a^4+4a^2b^2-b^4\right)x}{\left(a^2+b^2\right)^4}+\frac{2b^2\left(3a^4-6a^2b^2-b^4\right)x^2}{\left(a^2+b^2\right)^3}+\frac{4ab^2\left(a^4-4a^2b^2-b^4\right)x^3}{\left(a^2+b^2\right)^4}+\frac{b^2\left(a^4-6a^2b^2+b^4\right)x^4}{\left(a^2+b^2\right)^4}}{(a+x)^4(b^2+x^2)}\,d\right.}{2bd}$$

$$= -\frac{\cos^2(c+dx)\left(4ab(a^2-b^2)+(a^4-6a^2b^2+b^4)\tan(c+dx)\right)}{2\left(a^2+b^2\right)^4 d} - \frac{\text{Subst}\left(\int \left(-\frac{2a^2b^2}{(a^2+b^2)^2(a+x)^4}+\frac{4ab^2(-a^2+b^2)}{(a^2+b^2)^3(a+x)^3}-\frac{2(3a^4b^2-8a^2b^4+b^6)}{(a^2+b^2)^4(a+x)^2}-\frac{8ab^2(a^4-5a^2b^2+2b^4)}{(a^2+b^2)^5(a+x)}+\frac{b^2(-a^6+25a^4b^2-35\right.\right.}{2bd}$$

$$= \frac{4ab(a^4-5a^2b^2+2b^4)\log(a+b\tan(c+dx))}{(a^2+b^2)^5 d} - \frac{a^2b}{3(a^2+b^2)^2 d(a+b\tan(c+dx))^3} - \frac{ab(a^2-b^2)}{(a^2+b^2)^3 d(a+b\tan(c+dx))^2} - \frac{b(3a^4-8a^2b^2+b^4)}{(a^2+b^2)^4 d(a+b\tan(c+dx))} - \frac{\cos^2(c+dx)\left(4ab(a^2-b^2)+(a^4-6a^2b^2+b^4)\tan(c+dx)\right)}{2(a^2+b^2)^4 d} - \frac{b\text{Subst}\left(\int \frac{-a^6+25a^4b^2-35a^2b^4+3b^6+8a(a^4-5a^2b^2+2b^4)x}{b^2+x^2}dx, x, b\tan(c+dx)\right)}{2(a^2+b^2)^5 d}$$

$$= \frac{4ab(a^4-5a^2b^2+2b^4)\log(a+b\tan(c+dx))}{(a^2+b^2)^5 d} - \frac{a^2b}{3(a^2+b^2)^2 d(a+b\tan(c+dx))^3} - \frac{ab(a^2-b^2)}{(a^2+b^2)^3 d(a+b\tan(c+dx))^2} - \frac{b(3a^4-8a^2b^2+b^4)}{(a^2+b^2)^4 d(a+b\tan(c+dx))} - \frac{\cos^2(c+dx)\left(4ab(a^2-b^2)+(a^4-6a^2b^2+b^4)\tan(c+dx)\right)}{2(a^2+b^2)^4 d} - \frac{(4ab(a^4-5a^2b^2+2b^4))\text{Subst}\left(\int \frac{x}{b^2+x^2}dx, x, b\tan(c+dx)\right)}{(a^2+b^2)^5 d} + \frac{(b(a^6-25a^4b^2+35a^2b^4-3b^6))\text{Subst}\left(\int \frac{1}{b^2+x^2}dx, x, b\tan(c+dx)\right)}{2(a^2+b^2)^5 d}$$

$$= \frac{(a^6-25a^4b^2+35a^2b^4-3b^6)x}{2(a^2+b^2)^5} + \frac{4ab(a^4-5a^2b^2+2b^4)\log(\cos(c+dx))}{(a^2+b^2)^5 d} + \frac{4ab(a^4-5a^2b^2+2b^4)\log(a+b\tan(c+dx))}{(a^2+b^2)^5 d} - \frac{a^2b}{3(a^2+b^2)^2 d(a+b\tan(c+dx))^3} - \frac{ab(a^2-b^2)}{(a^2+b^2)^3 d(a+b\tan(c+dx))^2} - \frac{b(3a^4-8a^2b^2+b^4)}{(a^2+b^2)^4 d(a+b\tan(c+dx))} - \frac{\cos^2(c+dx)\left(4ab(a^2-b^2)+(a^4-6a^2b^2+b^4)\tan(c+dx)\right)}{2(a^2+b^2)^4 d}$$

Mathematica [A] (verified)

Time = 3.95 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.50

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^4}dx = -\frac{b\left(\frac{3(a^2+b^2)(a^4-6a^2b^2+b^4)\arctan(\tan(c+dx))}{b} + 12a(a-b)(a+b)(a^2+b^2)\cos^2(c+dx) + 3\left(4a^5-20a^3b^2+8ab\right.\right.}{}$$

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -1/6*(b*((3*(a^2 + b^2)*(a^4 - 6*a^2*b^2 + b^4)*ArcTan[Tan[c + d*x]])/b + 1
2*a*(a - b)*(a + b)*(a^2 + b^2)*Cos[c + d*x]^2 + 3*(4*a^5 - 20*a^3*b^2 + 8*
a*b^4 + (-a^6 + 15*a^4*b^2 - 15*a^2*b^4 + b^6)/Sqrt[-b^2])*Log[Sqrt[-b^2] -
 b*Tan[c + d*x]] - 24*a*(a^4 - 5*a^2*b^2 + 2*b^4)*Log[a + b*Tan[c + d*x]] +
 3*(4*a^5 - 20*a^3*b^2 + 8*a*b^4 + (a^6 - 15*a^4*b^2 + 15*a^2*b^4 - b^6)/Sq
rt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + (3*(a^2 + b^2)*(a^4 - 6*a^2*b^
2 + b^4)*Sin[2*(c + d*x)])/(2*b) + (2*a^2*(a^2 + b^2)^3)/(a + b*Tan[c + d*x
])^3 + (6*a*(a - b)*(a + b)*(a^2 + b^2)^2)/(a + b*Tan[c + d*x])^2 + (6*(a^2
 + b^2)*(3*a^4 - 8*a^2*b^2 + b^4))/(a + b*Tan[c + d*x])))/((a^2 + b^2)^5*d)
```

Maple [A] (verified)

Time = 17.84 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{\frac{\left(-\frac{1}{2}a^6+\frac{5}{2}a^4b^2+\frac{5}{2}a^2b^4-\frac{1}{2}b^6\right)\tan(dx+c)-2a^5b+2a\,b^5}{1+\tan^2(dx+c)}+\frac{\left(-8a^5b+40a^3b^3-16a\,b^5\right)\ln\left(1+\tan^2(dx+c)\right)}{4}+\frac{\left(a^6-25a^4b^2+35a^2b^4-3b^6\right)}{2}}{\left(a^2+b^2\right)^5}$
default	$\frac{\frac{\left(-\frac{1}{2}a^6+\frac{5}{2}a^4b^2+\frac{5}{2}a^2b^4-\frac{1}{2}b^6\right)\tan(dx+c)-2a^5b+2a\,b^5}{1+\tan^2(dx+c)}+\frac{\left(-8a^5b+40a^3b^3-16a\,b^5\right)\ln\left(1+\tan^2(dx+c)\right)}{4}+\frac{\left(a^6-25a^4b^2+35a^2b^4-3b^6\right)}{2}}{\left(a^2+b^2\right)^5}$
risch	$-\frac{3ixb}{2(5ia^4b-10ia^2b^3+ib^5-a^5+10a^3b^2-5a\,b^4)}-\frac{xa}{2(5ia^4b-10ia^2b^3+ib^5-a^5+10a^3b^2-5a\,b^4)}+\frac{i\mathrm{e}^{2i(dx+c)}}{8(-4ia^3b+4ia\,b^3+a^4-6}$

```
[In] int(sin(d*x+c)^2/(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a^2+b^2)^5*(((-1/2*a^6+5/2*a^4*b^2+5/2*a^2*b^4-1/2*b^6)*tan(d*x+c)-
2*a^5*b+2*a*b^5)/(1+tan(d*x+c)^2)+1/4*(-8*a^5*b+40*a^3*b^3-16*a*b^5)*ln(1+t
an(d*x+c)^2)+1/2*(a^6-25*a^4*b^2+35*a^2*b^4-3*b^6)*arctan(tan(d*x+c)))-1/3*
a^2*b/(a^2+b^2)^2/(a+b*tan(d*x+c))^3-b*(3*a^4-8*a^2*b^2+b^4)/(a^2+b^2)^4/(a
+b*tan(d*x+c))-a*b*(a^2-b^2)/(a^2+b^2)^3/(a+b*tan(d*x+c))^2+4*a*b*(a^4-5*a^
2*b^2+2*b^4)/(a^2+b^2)^5*ln(a+b*tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 802 vs. 2(258) = 516.

Time = 0.33 (sec) , antiderivative size = 802, normalized size of antiderivative = 3.04

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx =$$

$$-\frac{3\left(a^8b+4\,a^6b^3+6\,a^4b^5+4\,a^2b^7+b^9\right)\cos\left(dx+c\right)^5+\left(3\,a^8b+111\,a^6b^3-231\,a^4b^5+65\,a^2b^7-12\,b^9-3\,(\right.}{}$$

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/6*(3*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cos(d*x + c)^5 +
(3*a^8*b + 111*a^6*b^3 - 231*a^4*b^5 + 65*a^2*b^7 - 12*b^9 - 3*(a^9 - 28*a^
7*b^2 + 110*a^5*b^4 - 108*a^3*b^6 + 9*a*b^8)*d*x)*cos(d*x + c)^3 - 3*(25*a^
6*b^3 - 51*a^4*b^5 + 25*a^2*b^7 - 3*b^9 + 3*(a^7*b^2 - 25*a^5*b^4 + 35*a^3*
b^6 - 3*a*b^8)*d*x)*cos(d*x + c) - 12*((a^8*b - 8*a^6*b^3 + 17*a^4*b^5 - 6*
a^2*b^7)*cos(d*x + c)^3 + 3*(a^6*b^3 - 5*a^4*b^5 + 2*a^2*b^7)*cos(d*x + c)
+ (a^5*b^4 - 5*a^3*b^6 + 2*a*b^8 + (3*a^7*b^2 - 16*a^5*b^4 + 11*a^3*b^6 - 2
*a*b^8)*cos(d*x + c)^2)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) +
 (a^2 - b^2)*cos(d*x + c)^2 + b^2) - (32*a^5*b^4 - 66*a^3*b^6 + 6*a*b^8 - 3
*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cos(d*x + c)^4 + 3*(a^6*
b^3 - 25*a^4*b^5 + 35*a^2*b^7 - 3*b^9)*d*x + (45*a^7*b^2 - 143*a^5*b^4 + 21
9*a^3*b^6 - 9*a*b^8 + 3*(3*a^8*b - 76*a^6*b^3 + 130*a^4*b^5 - 44*a^2*b^7 +
3*b^9)*d*x)*cos(d*x + c)^2)*sin(d*x + c))/((a^13 + 2*a^11*b^2 - 5*a^9*b^4 -
 20*a^7*b^6 - 25*a^5*b^8 - 14*a^3*b^10 - 3*a*b^12)*d*cos(d*x + c)^3 + 3*(a^
11*b^2 + 5*a^9*b^4 + 10*a^7*b^6 + 10*a^5*b^8 + 5*a^3*b^10 + a*b^12)*d*cos(d
*x + c) + ((3*a^12*b + 14*a^10*b^3 + 25*a^8*b^5 + 20*a^6*b^7 + 5*a^4*b^9 -
2*a^2*b^11 - b^13)*d*cos(d*x + c)^2 + (a^10*b^3 + 5*a^8*b^5 + 10*a^6*b^7 +
10*a^4*b^9 + 5*a^2*b^11 + b^13)*d)*sin(d*x + c))
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \text{Exception raised: AttributeError}$$

```
[In] integrate(sin(d*x+c)**2/(a+b*tan(d*x+c))**4,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 662 vs. $2(258) = 516$.

Time = 0.34 (sec) , antiderivative size = 662, normalized size of antiderivative = 2.51

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx$$
$$= \frac{\frac{3\,(a^6-25\,a^4b^2+35\,a^2b^4-3\,b^6)(dx+c)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}} + \frac{24\,(a^5b-5\,a^3b^3+2\,ab^5)\log(b\tan(dx+c)+a)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}} - \frac{12\,(a^5b-5\,a^3b^3+2\,ab^5)\log\left(\tan(dx+c)^2+1\right)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}} - \cdots}{}$$

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/6*(3*(a^6 - 25*a^4*b^2 + 35*a^2*b^4 - 3*b^6)*(d*x + c)/(a^10 + 5*a^8*b^2
+ 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) + 24*(a^5*b - 5*a^3*b^3 + 2*a
*b^5)*log(b*tan(d*x + c) + a)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 +
 5*a^2*b^8 + b^10) - 12*(a^5*b - 5*a^3*b^3 + 2*a*b^5)*log(tan(d*x + c)^2 +
1)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) - (38*a^
6*b - 56*a^4*b^3 + 2*a^2*b^5 + 3*(7*a^4*b^3 - 22*a^2*b^5 + 3*b^7)*tan(d*x +
 c)^4 + 3*(17*a^5*b^2 - 46*a^3*b^4 + a*b^6)*tan(d*x + c)^3 + (35*a^6*b - 44
*a^4*b^3 - 73*a^2*b^5 + 6*b^7)*tan(d*x + c)^2 + 3*(a^7 + 20*a^5*b^2 - 43*a^
3*b^4 + 2*a*b^6)*tan(d*x + c))/(a^11 + 4*a^9*b^2 + 6*a^7*b^4 + 4*a^5*b^6 +
a^3*b^8 + (a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*tan(d*x + c)
^5 + 3*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10)*tan(d*x + c)^
4 + (3*a^10*b + 13*a^8*b^3 + 22*a^6*b^5 + 18*a^4*b^7 + 7*a^2*b^9 + b^11)*ta
n(d*x + c)^3 + (a^11 + 7*a^9*b^2 + 18*a^7*b^4 + 22*a^5*b^6 + 13*a^3*b^8 + 3
*a*b^10)*tan(d*x + c)^2 + 3*(a^10*b + 4*a^8*b^3 + 6*a^6*b^5 + 4*a^4*b^7 + a
^2*b^9)*tan(d*x + c)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 642 vs. $2(258) = 516$.

Time = 0.83 (sec) , antiderivative size = 642, normalized size of antiderivative = 2.43

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx$$
$$= \frac{\frac{3\left(a^6-25a^4b^2+35a^2b^4-3b^6\right)(dx+c)}{a^{10}+5a^8b^2+10a^6b^4+10a^4b^6+5a^2b^8+b^{10}} - \frac{12\left(a^5b-5a^3b^3+2ab^5\right)\log\left(\tan(dx+c)^2+1\right)}{a^{10}+5a^8b^2+10a^6b^4+10a^4b^6+5a^2b^8+b^{10}} + \frac{24\left(a^5b^2-5a^3b^4+2ab^6\right)\log(|b\tan(dx+c)+a|)}{a^{10}b+5a^8b^3+10a^6b^5+10a^4b^7+5a^2b^9+b^{11}} +}{}$$

```
[In] integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/6*(3*(a^6 - 25*a^4*b^2 + 35*a^2*b^4 - 3*b^6)*(d*x + c)/(a^10 + 5*a^8*b^2
+ 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) - 12*(a^5*b - 5*a^3*b^3 + 2*a
*b^5)*log(tan(d*x + c)^2 + 1)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 +
 5*a^2*b^8 + b^10) + 24*(a^5*b^2 - 5*a^3*b^4 + 2*a*b^6)*log(abs(b*tan(d*x +
 c) + a))/(a^10*b + 5*a^8*b^3 + 10*a^6*b^5 + 10*a^4*b^7 + 5*a^2*b^9 + b^11)
 + 3*(4*a^5*b*tan(d*x + c)^2 - 20*a^3*b^3*tan(d*x + c)^2 + 8*a*b^5*tan(d*x
+ c)^2 - a^6*tan(d*x + c) + 5*a^4*b^2*tan(d*x + c) + 5*a^2*b^4*tan(d*x + c)
 - b^6*tan(d*x + c) - 20*a^3*b^3 + 12*a*b^5)/((a^10 + 5*a^8*b^2 + 10*a^6*b^
4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*(tan(d*x + c)^2 + 1)) - 2*(22*a^5*b^4*ta
n(d*x + c)^3 - 110*a^3*b^6*tan(d*x + c)^3 + 44*a*b^8*tan(d*x + c)^3 + 75*a^
6*b^3*tan(d*x + c)^2 - 345*a^4*b^5*tan(d*x + c)^2 + 111*a^2*b^7*tan(d*x + c
)^2 + 3*b^9*tan(d*x + c)^2 + 87*a^7*b^2*tan(d*x + c) - 357*a^5*b^4*tan(d*x
+ c) + 87*a^3*b^6*tan(d*x + c) + 3*a*b^8*tan(d*x + c) + 35*a^8*b - 119*a^6*
b^3 + 23*a^4*b^5 + a^2*b^7)/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 +
5*a^2*b^8 + b^10)*(b*tan(d*x + c) + a)^3))/d
```

Mupad [B] (verification not implemented)

Time = 5.61 (sec) , antiderivative size = 597, normalized size of antiderivative = 2.26

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(\frac{4\,a\,b}{{\left(a^2+b^2\right)}^3}-\frac{28\,a\,b^3}{{\left(a^2+b^2\right)}^4}+\frac{32\,a\,b^5}{{\left(a^2+b^2\right)}^5}\right)}{d} - \frac{\frac{{\tan(c+d\,x)}^2\,\left(35\,a^4\,b-79\,a^2\,b^3+6\,b^5\right)}{6\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{{\tan(c+d\,x)}^4\,\left(7\,a^4\,b^3-22\,a^2\,b^5+3\,b^7\right)}{2\,\left(a^8+4\,a^6\,b^2+6\,a^4\,b^4+4\,a^2\,b^6+b^8\right)}+\frac{{\tan(c+d\,x)}^3\,\left(17\,a^5\,b^2-46\,a^3\,b^4+a\,b^6\right)}{2\,\left(a^8+4\,a^6\,b^2+6\,a^4\,b^4+4\,a^2\,b^6+b^8\right)}+\frac{a^2\,\left(19\,a^4\,b-\right.}{3\,\left(a^2+b^2\right)\,\left(a^6+3\right.}}{d\,\left(a^3+{\tan\left(c+d\,x\right)}^2\,\left(a^3+3\,a\,b^2\right)+{\tan\left(c+d\,x\right)}^3\,\left(3\,a^2\,b+b^3\right)+b^3\,{\tan\left(c+d\,x\right)}^5+3\,a\,b^2\right.} - \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(3\,b+a\,1\mathrm{i}\right)}{4\,d\,\left(a^5+a^4\,b\,5\mathrm{i}-10\,a^3\,b^2-a^2\,b^3\,10\mathrm{i}+5\,a\,b^4+b^5\,1\mathrm{i}\right)} + \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(-3\,b+a\,1\mathrm{i}\right)}{4\,d\,\left(a^5-a^4\,b\,5\mathrm{i}-10\,a^3\,b^2+a^2\,b^3\,10\mathrm{i}+5\,a\,b^4-b^5\,1\mathrm{i}\right)}$$

```
[In] int(sin(c + d*x)^2/(a + b*tan(c + d*x))^4,x)
```

```
[Out] (log(a + b*tan(c + d*x))*((4*a*b)/(a^2 + b^2)^3 - (28*a*b^3)/(a^2 + b^2)^4
+ (32*a*b^5)/(a^2 + b^2)^5))/d - ((tan(c + d*x)^2*(35*a^4*b + 6*b^5 - 79*a^
2*b^3))/(6*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (tan(c + d*x)^4*(3*b^7 -
22*a^2*b^5 + 7*a^4*b^3))/(2*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)
) + (tan(c + d*x)^3*(a*b^6 - 46*a^3*b^4 + 17*a^5*b^2))/(2*(a^8 + b^8 + 4*a^
2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (a^2*(19*a^4*b + b^5 - 28*a^2*b^3))/(3*(a
^2 + b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (a*tan(c + d*x)*(a^6 + 2*b
^6 - 43*a^2*b^4 + 20*a^4*b^2))/(2*(a^2 + b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^
4*b^2)))/(d*(a^3 + tan(c + d*x)^2*(3*a*b^2 + a^3) + tan(c + d*x)^3*(3*a^2*b
 + b^3) + b^3*tan(c + d*x)^5 + 3*a*b^2*tan(c + d*x)^4 + 3*a^2*b*tan(c + d*x
))) - (log(tan(c + d*x) - 1i)*(a*1i + 3*b))/(4*d*(5*a*b^4 + a^4*b*5i + a^5
+ b^5*1i - a^2*b^3*10i - 10*a^3*b^2)) + (log(tan(c + d*x) + 1i)*(a*1i - 3*b
))/(4*d*(5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2))
```

3.75 $\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 116

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx = -\frac{\cot(c+dx)}{a^4d} - \frac{4b\log(\tan(c+dx))}{a^5d} + \frac{4b\log(a+b\tan(c+dx))}{a^5d} - \frac{b}{3a^2d(a+b\tan(c+dx))^3} - \frac{b}{a^3d(a+b\tan(c+dx))^2} - \frac{3b}{a^4d(a+b\tan(c+dx))}$$

```
[Out] -cot(d*x+c)/a^4/d-4*b*ln(tan(d*x+c))/a^5/d+4*b*ln(a+b*tan(d*x+c))/a^5/d-1/3
*b/a^2/d/(a+b*tan(d*x+c))^3-b/a^3/d/(a+b*tan(d*x+c))^2-3*b/a^4/d/(a+b*tan(d
*x+c))
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3597, 46}

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx = -\frac{4b\log(\tan(c+dx))}{a^5d} + \frac{4b\log(a+b\tan(c+dx))}{a^5d} - \frac{3b}{a^4d(a+b\tan(c+dx))} - \frac{\cot(c+dx)}{a^4d} - \frac{b}{a^3d(a+b\tan(c+dx))^2} - \frac{b}{3a^2d(a+b\tan(c+dx))^3}$$

```
[In] Int[Csc[c + d*x]^2/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -(Cot[c + d*x]/(a^4*d)) - (4*b*Log[Tan[c + d*x]])/(a^5*d) + (4*b*Log[a + b*
Tan[c + d*x]])/(a^5*d) - b/(3*a^2*d*(a + b*Tan[c + d*x])^3) - b/(a^3*d*(a +
 b*Tan[c + d*x])^2) - (3*b)/(a^4*d*(a + b*Tan[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{b\text{Subst}\left(\int \frac{1}{x^2(a+x)^4}\,dx, x, b\tan(c+dx)\right)}{d} \\
&= \frac{b\text{Subst}\left(\int \left(\frac{1}{a^4x^2}-\frac{4}{a^5x}+\frac{1}{a^2(a+x)^4}+\frac{2}{a^3(a+x)^3}+\frac{3}{a^4(a+x)^2}+\frac{4}{a^5(a+x)}\right)dx, x, b\tan(c+dx)\right)}{d} \\
&= -\frac{\cot(c+dx)}{a^4d}-\frac{4b\log(\tan(c+dx))}{a^5d}+\frac{4b\log(a+b\tan(c+dx))}{a^5d} \\
&\quad -\frac{b}{3a^2d(a+b\tan(c+dx))^3}-\frac{b}{a^3d(a+b\tan(c+dx))^2}-\frac{3b}{a^4d(a+b\tan(c+dx))}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 259 vs. $2(116) = 232$.

Time = 3.28 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.23

$$
\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \frac{\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(-3a(b+a\cot(c+dx))^3\sin^2(c+dx)+\frac{a^2b^4\tan(c+dx)}{a^2+b^2}+\frac{b^2(18a^4}{}\right.}{}
$$

```
[In] Integrate[Csc[c + d*x]^2/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])*(-3*a*(b + a*Cot[c + d*x]
)^3*Sin[c + d*x]^2 + (a^2*b^4*Tan[c + d*x])/(a^2 + b^2) + (b^2*(18*a^4 + 23
*a^2*b^2 + 9*b^4)*(a*Cos[c + d*x] + b*Sin[c + d*x])^2*Tan[c + d*x])/(a^2 +
```

```
b^2)^2 - (2*a^2*b^3*(3*a^2 + 2*b^2)*(a + b*Tan[c + d*x]))/(a^2 + b^2)^2 - 1
2*b*Cos[c + d*x]^2*Log[Sin[c + d*x]]*(a + b*Tan[c + d*x])^3 + 12*b*Cos[c +
d*x]^2*Log[a*Cos[c + d*x] + b*Sin[c + d*x]]*(a + b*Tan[c + d*x])^3))/(3*a^5
*d*(a + b*Tan[c + d*x])^4)
```

Maple [A] (verified)

Time = 2.24 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{-\frac{1}{a^4\tan(dx+c)}-\frac{4b\ln(\tan(dx+c))}{a^5}-\frac{b}{3a^2(a+b\tan(dx+c))^3}+\frac{4b\ln(a+b\tan(dx+c))}{a^5}-\frac{3b}{a^4(a+b\tan(dx+c))}-\frac{b}{a^3(a+b\tan(dx+c))^2}}{d}$
default	$\frac{-\frac{1}{a^4\tan(dx+c)}-\frac{4b\ln(\tan(dx+c))}{a^5}-\frac{b}{3a^2(a+b\tan(dx+c))^3}+\frac{4b\ln(a+b\tan(dx+c))}{a^5}-\frac{3b}{a^4(a+b\tan(dx+c))}-\frac{b}{a^3(a+b\tan(dx+c))^2}}{d}$
risch	$-\frac{2i(96ia^3b^3e^{2i(dx+c)}+36ia^5b\,e^{4i(dx+c)}-120ia\,b^5e^{4i(dx+c)}-3a^6e^{6i(dx+c)}+63a^4b^2e^{6i(dx+c)}-120a^2b^4e^{6i(dx+c)}+12b^6e^{6i}}{}$

```
[In] int(csc(d*x+c)^2/(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a^4/tan(d*x+c)-4/a^5*b*ln(tan(d*x+c))-1/3/a^2*b/(a+b*tan(d*x+c))^3+
4/a^5*b*ln(a+b*tan(d*x+c))-3/a^4*b/(a+b*tan(d*x+c))-1/a^3*b/(a+b*tan(d*x+c)
)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 874 vs. 2(114) = 228.

Time = 0.30 (sec) , antiderivative size = 874, normalized size of antiderivative = 7.53

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx = -\frac{13\,a^6b^4+15\,a^4b^6+6\,a^2b^8-(3\,a^{10}+18\,a^8b^2-49\,a^6b^4-84\,a^4b^6-36\,a^2b^8)\cos(dx+c)^4+(9\,a^8b^2-71\,a}{}$$

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(13*a^6*b^4 + 15*a^4*b^6 + 6*a^2*b^8 - (3*a^10 + 18*a^8*b^2 - 49*a^6*b
^4 - 84*a^4*b^6 - 36*a^2*b^8)*cos(d*x + c)^4 + (9*a^8*b^2 - 71*a^6*b^4 - 10
2*a^4*b^6 - 42*a^2*b^8)*cos(d*x + c)^2 + 6*(a^6*b^4 + 3*a^4*b^6 + 3*a^2*b^8
 + b^10 - (3*a^8*b^2 + 8*a^6*b^4 + 6*a^4*b^6 - b^10)*cos(d*x + c)^4 + (3*a^
8*b^2 + 7*a^6*b^4 + 3*a^4*b^6 - 3*a^2*b^8 - 2*b^10)*cos(d*x + c)^2 + ((a^9*
b - 6*a^5*b^5 - 8*a^3*b^7 - 3*a*b^9)*cos(d*x + c)^3 + 3*(a^7*b^3 + 3*a^5*b^
5 + 3*a^3*b^7 + a*b^9)*cos(d*x + c))*sin(d*x + c))*log(2*a*b*cos(d*x + c)*s
in(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 6*(a^6*b^4 + 3*a^4*b^6 +
3*a^2*b^8 + b^10 - (3*a^8*b^2 + 8*a^6*b^4 + 6*a^4*b^6 - b^10)*cos(d*x + c)^
```

```
4 + (3*a^8*b^2 + 7*a^6*b^4 + 3*a^4*b^6 - 3*a^2*b^8 - 2*b^10)*cos(d*x + c)^2
 + ((a^9*b - 6*a^5*b^5 - 8*a^3*b^7 - 3*a*b^9)*cos(d*x + c)^3 + 3*(a^7*b^3 +
 3*a^5*b^5 + 3*a^3*b^7 + a*b^9)*cos(d*x + c))*sin(d*x + c))*log(-1/4*cos(d*
x + c)^2 + 1/4) - ((9*a^9*b + 78*a^7*b^3 + 69*a^5*b^5 + 4*a^3*b^7 - 12*a*b^
9)*cos(d*x + c)^3 - 3*(9*a^7*b^3 + 3*a^5*b^5 - 6*a^3*b^7 - 4*a*b^9)*cos(d*x
 + c))*sin(d*x + c))/((3*a^13*b + 8*a^11*b^3 + 6*a^9*b^5 - a^5*b^9)*d*cos(d
*x + c)^4 - (3*a^13*b + 7*a^11*b^3 + 3*a^9*b^5 - 3*a^7*b^7 - 2*a^5*b^9)*d*c
os(d*x + c)^2 - (a^11*b^3 + 3*a^9*b^5 + 3*a^7*b^7 + a^5*b^9)*d - ((a^14 - 6
*a^10*b^4 - 8*a^8*b^6 - 3*a^6*b^8)*d*cos(d*x + c)^3 + 3*(a^12*b^2 + 3*a^10*
b^4 + 3*a^8*b^6 + a^6*b^8)*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \int \frac{\csc^2{(c+dx)}}{(a+b\tan{(c+dx)})^4}\,dx$$

```
[In] integrate(csc(d*x+c)**2/(a+b*tan(d*x+c))**4,x)
[Out] Integral(csc(c + d*x)**2/(a + b*tan(c + d*x))**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.21

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx = -\frac{\frac{12\,b^3\tan(dx+c)^3+30\,ab^2\tan(dx+c)^2+22\,a^2b\tan(dx+c)+3\,a^3}{a^4b^3\tan(dx+c)^4+3\,a^5b^2\tan(dx+c)^3+3\,a^6b\tan(dx+c)^2+a^7\tan(dx+c)} - \frac{12\,b\log(b\tan(dx+c)+a)}{a^5} + \frac{12\,b\log(\tan(dx+c))}{a^5}}{3\,d}$$

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^4,x, algorithm="maxima")
[Out] -1/3*((12*b^3*tan(d*x + c)^3 + 30*a*b^2*tan(d*x + c)^2 + 22*a^2*b*tan(d*x +
 c) + 3*a^3)/(a^4*b^3*tan(d*x + c)^4 + 3*a^5*b^2*tan(d*x + c)^3 + 3*a^6*b*t
an(d*x + c)^2 + a^7*tan(d*x + c)) - 12*b*log(b*tan(d*x + c) + a)/a^5 + 12*b
*log(tan(d*x + c))/a^5)/d
```

Giac [A] (verification not implemented)

none

Time = 0.67 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.11

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \frac{\frac{12\,b\log(|b\tan(dx+c)+a|)}{a^5} - \frac{12\,b\log(|\tan(dx+c)|)}{a^5} + \frac{3\,(4\,b\tan(dx+c)-a)}{a^5\tan(dx+c)} - \frac{22\,b^4\tan(dx+c)^3+75\,ab^3\tan(dx+c)^2+87\,a^2b^2\tan(dx+c)+35\,a^3b}{(b\tan(dx+c)+a)^3a^5}}{3\,d}$$

```
[In] integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(12*b*log(abs(b*tan(d*x + c) + a))/a^5 - 12*b*log(abs(tan(d*x + c)))/a^
5 + 3*(4*b*tan(d*x + c) - a)/(a^5*tan(d*x + c)) - (22*b^4*tan(d*x + c)^3 +
75*a*b^3*tan(d*x + c)^2 + 87*a^2*b^2*tan(d*x + c) + 35*a^3*b)/((b*tan(d*x +
 c) + a)^3*a^5))/d
```

Mupad [B] (verification not implemented)

Time = 5.32 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.13

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \frac{8\,b\,\mathrm{atanh}\left(\frac{2\,b\tan(c+d\,x)}{a}+1\right)}{a^5\,d} - \frac{\frac{1}{a}+\frac{10\,b^2\tan(c+d\,x)^2}{a^3}+\frac{4\,b^3\tan(c+d\,x)^3}{a^4}+\frac{22\,b\tan(c+d\,x)}{3\,a^2}}{d\,\left(a^3\tan\left(c+d\,x\right)+3\,a^2\,b\tan\left(c+d\,x\right)^2+3\,a\,b^2\tan\left(c+d\,x\right)^3+b^3\tan\left(c+d\,x\right)^4\right)}$$

```
[In] int(1/(sin(c + d*x)^2*(a + b*tan(c + d*x))^4),x)
```

```
[Out] (8*b*atanh((2*b*tan(c + d*x))/a + 1))/(a^5*d) - (1/a + (10*b^2*tan(c + d*x)
^2)/a^3 + (4*b^3*tan(c + d*x)^3)/a^4 + (22*b*tan(c + d*x))/(3*a^2))/(d*(a^3
*tan(c + d*x) + b^3*tan(c + d*x)^4 + 3*a^2*b*tan(c + d*x)^2 + 3*a*b^2*tan(c
 + d*x)^3))
```

3.76 $\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 205

$$\begin{aligned}\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx = &-\frac{(a^2+10b^2)\cot(c+dx)}{a^6d}+\frac{2b\cot^2(c+dx)}{a^5d}\\ &-\frac{\cot^3(c+dx)}{3a^4d}-\frac{4b(a^2+5b^2)\log(\tan(c+dx))}{a^7d}\\ &+\frac{4b(a^2+5b^2)\log(a+b\tan(c+dx))}{a^7d}\\ &-\frac{b(a^2+b^2)}{3a^4d(a+b\tan(c+dx))^3}\\ &-\frac{b(a^2+2b^2)}{a^5d(a+b\tan(c+dx))^2}-\frac{b(3a^2+10b^2)}{a^6d(a+b\tan(c+dx))}\end{aligned}$$

```
[Out] -(a^2+10*b^2)*cot(d*x+c)/a^6/d+2*b*cot(d*x+c)^2/a^5/d-1/3*cot(d*x+c)^3/a^4/
d-4*b*(a^2+5*b^2)*ln(tan(d*x+c))/a^7/d+4*b*(a^2+5*b^2)*ln(a+b*tan(d*x+c))/a
^7/d-1/3*b*(a^2+b^2)/a^4/d/(a+b*tan(d*x+c))^3-b*(a^2+2*b^2)/a^5/d/(a+b*tan(
d*x+c))^2-b*(3*a^2+10*b^2)/a^6/d/(a+b*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used

$= \{3597, 908\}$

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \frac{2b\cot^2(c+dx)}{a^5d} - \frac{\cot^3(c+dx)}{3a^4d} - \frac{4b(a^2+5b^2)\log(\tan(c+dx))}{a^7d} + \frac{4b(a^2+5b^2)\log(a+b\tan(c+dx))}{a^7d} - \frac{b(3a^2+10b^2)}{a^6d(a+b\tan(c+dx))} - \frac{(a^2+10b^2)\cot(c+dx)}{a^6d} - \frac{b(a^2+2b^2)}{a^5d(a+b\tan(c+dx))^2} - \frac{b(a^2+b^2)}{3a^4d(a+b\tan(c+dx))^3}$$

```
[In] Int[Csc[c + d*x]^4/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -(((a^2 + 10*b^2)*Cot[c + d*x])/(a^6*d)) + (2*b*Cot[c + d*x]^2)/(a^5*d) - C
ot[c + d*x]^3/(3*a^4*d) - (4*b*(a^2 + 5*b^2)*Log[Tan[c + d*x]])/(a^7*d) + (
4*b*(a^2 + 5*b^2)*Log[a + b*Tan[c + d*x]])/(a^7*d) - (b*(a^2 + b^2))/(3*a^4
*d*(a + b*Tan[c + d*x])^3) - (b*(a^2 + 2*b^2))/(a^5*d*(a + b*Tan[c + d*x])^
2) - (b*(3*a^2 + 10*b^2))/(a^6*d*(a + b*Tan[c + d*x]))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{b\text{Subst}\left(\int \frac{b^2+x^2}{x^4(a+x)^4}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{b\text{Subst}\left(\int \left(\frac{b^2}{a^4x^4} - \frac{4b^2}{a^5x^3} + \frac{a^2+10b^2}{a^6x^2} - \frac{4(a^2+5b^2)}{a^7x} + \frac{a^2+b^2}{a^4(a+x)^4} + \frac{2(a^2+2b^2)}{a^5(a+x)^3} + \frac{3a^2+10b^2}{a^6(a+x)^2} + \frac{4(a^2+5b^2)}{a^7(a+x)}\right) dx, x, b\tan\right.}{d}$$

$$= -\frac{(a^2+10b^2)\cot(c+dx)}{a^6d} + \frac{2b\cot^2(c+dx)}{a^5d} - \frac{\cot^3(c+dx)}{3a^4d} - \frac{4b(a^2+5b^2)\log(\tan(c+dx))}{a^7d} + \frac{4b(a^2+5b^2)\log(a+b\tan(c+dx))}{a^7d} - \frac{b(a^2+b^2)}{3a^4d(a+b\tan(c+dx))^3} - \frac{b(a^2+2b^2)}{a^5d(a+b\tan(c+dx))^2} - \frac{b(3a^2+10b^2)}{a^6d(a+b\tan(c+dx))}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 528 vs. $2(205) = 410$.

Time = 2.88 (sec) , antiderivative size = 528, normalized size of antiderivative = 2.58

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx$$
$$= \frac{\sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(-192b(a^2+5b^2)\log(\sin(c+dx))(a\cos(c+dx)+b\sin(c+d\right.}{}$$

```
[In] Integrate[Csc[c + d*x]^4/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])*(-192*b*(a^2 + 5*b^2)*Log
[Sin[c + d*x]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^3 + 192*b*(a^2 + 5*b^2)*Lo
g[a*Cos[c + d*x] + b*Sin[c + d*x]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^3 - (C
sc[c + d*x]^3*(8*a^8 - 4*a^6*b^2 - 50*a^4*b^4 - 190*a^2*b^6 - 150*b^8 + 3*(
3*a^8 + 10*a^6*b^2 + 45*a^4*b^4 + 115*a^2*b^6 + 75*b^8)*Cos[2*(c + d*x)] +
6*(2*a^6*b^2 - 17*a^4*b^4 - 35*a^2*b^6 - 15*b^8)*Cos[4*(c + d*x)] - a^8*Cos
[6*(c + d*x)] - 22*a^6*b^2*Cos[6*(c + d*x)] + 17*a^4*b^4*Cos[6*(c + d*x)] +
 55*a^2*b^6*Cos[6*(c + d*x)] + 15*b^8*Cos[6*(c + d*x)] - 3*a^7*b*Sin[2*(c +
 d*x)] + 3*a^5*b^3*Sin[2*(c + d*x)] - 75*a^3*b^5*Sin[2*(c + d*x)] - 75*a*b^
7*Sin[2*(c + d*x)] - 6*a^7*b*Sin[4*(c + d*x)] + 84*a^5*b^3*Sin[4*(c + d*x)]
 + 156*a^3*b^5*Sin[4*(c + d*x)] + 60*a*b^7*Sin[4*(c + d*x)] - 3*a^7*b*Sin[6
*(c + d*x)] - 65*a^5*b^3*Sin[6*(c + d*x)] - 79*a^3*b^5*Sin[6*(c + d*x)] - 1
5*a*b^7*Sin[6*(c + d*x)]))/(a^2 + b^2)))/(48*a^7*d*(a + b*Tan[c + d*x])^4)
```

Maple [A] (verified)

Time = 4.26 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{-\frac{b(3a^2+10b^2)}{a^6(a+b\tan(dx+c))}-\frac{(a^2+b^2)b}{3a^4(a+b\tan(dx+c))^3}-\frac{b(a^2+2b^2)}{a^5(a+b\tan(dx+c))^2}+\frac{4b(a^2+5b^2)\ln(a+b\tan(dx+c))}{a^7}-\frac{1}{3a^4\tan(dx+c)^3}-\frac{a^2+10}{a^6\tan(d}}{d}$
default	$\frac{-\frac{b(3a^2+10b^2)}{a^6(a+b\tan(dx+c))}-\frac{(a^2+b^2)b}{3a^4(a+b\tan(dx+c))^3}-\frac{b(a^2+2b^2)}{a^5(a+b\tan(dx+c))^2}+\frac{4b(a^2+5b^2)\ln(a+b\tan(dx+c))}{a^7}-\frac{1}{3a^4\tan(dx+c)^3}-\frac{a^2+10}{a^6\tan(d}}{d}$
risch	$-\frac{4i(ia^7-30b^7-26a^4b^3-56b^5a^2-a^6b+32ia^5b^2e^{6i(dx+c)}+280ia^3b^4e^{6i(dx+c)}-300ia\,b^6e^{4i(dx+c)}-300ia^3b^4e^{4i(dx+c)}-4}{}$

```
[In] int(csc(d*x+c)^4/(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-b*(3*a^2+10*b^2)/a^6/(a+b*tan(d*x+c))-1/3*(a^2+b^2)*b/a^4/(a+b*tan(d*
x+c))^3-b*(a^2+2*b^2)/a^5/(a+b*tan(d*x+c))^2+4*b*(a^2+5*b^2)/a^7*ln(a+b*tan
```

```
(d*x+c))-1/3/a^4/tan(d*x+c)^3-(a^2+10*b^2)/a^6/tan(d*x+c)+2/a^5*b/tan(d*x+c
)^2-4*b*(a^2+5*b^2)/a^7*ln(tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1235 vs. $2(201) = 402$.

Time = 0.32 (sec) , antiderivative size = 1235, normalized size of antiderivative = 6.02

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \text{Too large to display}$$

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(19*a^6*b^4 + 51*a^4*b^6 + 30*a^2*b^8 + 2*(a^10 + 23*a^8*b^2 - 22*a^6*b
^4 - 138*a^4*b^6 - 90*a^2*b^8)*cos(d*x + c)^6 - 3*(a^10 + 25*a^8*b^2 - 46*a
^6*b^4 - 206*a^4*b^6 - 130*a^2*b^8)*cos(d*x + c)^4 + 3*(9*a^8*b^2 - 38*a^6*
b^4 - 131*a^4*b^6 - 80*a^2*b^8)*cos(d*x + c)^2 + 6*(a^6*b^4 + 7*a^4*b^6 + 1
1*a^2*b^8 + 5*b^10 + (3*a^8*b^2 + 20*a^6*b^4 + 26*a^4*b^6 + 4*a^2*b^8 - 5*b
^10)*cos(d*x + c)^6 - 3*(2*a^8*b^2 + 13*a^6*b^4 + 15*a^4*b^6 - a^2*b^8 - 5*
b^10)*cos(d*x + c)^4 + 3*(a^8*b^2 + 6*a^6*b^4 + 4*a^4*b^6 - 6*a^2*b^8 - 5*b
^10)*cos(d*x + c)^2 - ((a^9*b + 4*a^7*b^3 - 10*a^5*b^5 - 28*a^3*b^7 - 15*a*
b^9)*cos(d*x + c)^5 - (a^9*b + a^7*b^3 - 31*a^5*b^5 - 61*a^3*b^7 - 30*a*b^9
)*cos(d*x + c)^3 - 3*(a^7*b^3 + 7*a^5*b^5 + 11*a^3*b^7 + 5*a*b^9)*cos(d*x +
 c))*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*
x + c)^2 + b^2) - 6*(a^6*b^4 + 7*a^4*b^6 + 11*a^2*b^8 + 5*b^10 + (3*a^8*b^2
 + 20*a^6*b^4 + 26*a^4*b^6 + 4*a^2*b^8 - 5*b^10)*cos(d*x + c)^6 - 3*(2*a^8*
b^2 + 13*a^6*b^4 + 15*a^4*b^6 - a^2*b^8 - 5*b^10)*cos(d*x + c)^4 + 3*(a^8*b
^2 + 6*a^6*b^4 + 4*a^4*b^6 - 6*a^2*b^8 - 5*b^10)*cos(d*x + c)^2 - ((a^9*b +
 4*a^7*b^3 - 10*a^5*b^5 - 28*a^3*b^7 - 15*a*b^9)*cos(d*x + c)^5 - (a^9*b +
a^7*b^3 - 31*a^5*b^5 - 61*a^3*b^7 - 30*a*b^9)*cos(d*x + c)^3 - 3*(a^7*b^3 +
 7*a^5*b^5 + 11*a^3*b^7 + 5*a*b^9)*cos(d*x + c))*sin(d*x + c))*log(-1/4*cos
(d*x + c)^2 + 1/4) + (2*(3*a^9*b + 77*a^7*b^3 + 142*a^5*b^5 + 34*a^3*b^7 -
30*a*b^9)*cos(d*x + c)^5 - (3*a^9*b + 193*a^7*b^3 + 350*a^5*b^5 + 26*a^3*b^
7 - 120*a*b^9)*cos(d*x + c)^3 + 3*(15*a^7*b^3 + 23*a^5*b^5 - 14*a^3*b^7 - 2
0*a*b^9)*cos(d*x + c))*sin(d*x + c))/((3*a^13*b + 5*a^11*b^3 + a^9*b^5 - a^
7*b^7)*d*cos(d*x + c)^6 - 3*(2*a^13*b + 3*a^11*b^3 - a^7*b^7)*d*cos(d*x + c
)^4 + 3*(a^13*b + a^11*b^3 - a^9*b^5 - a^7*b^7)*d*cos(d*x + c)^2 + (a^11*b^
3 + 2*a^9*b^5 + a^7*b^7)*d - ((a^14 - a^12*b^2 - 5*a^10*b^4 - 3*a^8*b^6)*d*
cos(d*x + c)^5 - (a^14 - 4*a^12*b^2 - 11*a^10*b^4 - 6*a^8*b^6)*d*cos(d*x +
c)^3 - 3*(a^12*b^2 + 2*a^10*b^4 + a^8*b^6)*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \int \frac{\csc^4{(c+dx)}}{(a+b\tan{(c+dx)})^4}\,dx$$

```
[In] integrate(csc(d*x+c)**4/(a+b*tan(d*x+c))**4,x)
```

```
[Out] Integral(csc(c + d*x)**4/(a + b*tan(c + d*x))**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.11

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \frac{\frac{3\,a^4b\tan(dx+c)-12\,(a^2b^3+5\,b^5)\tan(dx+c)^5-a^5-30\,(a^3b^2+5\,ab^4)\tan(dx+c)^4-22\,(a^4b+5\,a^2b^3)\tan(dx+c)^3-3\,(a^5+5\,a^3b^2)\tan(dx+c)^2}{a^6b^3\tan(dx+c)^6+3\,a^7b^2\tan(dx+c)^5+3\,a^8b\tan(dx+c)^4+a^9\tan(dx+c)^3}+\frac{12}{}}{3\,d}$$

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*((3*a^4*b*tan(d*x + c) - 12*(a^2*b^3 + 5*b^5)*tan(d*x + c)^5 - a^5 - 30
*(a^3*b^2 + 5*a*b^4)*tan(d*x + c)^4 - 22*(a^4*b + 5*a^2*b^3)*tan(d*x + c)^3
 - 3*(a^5 + 5*a^3*b^2)*tan(d*x + c)^2)/(a^6*b^3*tan(d*x + c)^6 + 3*a^7*b^2*
tan(d*x + c)^5 + 3*a^8*b*tan(d*x + c)^4 + a^9*tan(d*x + c)^3) + 12*(a^2*b +
 5*b^3)*log(b*tan(d*x + c) + a)/a^7 - 12*(a^2*b + 5*b^3)*log(tan(d*x + c))/
a^7)/d
```

Giac [A] (verification not implemented)

none

Time = 0.70 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.08

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx = -\frac{\frac{12\,(a^2b+5\,b^3)\log(|\tan(dx+c)|)}{a^7}-\frac{12\,(a^2b^2+5\,b^4)\log(|b\tan(dx+c)+a|)}{a^7b}+\frac{12\,a^2b^3\tan(dx+c)^5+60\,b^5\tan(dx+c)^5+30\,a^3b^2\tan(dx+c)^4+15}{}}{3\,d}$$

```
[In] integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(12*(a^2*b + 5*b^3)*log(abs(tan(d*x + c)))/a^7 - 12*(a^2*b^2 + 5*b^4)*
log(abs(b*tan(d*x + c) + a))/(a^7*b) + (12*a^2*b^3*tan(d*x + c)^5 + 60*b^5*
```

```
tan(d*x + c)^5 + 30*a^3*b^2*tan(d*x + c)^4 + 150*a*b^4*tan(d*x + c)^4 + 22*
a^4*b*tan(d*x + c)^3 + 110*a^2*b^3*tan(d*x + c)^3 + 3*a^5*tan(d*x + c)^2 +
15*a^3*b^2*tan(d*x + c)^2 - 3*a^4*b*tan(d*x + c) + a^5)/((b*tan(d*x + c)^2
+ a*tan(d*x + c))^3*a^6))/d
```

Mupad [B] (verification not implemented)

Time = 5.64 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.13

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \frac{8\,b\,\mathrm{atanh}\left(\frac{4\,b\,(a^2+5\,b^2)\,(a+2\,b\tan(c+d\,x))}{a\,(4\,a^2\,b+20\,b^3)}\right)\,(a^2+5\,b^2)}{a^7\,d} - \frac{\frac{1}{3\,a}+\frac{\tan(c+d\,x)^2\,(a^2+5\,b^2)}{a^3}-\frac{b\tan(c+d\,x)}{a^2}+\frac{22\,b\tan(c+d\,x)^3\,(a^2+5\,b^2)}{3\,a^4}+\frac{10\,b^2\tan(c+d\,x)^4\,(a^2+5\,b^2)}{a^5}+\frac{4\,b^3\tan(c+d\,x)^5\,(a^2+5\,b^2)}{a^6}}{d\,\left(a^3\tan\left(c+d\,x\right)^3+3\,a^2\,b\tan\left(c+d\,x\right)^4+3\,a\,b^2\tan\left(c+d\,x\right)^5+b^3\tan\left(c+d\,x\right)^6\right)}$$

```
[In] int(1/(sin(c + d*x)^4*(a + b*tan(c + d*x))^4),x)
```

```
[Out] (8*b*atanh((4*b*(a^2 + 5*b^2)*(a + 2*b*tan(c + d*x)))/(a*(4*a^2*b + 20*b^3)
))*(a^2 + 5*b^2))/(a^7*d) - (1/(3*a) + (tan(c + d*x)^2*(a^2 + 5*b^2))/a^3 -
 (b*tan(c + d*x))/a^2 + (22*b*tan(c + d*x)^3*(a^2 + 5*b^2))/(3*a^4) + (10*b
^2*tan(c + d*x)^4*(a^2 + 5*b^2))/a^5 + (4*b^3*tan(c + d*x)^5*(a^2 + 5*b^2))
/a^6)/(d*(a^3*tan(c + d*x)^3 + b^3*tan(c + d*x)^6 + 3*a^2*b*tan(c + d*x)^4
+ 3*a*b^2*tan(c + d*x)^5))
```

3.77 $\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 300

$$\begin{aligned}\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^4}\,dx = &-\frac{(a^4+20a^2b^2+35b^4)\cot(c+dx)}{a^8d}+\frac{2b(2a^2+5b^2)\cot^2(c+dx)}{a^7d}\\ &-\frac{2(a^2+5b^2)\cot^3(c+dx)}{3a^6d}+\frac{b\cot^4(c+dx)}{a^5d}-\frac{\cot^5(c+dx)}{5a^4d}\\ &-\frac{4b(a^4+10a^2b^2+14b^4)\log(\tan(c+dx))}{a^9d}\\ &+\frac{4b(a^4+10a^2b^2+14b^4)\log(a+b\tan(c+dx))}{a^9d}\\ &-\frac{b(a^2+b^2)^2}{3a^6d(a+b\tan(c+dx))^3}-\frac{b(a^2+b^2)(a^2+3b^2)}{a^7d(a+b\tan(c+dx))^2}\\ &-\frac{b(3a^4+20a^2b^2+21b^4)}{a^8d(a+b\tan(c+dx))}\end{aligned}$$

```
[Out] -(a^4+20*a^2*b^2+35*b^4)*cot(d*x+c)/a^8/d+2*b*(2*a^2+5*b^2)*cot(d*x+c)^2/a^
7/d-2/3*(a^2+5*b^2)*cot(d*x+c)^3/a^6/d+b*cot(d*x+c)^4/a^5/d-1/5*cot(d*x+c)^
5/a^4/d-4*b*(a^4+10*a^2*b^2+14*b^4)*ln(tan(d*x+c))/a^9/d+4*b*(a^4+10*a^2*b^
2+14*b^4)*ln(a+b*tan(d*x+c))/a^9/d-1/3*b*(a^2+b^2)^2/a^6/d/(a+b*tan(d*x+c))
^3-b*(a^2+b^2)*(a^2+3*b^2)/a^7/d/(a+b*tan(d*x+c))^2-b*(3*a^4+20*a^2*b^2+21*
b^4)/a^8/d/(a+b*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3597, 908}

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \frac{b\cot^4(c+dx)}{a^5d} - \frac{\cot^5(c+dx)}{5a^4d} - \frac{b(a^2+b^2)\,(a^2+3b^2)}{a^7d(a+b\tan(c+dx))^2} + \frac{2b(2a^2+5b^2)\cot^2(c+dx)}{a^7d} - \frac{b(a^2+b^2)^2}{3a^6d(a+b\tan(c+dx))^3} - \frac{2(a^2+5b^2)\cot^3(c+dx)}{3a^6d} - \frac{4b(a^4+10a^2b^2+14b^4)\log(\tan(c+dx))}{a^9d} + \frac{4b(a^4+10a^2b^2+14b^4)\log(a+b\tan(c+dx))}{a^9d} - \frac{b(3a^4+20a^2b^2+21b^4)}{a^8d(a+b\tan(c+dx))} - \frac{(a^4+20a^2b^2+35b^4)\cot(c+dx)}{a^8d}$$

```
[In] Int[Csc[c + d*x]^6/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] -(((a^4 + 20*a^2*b^2 + 35*b^4)*Cot[c + d*x])/(a^8*d)) + (2*b*(2*a^2 + 5*b^2
)*Cot[c + d*x]^2)/(a^7*d) - (2*(a^2 + 5*b^2)*Cot[c + d*x]^3)/(3*a^6*d) + (b
*Cot[c + d*x]^4)/(a^5*d) - Cot[c + d*x]^5/(5*a^4*d) - (4*b*(a^4 + 10*a^2*b^
2 + 14*b^4)*Log[Tan[c + d*x]])/(a^9*d) + (4*b*(a^4 + 10*a^2*b^2 + 14*b^4)*L
og[a + b*Tan[c + d*x]])/(a^9*d) - (b*(a^2 + b^2)^2)/(3*a^6*d*(a + b*Tan[c +
 d*x])^3) - (b*(a^2 + b^2)*(a^2 + 3*b^2))/(a^7*d*(a + b*Tan[c + d*x])^2) -
(b*(3*a^4 + 20*a^2*b^2 + 21*b^4))/(a^8*d*(a + b*Tan[c + d*x]))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{b\text{Subst}\left(\int \frac{(b^2+x^2)^2}{x^6(a+x)^4}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{b\text{Subst}\left(\int \left(\frac{b^4}{a^4x^6} - \frac{4b^4}{a^5x^5} + \frac{2b^2(a^2+5b^2)}{a^6x^4} - \frac{4(2a^2b^2+5b^4)}{a^7x^3} + \frac{a^4+20a^2b^2+35b^4}{a^8x^2} - \frac{4(a^4+10a^2b^2+14b^4)}{a^9x} + \frac{(a^2+b^2)^2}{a^6(a+x)^4} + \right.\right.}{d}$$

$$= -\frac{(a^4+20a^2b^2+35b^4)\cot(c+dx)}{a^8d} + \frac{2b(2a^2+5b^2)\cot^2(c+dx)}{a^7d} - \frac{2(a^2+5b^2)\cot^3(c+dx)}{3a^6d} + \frac{b\cot^4(c+dx)}{a^5d} - \frac{\cot^5(c+dx)}{5a^4d} - \frac{4b(a^4+10a^2b^2+14b^4)\log(\tan(c+dx))}{a^9d} + \frac{4b(a^4+10a^2b^2+14b^4)\log(a+b\tan(c+dx))}{a^9d} - \frac{b(a^2+b^2)^2}{3a^6d(a+b\tan(c+dx))^3} - \frac{b(a^2+b^2)(a^2+3b^2)}{a^7d(a+b\tan(c+dx))^2} - \frac{b(3a^4+20a^2b^2+21b^4)}{a^8d(a+b\tan(c+dx))}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 673 vs. $2(300) = 600$.

Time = 3.10 (sec) , antiderivative size = 673, normalized size of antiderivative = 2.24

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \frac{\sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(-7680b(a^4+10a^2b^2+14b^4)\log(\sin(c+dx))(a\cos(c+dx)+\right.}{}$$

```
[In] Integrate[Csc[c + d*x]^6/(a + b*Tan[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])*(-7680*b*(a^4 + 10*a^2*b^
2 + 14*b^4)*Log[Sin[c + d*x]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^3 + 7680*b*
(a^4 + 10*a^2*b^2 + 14*b^4)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]]*(a*Cos[c +
 d*x] + b*Sin[c + d*x])^3 + Csc[c + d*x]^5*(-200*a^8 + 380*a^6*b^2 + 3070*a
^4*b^4 + 11375*a^2*b^6 + 11025*b^8 - 4*(52*a^8 + 194*a^6*b^2 + 1510*a^4*b^4
 + 5705*a^2*b^6 + 4410*b^8)*Cos[2*(c + d*x)] + 4*(4*a^8 - 16*a^6*b^2 + 1010
*a^4*b^4 + 4585*a^2*b^6 + 2205*b^8)*Cos[4*(c + d*x)] + 16*a^8*Cos[6*(c + d*
x)] + 776*a^6*b^2*Cos[6*(c + d*x)] - 1000*a^4*b^4*Cos[6*(c + d*x)] - 8540*a
^2*b^6*Cos[6*(c + d*x)] - 2520*b^8*Cos[6*(c + d*x)] - 8*a^8*Cos[8*(c + d*x)
] - 316*a^6*b^2*Cos[8*(c + d*x)] - 70*a^4*b^4*Cos[8*(c + d*x)] + 1645*a^2*b
^6*Cos[8*(c + d*x)] + 315*b^8*Cos[8*(c + d*x)] + 264*a^7*b*Sin[2*(c + d*x)]
 + 372*a^5*b^3*Sin[2*(c + d*x)] + 4830*a^3*b^5*Sin[2*(c + d*x)] + 1470*a*b^
```

```
7*Sin[2*(c + d*x)] + 144*a^7*b*Sin[4*(c + d*x)] - 2476*a^5*b^3*Sin[4*(c + d
*x)] - 9730*a^3*b^5*Sin[4*(c + d*x)] - 1470*a*b^7*Sin[4*(c + d*x)] - 24*a^7
*b*Sin[6*(c + d*x)] + 2756*a^5*b^3*Sin[6*(c + d*x)] + 7670*a^3*b^5*Sin[6*(c
 + d*x)] + 630*a*b^7*Sin[6*(c + d*x)] - 24*a^7*b*Sin[8*(c + d*x)] - 922*a^5
*b^3*Sin[8*(c + d*x)] - 2095*a^3*b^5*Sin[8*(c + d*x)] - 105*a*b^7*Sin[8*(c
+ d*x)])))/(1920*a^9*d*(a + b*Tan[c + d*x])^4)
```

Maple [A] (verified)

Time = 7.96 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{-\frac{b\left(3a^4+20a^2b^2+21b^4\right)}{a^8(a+b\tan(dx+c))}-\frac{\left(a^4+2a^2b^2+b^4\right)b}{3a^6(a+b\tan(dx+c))^3}-\frac{b\left(a^4+4a^2b^2+3b^4\right)}{a^7(a+b\tan(dx+c))^2}+\frac{4b\left(a^4+10a^2b^2+14b^4\right)\ln(a+b\tan(dx+c))}{a^9}-\frac{1}{5a^4\tan(dx+c)^5}}{d}$
default	$\frac{-\frac{b\left(3a^4+20a^2b^2+21b^4\right)}{a^8(a+b\tan(dx+c))}-\frac{\left(a^4+2a^2b^2+b^4\right)b}{3a^6(a+b\tan(dx+c))^3}-\frac{b\left(a^4+4a^2b^2+3b^4\right)}{a^7(a+b\tan(dx+c))^2}+\frac{4b\left(a^4+10a^2b^2+14b^4\right)\ln(a+b\tan(dx+c))}{a^9}-\frac{1}{5a^4\tan(dx+c)^5}}{d}$
risch	Expression too large to display

```
[In] int(csc(d*x+c)^6/(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-b*(3*a^4+20*a^2*b^2+21*b^4)/a^8/(a+b*tan(d*x+c))-1/3*(a^4+2*a^2*b^2+b
^4)*b/a^6/(a+b*tan(d*x+c))^3-b*(a^4+4*a^2*b^2+3*b^4)/a^7/(a+b*tan(d*x+c))^2
+4*b*(a^4+10*a^2*b^2+14*b^4)/a^9*ln(a+b*tan(d*x+c))-1/5/a^4/tan(d*x+c)^5-1/
3*(2*a^2+10*b^2)/a^6/tan(d*x+c)^3-(a^4+20*a^2*b^2+35*b^4)/a^8/tan(d*x+c)+1/
a^5*b/tan(d*x+c)^4+2*b*(2*a^2+5*b^2)/a^7/tan(d*x+c)^2-4*b*(a^4+10*a^2*b^2+1
4*b^4)/a^9*ln(tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1536 vs. $2(294) = 588$.

Time = 0.37 (sec) , antiderivative size = 1536, normalized size of antiderivative = 5.12

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \text{Too large to display}$$

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/15*(110*a^6*b^4 + 510*a^4*b^6 + 420*a^2*b^8 - 4*(2*a^10 + 81*a^8*b^2 + 2
9*a^6*b^4 - 660*a^4*b^6 - 630*a^2*b^8)*cos(d*x + c)^8 + 2*(10*a^10 + 423*a^
8*b^2 - 47*a^6*b^4 - 4320*a^4*b^6 - 3990*a^2*b^8)*cos(d*x + c)^6 - 15*(a^10
 + 47*a^8*b^2 - 44*a^6*b^4 - 658*a^4*b^6 - 588*a^2*b^8)*cos(d*x + c)^4 + 20
*(9*a^8*b^2 - 28*a^6*b^4 - 219*a^4*b^6 - 189*a^2*b^8)*cos(d*x + c)^2 + 30*(
a^6*b^4 + 11*a^4*b^6 + 24*a^2*b^8 + 14*b^10 - (3*a^8*b^2 + 32*a^6*b^4 + 61*
a^4*b^6 + 18*a^2*b^8 - 14*b^10)*cos(d*x + c)^8 + (9*a^8*b^2 + 95*a^6*b^4 +
```

```
172*a^4*b^6 + 30*a^2*b^8 - 56*b^10)*cos(d*x + c)^6 - 3*(3*a^8*b^2 + 31*a^6*
b^4 + 50*a^4*b^6 - 6*a^2*b^8 - 28*b^10)*cos(d*x + c)^4 + (3*a^8*b^2 + 29*a^
6*b^4 + 28*a^4*b^6 - 54*a^2*b^8 - 56*b^10)*cos(d*x + c)^2 + ((a^9*b + 8*a^7
*b^3 - 9*a^5*b^5 - 58*a^3*b^7 - 42*a*b^9)*cos(d*x + c)^7 - (2*a^9*b + 13*a^
7*b^3 - 51*a^5*b^5 - 188*a^3*b^7 - 126*a*b^9)*cos(d*x + c)^5 + (a^9*b + 2*a
^7*b^3 - 75*a^5*b^5 - 202*a^3*b^7 - 126*a*b^9)*cos(d*x + c)^3 + 3*(a^7*b^3
+ 11*a^5*b^5 + 24*a^3*b^7 + 14*a*b^9)*cos(d*x + c))*sin(d*x + c))*log(2*a*b
*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 30*(a^6*b^
4 + 11*a^4*b^6 + 24*a^2*b^8 + 14*b^10 - (3*a^8*b^2 + 32*a^6*b^4 + 61*a^4*b^
6 + 18*a^2*b^8 - 14*b^10)*cos(d*x + c)^8 + (9*a^8*b^2 + 95*a^6*b^4 + 172*a^
4*b^6 + 30*a^2*b^8 - 56*b^10)*cos(d*x + c)^6 - 3*(3*a^8*b^2 + 31*a^6*b^4 +
50*a^4*b^6 - 6*a^2*b^8 - 28*b^10)*cos(d*x + c)^4 + (3*a^8*b^2 + 29*a^6*b^4
+ 28*a^4*b^6 - 54*a^2*b^8 - 56*b^10)*cos(d*x + c)^2 + ((a^9*b + 8*a^7*b^3 -
 9*a^5*b^5 - 58*a^3*b^7 - 42*a*b^9)*cos(d*x + c)^7 - (2*a^9*b + 13*a^7*b^3
- 51*a^5*b^5 - 188*a^3*b^7 - 126*a*b^9)*cos(d*x + c)^5 + (a^9*b + 2*a^7*b^3
 - 75*a^5*b^5 - 202*a^3*b^7 - 126*a*b^9)*cos(d*x + c)^3 + 3*(a^7*b^3 + 11*a
^5*b^5 + 24*a^3*b^7 + 14*a*b^9)*cos(d*x + c))*sin(d*x + c))*log(-1/4*cos(d*
x + c)^2 + 1/4) - 2*(2*(6*a^9*b + 259*a^7*b^3 + 783*a^5*b^5 + 340*a^3*b^7 -
 210*a*b^9)*cos(d*x + c)^7 - (15*a^9*b + 1141*a^7*b^3 + 3546*a^5*b^5 + 1270
*a^3*b^7 - 1260*a*b^9)*cos(d*x + c)^5 + 5*(151*a^7*b^3 + 483*a^5*b^5 + 100*
a^3*b^7 - 252*a*b^9)*cos(d*x + c)^3 - 15*(9*a^7*b^3 + 29*a^5*b^5 - 6*a^3*b^
7 - 28*a*b^9)*cos(d*x + c))*sin(d*x + c))/((3*a^13*b + 2*a^11*b^3 - a^9*b^5
)*d*cos(d*x + c)^8 - (9*a^13*b + 5*a^11*b^3 - 4*a^9*b^5)*d*cos(d*x + c)^6 +
 3*(3*a^13*b + a^11*b^3 - 2*a^9*b^5)*d*cos(d*x + c)^4 - (3*a^13*b - a^11*b^
3 - 4*a^9*b^5)*d*cos(d*x + c)^2 - (a^11*b^3 + a^9*b^5)*d - ((a^14 - 2*a^12*
b^2 - 3*a^10*b^4)*d*cos(d*x + c)^7 - (2*a^14 - 7*a^12*b^2 - 9*a^10*b^4)*d*c
os(d*x + c)^5 + (a^14 - 8*a^12*b^2 - 9*a^10*b^4)*d*cos(d*x + c)^3 + 3*(a^12
*b^2 + a^10*b^4)*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^4}\,dx=\int \frac{\csc^6{(c+dx)}}{(a+b\tan{(c+dx)})^4}\,dx$$

[In] integrate(csc(d*x+c)**6/(a+b*tan(d*x+c))**4,x)

[Out] Integral(csc(c + d*x)**6/(a + b*tan(c + d*x))**4, x)

Maxima [A] (verification not implemented)

none

Time = 0.65 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.08

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^4}\,dx$$
$$= \frac{\frac{6\,a^6b\tan(dx+c)-60\,(a^4b^3+10\,a^2b^5+14\,b^7)\tan(dx+c)^7-3\,a^7-150\,(a^5b^2+10\,a^3b^4+14\,ab^6)\tan(dx+c)^6-110\,(a^6b+10\,a^4b^3+14\,a^2b^5)\tan(dx+c)^5-\cdots}{a^8b^3\tan(dx+c)^8+3\,a^9b^2\tan(dx+c)^7+3\,a^{10}b\tan(dx+c)^6+a^{1}\cdots}}{\cdots}$$

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/15*((6*a^6*b*tan(d*x + c) - 60*(a^4*b^3 + 10*a^2*b^5 + 14*b^7)*tan(d*x +
c)^7 - 3*a^7 - 150*(a^5*b^2 + 10*a^3*b^4 + 14*a*b^6)*tan(d*x + c)^6 - 110*(
a^6*b + 10*a^4*b^3 + 14*a^2*b^5)*tan(d*x + c)^5 - 15*(a^7 + 10*a^5*b^2 + 14
*a^3*b^4)*tan(d*x + c)^4 + 6*(5*a^6*b + 7*a^4*b^3)*tan(d*x + c)^3 - 2*(5*a^
7 + 7*a^5*b^2)*tan(d*x + c)^2)/(a^8*b^3*tan(d*x + c)^8 + 3*a^9*b^2*tan(d*x
+ c)^7 + 3*a^10*b*tan(d*x + c)^6 + a^11*tan(d*x + c)^5) + 60*(a^4*b + 10*a^
2*b^3 + 14*b^5)*log(b*tan(d*x + c) + a)/a^9 - 60*(a^4*b + 10*a^2*b^3 + 14*b
^5)*log(tan(d*x + c))/a^9)/d
```

Giac [A] (verification not implemented)

none

Time = 0.76 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.43

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^4}\,dx =$$
$$-\frac{\frac{60\,(a^4b+10\,a^2b^3+14\,b^5)\log(|\tan(dx+c)|)}{a^9}-\frac{60\,(a^4b^2+10\,a^2b^4+14\,b^6)\log(|b\tan(dx+c)+a|)}{a^9b}+\frac{5\,\left(22\,a^4b^4\tan(dx+c)^3+220\,a^2b^6\tan(dx+c)^3\cdots\right.}{\cdots}}{\cdots}$$

```
[In] integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/15*(60*(a^4*b + 10*a^2*b^3 + 14*b^5)*log(abs(tan(d*x + c)))/a^9 - 60*(a^
4*b^2 + 10*a^2*b^4 + 14*b^6)*log(abs(b*tan(d*x + c) + a))/(a^9*b) + 5*(22*a
^4*b^4*tan(d*x + c)^3 + 220*a^2*b^6*tan(d*x + c)^3 + 308*b^8*tan(d*x + c)^3
 + 75*a^5*b^3*tan(d*x + c)^2 + 720*a^3*b^5*tan(d*x + c)^2 + 987*a*b^7*tan(d
*x + c)^2 + 87*a^6*b^2*tan(d*x + c) + 792*a^4*b^4*tan(d*x + c) + 1059*a^2*b
^6*tan(d*x + c) + 35*a^7*b + 294*a^5*b^3 + 381*a^3*b^5)/((b*tan(d*x + c) +
a)^3*a^9) - (137*a^4*b*tan(d*x + c)^5 + 1370*a^2*b^3*tan(d*x + c)^5 + 1918*
b^5*tan(d*x + c)^5 - 15*a^5*tan(d*x + c)^4 - 300*a^3*b^2*tan(d*x + c)^4 - 5
25*a*b^4*tan(d*x + c)^4 + 60*a^4*b*tan(d*x + c)^3 + 150*a^2*b^3*tan(d*x + c
)^3 - 10*a^5*tan(d*x + c)^2 - 50*a^3*b^2*tan(d*x + c)^2 + 15*a^4*b*tan(d*x
+ c) - 3*a^5)/(a^9*tan(d*x + c)^5))/d
```

Mupad [B] (verification not implemented)

Time = 6.83 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.12

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \frac{8\,b\,\mathrm{atanh}\left(\frac{4\,b\,(a+2\,b\tan(c+d\,x))\,(a^4+10\,a^2\,b^2+14\,b^4)}{a\,(4\,a^4\,b+40\,a^2\,b^3+56\,b^5)}\right)\,(a^4+10\,a^2\,b^2+14\,b^4)}{a^9\,d} - \frac{\frac{1}{5\,a}+\frac{\tan(c+d\,x)^4\,(a^4+10\,a^2\,b^2+14\,b^4)}{a^5}+\frac{2\tan(c+d\,x)^2\,(5\,a^2+7\,b^2)}{15\,a^3}-\frac{2\,b\tan(c+d\,x)}{5\,a^2}+\frac{22\,b\tan(c+d\,x)^5\,(a^4+10\,a^2\,b^2+14\,b^4)}{3\,a^6}+\frac{10\,b^2}{}}{d\,\left(a^3\tan\left(c+d\,x\right)^5+3\,a^2\,b\tan\left(c+d\,x\right)^6+3\,a\,b^2\tan}$$

```
[In] int(1/(sin(c + d*x)^6*(a + b*tan(c + d*x))^4),x)
```

```
[Out] (8*b*atanh((4*b*(a + 2*b*tan(c + d*x))*(a^4 + 14*b^4 + 10*a^2*b^2))/(a*(4*a
^4*b + 56*b^5 + 40*a^2*b^3)))*(a^4 + 14*b^4 + 10*a^2*b^2))/(a^9*d) - (1/(5*
a) + (tan(c + d*x)^4*(a^4 + 14*b^4 + 10*a^2*b^2))/a^5 + (2*tan(c + d*x)^2*(
5*a^2 + 7*b^2))/(15*a^3) - (2*b*tan(c + d*x))/(5*a^2) + (22*b*tan(c + d*x)^
5*(a^4 + 14*b^4 + 10*a^2*b^2))/(3*a^6) + (10*b^2*tan(c + d*x)^6*(a^4 + 14*b
^4 + 10*a^2*b^2))/a^7 + (4*b^3*tan(c + d*x)^7*(a^4 + 14*b^4 + 10*a^2*b^2))/
a^8 - (2*b*tan(c + d*x)^3*(5*a^2 + 7*b^2))/(5*a^4))/(d*(a^3*tan(c + d*x)^5
+ b^3*tan(c + d*x)^8 + 3*a^2*b*tan(c + d*x)^6 + 3*a*b^2*tan(c + d*x)^7))
```

3.78 $\int \frac{\csc(x)}{1+\tan(x)} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 26

$$\int \frac{\csc(x)}{1+\tan(x)} \, dx = -\operatorname{arctanh}(\cos(x)) + \frac{\operatorname{arctanh}\left(\frac{\cos(x)-\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -arctanh(cos(x))+1/2*arctanh(1/2*(cos(x)-sin(x))*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {3599, 3189, 3855, 3153, 212}

$$\int \frac{\csc(x)}{1+\tan(x)} \, dx = \frac{\operatorname{arctanh}\left(\frac{\cos(x)-\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}} - \operatorname{arctanh}(\cos(x))$$

```
[In] Int[Csc[x]/(1 + Tan[x]),x]
```

```
[Out] -ArcTanh[Cos[x]] + ArcTanh[(Cos[x] - Sin[x])/Sqrt[2]]/Sqrt[2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
```

```
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3189

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[cos[c + d*x]^m*(sin[c + d*x]^n/(a*cos[c + d*x] + b*sin[c + d*x])),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cot(x)}{\cos(x)+\sin(x)}\,dx \\
&= \int \left(\csc(x)+\frac{1}{-\cos(x)-\sin(x)}\right)dx \\
&= \int \csc(x)\,dx+\int \frac{1}{-\cos(x)-\sin(x)}\,dx \\
&= -\operatorname{arctanh}(\cos(x))-\operatorname{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,-\cos(x)+\sin(x)\right) \\
&= -\operatorname{arctanh}(\cos(x))-\frac{\operatorname{arctanh}\left(\frac{-\cos(x)+\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.58

$$\int \frac{\csc(x)}{1+\tan(x)}\,dx = (1+i)(-1)^{3/4}\text{arctanh}\left(\frac{-1+\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right) - \log\left(\cos\left(\frac{x}{2}\right)\right) + \log\left(\sin\left(\frac{x}{2}\right)\right)$$

```
[In] Integrate[Csc[x]/(1 + Tan[x]),x]
[Out] (1 + I)*(-1)^(3/4)*ArcTanh[(-1 + Tan[x/2])/Sqrt[2]] - Log[Cos[x/2]] + Log[S
in[x/2]]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

method	result	size
default	$\ln\left(\tan\left(\frac{x}{2}\right)\right)-\sqrt{2}\,\text{arctanh}\left(\frac{\left(2\tan\left(\frac{x}{2}\right)-2\right)\sqrt{2}}{4}\right)$	26
risch	$\ln\left(e^{ix}-1\right)-\frac{\sqrt{2}\,\ln\left(e^{ix}-\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{2}+\frac{\sqrt{2}\,\ln\left(e^{ix}+\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)}{2}-\ln\left(e^{ix}+1\right)$	66

```
[In] int(csc(x)/(1+tan(x)),x,method=_RETURNVERBOSE)
[Out] ln(tan(1/2*x))-2^(1/2)*arctanh(1/4*(2*tan(1/2*x)-2)*2^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(24) = 48.

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.12

$$\int \frac{\csc(x)}{1+\tan(x)}\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{2\left(\sqrt{2}+\cos(x)\right)\sin(x)-2\sqrt{2}\cos(x)-3}{2\cos(x)\sin(x)+1}\right) - \frac{1}{2}\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\frac{1}{2}\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

```
[In] integrate(csc(x)/(1+tan(x)),x, algorithm="fricas")
[Out] 1/4*sqrt(2)*log((2*(sqrt(2) + cos(x))*sin(x) - 2*sqrt(2)*cos(x) - 3)/(2*cos
(x)*sin(x) + 1)) - 1/2*log(1/2*cos(x) + 1/2) + 1/2*log(-1/2*cos(x) + 1/2)
```

Sympy [F]

$$\int \frac{\csc(x)}{1+\tan(x)}\,dx = \int \frac{\csc(x)}{\tan(x)+1}\,dx$$

```
[In] integrate(csc(x)/(1+tan(x)),x)
[Out] Integral(csc(x)/(tan(x) + 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(24) = 48.

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

$$\int \frac{\csc(x)}{1+\tan(x)}\,dx = \frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sin(x)}{\cos(x)+1}+1}{\sqrt{2}+\frac{\sin(x)}{\cos(x)+1}-1}\right) + \log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

```
[In] integrate(csc(x)/(1+tan(x)),x, algorithm="maxima")
[Out] 1/2*sqrt(2)*log(-(sqrt(2) - sin(x)/(cos(x) + 1) + 1)/(sqrt(2) + sin(x)/(cos
(x) + 1) - 1)) + log(sin(x)/(cos(x) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.69

$$\int \frac{\csc(x)}{1+\tan(x)}\,dx = \frac{1}{2}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\tan\left(\frac{1}{2}x\right)-2\right|}{\left|2\sqrt{2}+2\tan\left(\frac{1}{2}x\right)-2\right|}\right) + \log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)$$

```
[In] integrate(csc(x)/(1+tan(x)),x, algorithm="giac")
[Out] 1/2*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(1/2*x) - 2)/abs(2*sqrt(2) + 2*tan(1/
2*x) - 2)) + log(abs(tan(1/2*x)))
```

Mupad [B] (verification not implemented)

Time = 4.56 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.46

$$\int \frac{\csc(x)}{1+\tan(x)}\,dx = \ln\left(\tan\left(\frac{x}{2}\right)\right) - \sqrt{2}\,\mathrm{atanh}\left(\frac{5\sqrt{2}\tan\left(\frac{x}{2}\right)+2\sqrt{2}}{7\tan\left(\frac{x}{2}\right)+3}\right)$$

```
[In] int(1/(sin(x)*(tan(x) + 1)),x)
```

```
[Out] log(tan(x/2)) - 2^(1/2)*atanh((5*2^(1/2)*tan(x/2) + 2*2^(1/2))/(7*tan(x/2)
+ 3))
```

3.79 $\int \sin^m(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 229

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{a^3\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sin^{1+m}(c+dx)}{d(1+m)\sqrt{\cos^2(c+dx)}} + \frac{3a^2b\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},\sin^2(c+dx)\right)\sin^{2+m}(c+dx)}{d(2+m)} + \frac{3ab^2\sqrt{\cos^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{3+m}{2},\frac{5+m}{2},\sin^2(c+dx)\right)\sec(c+dx)\sin^{3+m}(c+dx)}{d(3+m)} + \frac{b^3\,\text{Hypergeometric2F1}\left(2,\frac{4+m}{2},\frac{6+m}{2},\sin^2(c+dx)\right)\sin^{4+m}(c+dx)}{d(4+m)}$$

```
[Out] 3*a^2*b*hypergeom([1, 1+1/2*m],[2+1/2*m],sin(d*x+c)^2)*sin(d*x+c)^(2+m)/d/(
2+m)+b^3*hypergeom([2, 2+1/2*m],[3+1/2*m],sin(d*x+c)^2)*sin(d*x+c)^(4+m)/d/
(4+m)+a^3*cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*s
in(d*x+c)^(1+m)/d/(1+m)/(cos(d*x+c)^2)^(1/2)+3*a*b^2*hypergeom([3/2, 3/2+1/
2*m],[5/2+1/2*m],sin(d*x+c)^2)*sec(d*x+c)*sin(d*x+c)^(3+m)*(cos(d*x+c)^2)^(
1/2)/d/(3+m)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4486, 2722, 2644, 371, 2657}

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$=\frac{a^3\cos(c+dx)\sin^{m+1}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{d(m+1)\sqrt{\cos^2(c+dx)}}$$
$$+\frac{3a^2b\sin^{m+2}(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},\sin^2(c+dx)\right)}{d(m+2)}$$
$$+\frac{3ab^2\sqrt{\cos^2(c+dx)}\sec(c+dx)\sin^{m+3}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+3}{2},\frac{m+5}{2},\sin^2(c+dx)\right)}{d(m+3)}$$
$$+\frac{b^3\sin^{m+4}(c+dx)\,\text{Hypergeometric2F1}\left(2,\frac{m+4}{2},\frac{m+6}{2},\sin^2(c+dx)\right)}{d(m+4)}$$

```
[In] Int[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]
^2]*Sin[c + d*x]^(1 + m))/(d*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (3*a^2*b*Hyper
geometric2F1[1, (2 + m)/2, (4 + m)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + m))
/(d*(2 + m)) + (3*a*b^2*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3/2, (3 + m)
/2, (5 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*Sin[c + d*x]^(3 + m))/(d*(3 + m
)) + (b^3*Hypergeometric2F1[2, (4 + m)/2, (6 + m)/2, Sin[c + d*x]^2]*Sin[c
+ d*x]^(4 + m))/(d*(4 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
```

```
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\text{integral} = \int \left(a^3\sin^m(c+dx)+3a^2b\sec(c+dx)\sin^{1+m}(c+dx)+3ab^2\sec^2(c+dx)\sin^{2+m}(c+dx) + b^3\sec^3(c+dx)\sin^{3+m}(c+dx)\right)\,dx$$

$$= a^3\int \sin^m(c+dx)\,dx + \left(3a^2b\right)\int \sec(c+dx)\sin^{1+m}(c+dx)\,dx + \left(3ab^2\right)\int \sec^2(c+dx)\sin^{2+m}(c+dx)\,dx + b^3\int \sec^3(c+dx)\sin^{3+m}(c+dx)\,dx$$

$$= \frac{a^3\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sin^{1+m}(c+dx)}{d(1+m)\sqrt{\cos^2(c+dx)}} + \frac{3ab^2\sqrt{\cos^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{3+m}{2},\frac{5+m}{2},\sin^2(c+dx)\right)\sec(c+dx)\sin^{3+m}(c+dx)}{d(3+m)} + \frac{\left(3a^2b\right)\text{Subst}\left(\int \frac{x^{1+m}}{1-x^2}\,dx,x,\sin(c+dx)\right)}{d} + \frac{b^3\text{Subst}\left(\int \frac{x^{3+m}}{(1-x^2)^2}\,dx,x,\sin(c+dx)\right)}{d}$$

$$= \frac{a^3\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sin^{1+m}(c+dx)}{d(1+m)\sqrt{\cos^2(c+dx)}} + \frac{3a^2b\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},\sin^2(c+dx)\right)\sin^{2+m}(c+dx)}{d(2+m)} + \frac{3ab^2\sqrt{\cos^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{3+m}{2},\frac{5+m}{2},\sin^2(c+dx)\right)\sec(c+dx)\sin^{3+m}(c+dx)}{d(3+m)} + \frac{b^3\,\text{Hypergeometric2F1}\left(2,\frac{4+m}{2},\frac{6+m}{2},\sin^2(c+dx)\right)\sin^{4+m}(c+dx)}{d(4+m)}$$

Mathematica [A] (verified)

Time = 2.76 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.90

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{\sin^{1+m}(c+dx)\left(\frac{a^3\sqrt{\cos^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sec(c+dx)}{1+m}+b\sin(c+dx)\left(\frac{3a^2\,\text{Hypergeometric2F1}}{2}\right.\right.}{}$$

```
[In] Integrate[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (Sin[c + d*x]^(1 + m)*((a^3*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1
+ m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x])/(1 + m) + b*Sin[c + d*x]*(
(3*a^2*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, Sin[c + d*x]^2])/(2 + m)
+ b*((b*Hypergeometric2F1[2, (4 + m)/2, (6 + m)/2, Sin[c + d*x]^2]*Sin[c +
d*x]^2)/(4 + m) + (3*a*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3/2, (3 + m)/
2, (5 + m)/2, Sin[c + d*x]^2]*Tan[c + d*x])/(3 + m)))))/d
```

Maple [F]

$$\int \left(\sin^{m}\left(dx+c\right)\right)\left(a+b\tan\left(dx+c\right)\right)^{3}dx$$

```
[In] int(sin(d*x+c)^m*(a+b*tan(d*x+c))^3,x)
```

```
[Out] int(sin(d*x+c)^m*(a+b*tan(d*x+c))^3,x)
```

Fricas [F]

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^3\,dx = \int \left(b\tan\left(dx+c\right)+a\right)^{3}\sin\left(dx+c\right)^{m}\,dx$$

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*tan(d*x + c)^3 + 3*a*b^2*tan(d*x + c)^2 + 3*a^2*b*tan(d*x + c
) + a^3)*sin(d*x + c)^m, x)
```

Sympy [F]

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^3\,dx = \int (a+b\tan{(c+dx)})^3\sin^m{(c+dx)}\,dx$$

```
[In] integrate(sin(d*x+c)**m*(a+b*tan(d*x+c))**3,x)
[Out] Integral((a + b*tan(c + d*x))**3*sin(c + d*x)**m, x)
```

Maxima [F]

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^3\,dx = \int (b\tan{(dx+c)}+a)^3\sin{(dx+c)}^m\,dx$$

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^3*sin(d*x + c)^m, x)
```

Giac [F]

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^3\,dx = \int (b\tan{(dx+c)}+a)^3\sin{(dx+c)}^m\,dx$$

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^3,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^3*sin(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^3\,dx = \int \sin{(c+d\,x)}^m\,(a+b\tan(c+d\,x))^3\,dx$$

```
[In] int(sin(c + d*x)^m*(a + b*tan(c + d*x))^3,x)
[Out] int(sin(c + d*x)^m*(a + b*tan(c + d*x))^3, x)
```

3.80 $\int \sin^m(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 179

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{a^2\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sin^{1+m}(c+dx)}{d(1+m)\sqrt{\cos^2(c+dx)}}$$
$$+\frac{2ab\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},\sin^2(c+dx)\right)\sin^{2+m}(c+dx)}{d(2+m)}$$
$$+\frac{b^2\sqrt{\cos^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{3+m}{2},\frac{5+m}{2},\sin^2(c+dx)\right)\sec(c+dx)\sin^{3+m}(c+dx)}{d(3+m)}$$

```
[Out] 2*a*b*hypergeom([1, 1+1/2*m],[2+1/2*m],sin(d*x+c)^2)*sin(d*x+c)^(2+m)/d/(2+
m)+a^2*cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*sin(
d*x+c)^(1+m)/d/(1+m)/(cos(d*x+c)^2)^(1/2)+b^2*hypergeom([3/2, 3/2+1/2*m],[5
/2+1/2*m],sin(d*x+c)^2)*sec(d*x+c)*sin(d*x+c)^(3+m)*(cos(d*x+c)^2)^(1/2)/d/
(3+m)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used

$= \{4486, 2722, 2644, 371, 2657\}$

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{a^2\cos(c+dx)\sin^{m+1}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{d(m+1)\sqrt{\cos^2(c+dx)}}$$
$$+ \frac{2ab\sin^{m+2}(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},\sin^2(c+dx)\right)}{d(m+2)}$$
$$+ \frac{b^2\sqrt{\cos^2(c+dx)}\sec(c+dx)\sin^{m+3}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+3}{2},\frac{m+5}{2},\sin^2(c+dx)\right)}{d(m+3)}$$

```
[In] Int[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]
^2]*Sin[c + d*x]^(1 + m))/(d*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (2*a*b*Hyperge
ometric2F1[1, (2 + m)/2, (4 + m)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + m))/(
d*(2 + m)) + (b^2*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3/2, (3 + m)/2, (5
 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*Sin[c + d*x]^(3 + m))/(d*(3 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^2 \sin^m(c+dx) + 2ab\sec(c+dx)\sin^{1+m}(c+dx) + b^2\sec^2(c+dx)\sin^{2+m}(c+dx)\right)\,dx\\
&= a^2\int \sin^m(c+dx)\,dx + (2ab)\int \sec(c+dx)\sin^{1+m}(c+dx)\,dx + b^2\int \sec^2(c+dx)\sin^{2+m}(c+dx)\,dx\\
&= \frac{a^2\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sin^{1+m}(c+dx)}{d(1+m)\sqrt{\cos^2(c+dx)}} + \frac{b^2\sqrt{\cos^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{3+m}{2},\frac{5+m}{2},\sin^2(c+dx)\right)\sec(c+dx)\sin^{3+m}(c+dx)}{d(3+m)} + \frac{(2ab)\text{Subst}\left(\int \frac{x^{1+m}}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d}\\
&= \frac{a^2\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sin^{1+m}(c+dx)}{d(1+m)\sqrt{\cos^2(c+dx)}} + \frac{2ab\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},\sin^2(c+dx)\right)\sin^{2+m}(c+dx)}{d(2+m)} + \frac{b^2\sqrt{\cos^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{3+m}{2},\frac{5+m}{2},\sin^2(c+dx)\right)\sec(c+dx)\sin^{3+m}(c+dx)}{d(3+m)}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.93

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{\sin^{1+m}(c+dx)\left(\frac{a^2\sqrt{\cos^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sec(c+dx)}{1+m} + \frac{b\sin(c+dx)\left(2a(3+m)\,\text{Hypergeometric2F1}\left(\right.\right.}{}\right.}{d}$$

```
[In] Integrate[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (Sin[c + d*x]^(1 + m)*((a^2*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1
+ m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x])/(1 + m) + (b*Sin[c + d*x]*
(2*a*(3 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, Sin[c + d*x]^2] + b
*(2 + m)*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3/2, (3 + m)/2, (5 + m)/2,
Sin[c + d*x]^2]*Tan[c + d*x]))/((2 + m)*(3 + m))))/d
```

Maple [F]

$$\int \left(\sin^{m}\left(dx+c\right)\right) \left(a+b\tan\left(dx+c\right)\right)^{2} dx$$

```
[In] int(sin(d*x+c)^m*(a+b*tan(d*x+c))^2,x)
[Out] int(sin(d*x+c)^m*(a+b*tan(d*x+c))^2,x)
```

Fricas [F]

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^2\,dx = \int \left(b\tan\left(dx+c\right)+a\right)^{2}\sin\left(dx+c\right)^{m}\,dx$$

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
[Out] integral((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)*sin(d*x + c)^m, x)
```

Sympy [F]

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^2\,dx = \int \left(a+b\tan\left(c+dx\right)\right)^{2}\sin^{m}\left(c+dx\right)dx$$

```
[In] integrate(sin(d*x+c)**m*(a+b*tan(d*x+c))**2,x)
[Out] Integral((a + b*tan(c + d*x))**2*sin(c + d*x)**m, x)
```

Maxima [F]

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^2\,dx = \int \left(b\tan\left(dx+c\right)+a\right)^{2}\sin\left(dx+c\right)^{m}\,dx$$

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^2*sin(d*x + c)^m, x)
```

Giac [F]

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^2\,dx = \int (b\tan(dx+c)+a)^2 \sin(dx+c)^m\,dx$$

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^2,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^2*sin(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^2\,dx = \int \sin(c+d\,x)^m\,(a+b\tan(c+d\,x))^2\,dx$$

```
[In] int(sin(c + d*x)^m*(a + b*tan(c + d*x))^2,x)
[Out] int(sin(c + d*x)^m*(a + b*tan(c + d*x))^2, x)
```

3.81 $\int \sin^m(c+dx)(a+b\tan(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 109

$$\int \sin^m(c+dx)(a+b\tan(c+dx))\,dx = \frac{a\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sin^{1+m}(c+dx)}{d(1+m)\sqrt{\cos^2(c+dx)}} + \frac{b\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},\sin^2(c+dx)\right)\sin^{2+m}(c+dx)}{d(2+m)}$$

```
[Out] b*hypergeom([1, 1+1/2*m],[2+1/2*m],sin(d*x+c)^2)*sin(d*x+c)^(2+m)/d/(2+m)+a
*cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*sin(d*x+c)
^(1+m)/d/(1+m)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {4486, 2722, 2644, 371}

$$\int \sin^m(c+dx)(a+b\tan(c+dx))\,dx = \frac{a\cos(c+dx)\sin^{m+1}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{d(m+1)\sqrt{\cos^2(c+dx)}} + \frac{b\sin^{m+2}(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},\sin^2(c+dx)\right)}{d(m+2)}$$

```
[In] Int[Sin[c + d*x]^m*(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2
]*Sin[c + d*x]^(1 + m))/(d*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (b*Hypergeometri
c2F1[1, (2 + m)/2, (4 + m)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + m))/(d*(2 +
 m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a\sin^m(c+dx)+b\sec(c+dx)\sin^{1+m}(c+dx)\right)\,dx \\
&= a\int \sin^m(c+dx)\,dx+b\int \sec(c+dx)\sin^{1+m}(c+dx)\,dx \\
&= \frac{a\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sin^{1+m}(c+dx)}{d(1+m)\sqrt{\cos^2(c+dx)}} \\
&\quad+\frac{b\text{Subst}\left(\int \frac{x^{1+m}}{1-x^2}\,dx,x,\sin(c+dx)\right)}{d} \\
&= \frac{a\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sin^{1+m}(c+dx)}{d(1+m)\sqrt{\cos^2(c+dx)}} \\
&\quad+\frac{b\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},\sin^2(c+dx)\right)\sin^{2+m}(c+dx)}{d(2+m)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00

$$\int \sin^m(c+dx)(a+b\tan(c+dx))\,dx$$
$$=\frac{a\sqrt{\cos^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sec(c+dx)\sin^{1+m}(c+dx)}{d(1+m)}$$
$$+\frac{b\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},\sin^2(c+dx)\right)\sin^{2+m}(c+dx)}{d(2+m)}$$

```
[In] Integrate[Sin[c + d*x]^m*(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c
+ d*x]^2]*Sec[c + d*x]*Sin[c + d*x]^(1 + m))/(d*(1 + m)) + (b*Hypergeometri
c2F1[1, (2 + m)/2, (4 + m)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + m))/(d*(2 +
 m))
```

Maple [F]

$$\int (\sin^m(dx+c))\,(a+b\tan(dx+c))\,dx$$

```
[In] int(sin(d*x+c)^m*(a+b*tan(d*x+c)),x)
```

```
[Out] int(sin(d*x+c)^m*(a+b*tan(d*x+c)),x)
```

Fricas [F]

$$\int \sin^m(c+dx)(a+b\tan(c+dx))\,dx=\int (b\tan(dx+c)+a)\sin(dx+c)^m\,dx$$

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c) + a)*sin(d*x + c)^m, x)
```

Sympy [F]

$$\int \sin^m(c+dx)(a+b\tan(c+dx))\,dx = \int (a+b\tan{(c+dx)})\sin^m{(c+dx)}\,dx$$

```
[In] integrate(sin(d*x+c)**m*(a+b*tan(d*x+c)),x)
[Out] Integral((a + b*tan(c + d*x))*sin(c + d*x)**m, x)
```

Maxima [F]

$$\int \sin^m(c+dx)(a+b\tan(c+dx))\,dx = \int (b\tan{(dx+c)}+a)\sin{(dx+c)}^m\,dx$$

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c)),x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)*sin(d*x + c)^m, x)
```

Giac [F]

$$\int \sin^m(c+dx)(a+b\tan(c+dx))\,dx = \int (b\tan{(dx+c)}+a)\sin{(dx+c)}^m\,dx$$

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c)),x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)*sin(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^m(c+dx)(a+b\tan(c+dx))\,dx = \int \sin{(c+d\,x)}^m\,(a+b\tan(c+d\,x))\,dx$$

```
[In] int(sin(c + d*x)^m*(a + b*tan(c + d*x)),x)
[Out] int(sin(c + d*x)^m*(a + b*tan(c + d*x)), x)
```

3.82 $\int \frac{\sin^m(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 765

$$\int \frac{\sin^m(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{2^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1+m}{2},1+m,\frac{3+m}{2},-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\tan\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^m(1+\cdots}{ad(1+m)}$$
$$+\frac{2^{1+m}b\,\text{AppellF1}\left(\frac{2+m}{2},1+m,1,\frac{4+m}{2},-\tan^2\left(\frac{1}{2}(c+dx)\right),\frac{a^2\tan^2\left(\frac{1}{2}(c+dx)\right)}{\left(b-\sqrt{a^2+b^2}\right)^2}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+\cdots}{1+\tan^2\left(\frac{1}{2}(c\cdots}\right.}{\sqrt{a^2+b^2}\left(b-\sqrt{a^2+b^2}\right)d(2+m)}$$
$$-\frac{2^{1+m}b\,\text{AppellF1}\left(\frac{2+m}{2},1+m,1,\frac{4+m}{2},-\tan^2\left(\frac{1}{2}(c+dx)\right),\frac{a^2\tan^2\left(\frac{1}{2}(c+dx)\right)}{\left(b+\sqrt{a^2+b^2}\right)^2}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+\cdots}{1+\tan^2\left(\frac{1}{2}(c\cdots}\right.}{\sqrt{a^2+b^2}\left(b+\sqrt{a^2+b^2}\right)d(2+m)}$$
$$+\frac{2^{1+m}ab\,\text{AppellF1}\left(\frac{3+m}{2},1+m,1,\frac{5+m}{2},-\tan^2\left(\frac{1}{2}(c+dx)\right),\frac{a^2\tan^2\left(\frac{1}{2}(c+dx)\right)}{\left(b-\sqrt{a^2+b^2}\right)^2}\right)\tan^3\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c\cdots}{1+\tan^2\left(\frac{1}{2}(\cdots}\right.}{\sqrt{a^2+b^2}\left(b-\sqrt{a^2+b^2}\right)^2d(3+m)}$$
$$-\frac{2^{1+m}ab\,\text{AppellF1}\left(\frac{3+m}{2},1+m,1,\frac{5+m}{2},-\tan^2\left(\frac{1}{2}(c+dx)\right),\frac{a^2\tan^2\left(\frac{1}{2}(c+dx)\right)}{\left(b+\sqrt{a^2+b^2}\right)^2}\right)\tan^3\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c\cdots}{1+\tan^2\left(\frac{1}{2}(\cdots}\right.}{\sqrt{a^2+b^2}\left(b+\sqrt{a^2+b^2}\right)^2d(3+m)}$$

```
[Out] 2^(1+m)*hypergeom([1+m, 1/2+1/2*m],[3/2+1/2*m],-tan(1/2*d*x+1/2*c)^2)*tan(1
/2*d*x+1/2*c)*(tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2))^m*(1+tan(1/2*d*
x+1/2*c)^2)^m/a/d/(1+m)+2^(1+m)*b*AppellF1(1+1/2*m,1,1+m,2+1/2*m,a^2*tan(1/
2*d*x+1/2*c)^2/(b-(a^2+b^2)^(1/2))^2,-tan(1/2*d*x+1/2*c)^2)*tan(1/2*d*x+1/2
*c)^2*(tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2))^m*(1+tan(1/2*d*x+1/2*c)
```

```
^2)^m/d/(2+m)/(b-(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2)-2^(1+m)*b*AppellF1(1+1/2*
m,1,1+m,2+1/2*m,a^2*tan(1/2*d*x+1/2*c)^2/(b+(a^2+b^2)^(1/2))^2,-tan(1/2*d*x
+1/2*c)^2)*tan(1/2*d*x+1/2*c)^2*(tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2
))^m*(1+tan(1/2*d*x+1/2*c)^2)^m/d/(2+m)/(a^2+b^2)^(1/2)/(b+(a^2+b^2)^(1/2))
+2^(1+m)*a*b*AppellF1(3/2+1/2*m,1,1+m,5/2+1/2*m,a^2*tan(1/2*d*x+1/2*c)^2/(b
-(a^2+b^2)^(1/2))^2,-tan(1/2*d*x+1/2*c)^2)*tan(1/2*d*x+1/2*c)^3*(tan(1/2*d*
x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2))^m*(1+tan(1/2*d*x+1/2*c)^2)^m/d/(3+m)/(b-
(a^2+b^2)^(1/2))^2/(a^2+b^2)^(1/2)-2^(1+m)*a*b*AppellF1(3/2+1/2*m,1,1+m,5/2
+1/2*m,a^2*tan(1/2*d*x+1/2*c)^2/(b+(a^2+b^2)^(1/2))^2,-tan(1/2*d*x+1/2*c)^2
)*tan(1/2*d*x+1/2*c)^3*(tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2))^m*(1+t
an(1/2*d*x+1/2*c)^2)^m/d/(3+m)/(a^2+b^2)^(1/2)/(b+(a^2+b^2)^(1/2))^2
```

Rubi [A] (verified)

Time = 6.24 (sec) , antiderivative size = 765, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {12, 6851, 6860, 371, 973, 524}

$$\int \frac{\sin^m(c+dx)}{a+b\tan(c+dx)}\,dx$$

$$= \frac{b2^{m+1}\tan^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}\right)^m\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)^m \text{AppellF1}\left(\frac{m+2}{2},m+1,1,\frac{m+4}{2},-\tan^2\right.}{d(m+2)\sqrt{a^2+b^2}\left(b-\sqrt{a^2+b^2}\right)}$$

$$-\frac{b2^{m+1}\tan^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}\right)^m\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)^m \text{AppellF1}\left(\frac{m+2}{2},m+1,1,\frac{m+4}{2},-\tan\right.}{d(m+2)\sqrt{a^2+b^2}\left(\sqrt{a^2+b^2}+b\right)}$$

$$+\frac{ab2^{m+1}\tan^3\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}\right)^m\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)^m \text{AppellF1}\left(\frac{m+3}{2},m+1,1,\frac{m+5}{2},-\text{ta}\right.}{d(m+3)\sqrt{a^2+b^2}\left(b-\sqrt{a^2+b^2}\right)^2}$$

$$-\frac{ab2^{m+1}\tan^3\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}\right)^m\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)^m \text{AppellF1}\left(\frac{m+3}{2},m+1,1,\frac{m+5}{2},-\text{ta}\right.}{d(m+3)\sqrt{a^2+b^2}\left(\sqrt{a^2+b^2}+b\right)^2}$$

$$+\frac{2^{m+1}\tan\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}\right)^m\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)^m \text{Hypergeometric2F1}\left(\frac{m+1}{2},m+1,\frac{m+3}{2},\right.}{ad(m+1)}$$

```
[In] Int[Sin[c + d*x]^m/(a + b*Tan[c + d*x]),x]
```

```
[Out] (2^(1 + m)*Hypergeometric2F1[(1 + m)/2, 1 + m, (3 + m)/2, -Tan[(c + d*x)/2]
^2]*Tan[(c + d*x)/2]*(Tan[(c + d*x)/2]/(1 + Tan[(c + d*x)/2]^2))^m*(1 + Tan
[(c + d*x)/2]^2)^m)/(a*d*(1 + m)) + (2^(1 + m)*b*AppellF1[(2 + m)/2, 1 + m,
```

```
 1, (4 + m)/2, -Tan[(c + d*x)/2]^2, (a^2*Tan[(c + d*x)/2]^2)/(b - Sqrt[a^2
+ b^2])^2]*Tan[(c + d*x)/2]^2*(Tan[(c + d*x)/2]/(1 + Tan[(c + d*x)/2]^2))^m
*(1 + Tan[(c + d*x)/2]^2)^m)/(Sqrt[a^2 + b^2]*(b - Sqrt[a^2 + b^2])*d*(2 +
m)) - (2^(1 + m)*b*AppellF1[(2 + m)/2, 1 + m, 1, (4 + m)/2, -Tan[(c + d*x)/
2]^2, (a^2*Tan[(c + d*x)/2]^2)/(b + Sqrt[a^2 + b^2])^2]*Tan[(c + d*x)/2]^2*
(Tan[(c + d*x)/2]/(1 + Tan[(c + d*x)/2]^2))^m*(1 + Tan[(c + d*x)/2]^2)^m)/(
Sqrt[a^2 + b^2]*(b + Sqrt[a^2 + b^2])*d*(2 + m)) + (2^(1 + m)*a*b*AppellF1[
(3 + m)/2, 1 + m, 1, (5 + m)/2, -Tan[(c + d*x)/2]^2, (a^2*Tan[(c + d*x)/2]^
2)/(b - Sqrt[a^2 + b^2])^2]*Tan[(c + d*x)/2]^3*(Tan[(c + d*x)/2]/(1 + Tan[(
c + d*x)/2]^2))^m*(1 + Tan[(c + d*x)/2]^2)^m)/(Sqrt[a^2 + b^2]*(b - Sqrt[a^
2 + b^2])^2*d*(3 + m)) - (2^(1 + m)*a*b*AppellF1[(3 + m)/2, 1 + m, 1, (5 +
m)/2, -Tan[(c + d*x)/2]^2, (a^2*Tan[(c + d*x)/2]^2)/(b + Sqrt[a^2 + b^2])^2
]*Tan[(c + d*x)/2]^3*(Tan[(c + d*x)/2]/(1 + Tan[(c + d*x)/2]^2))^m*(1 + Tan
[(c + d*x)/2]^2)^m)/(Sqrt[a^2 + b^2]*(b + Sqrt[a^2 + b^2])^2*d*(3 + m))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 973

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[d*((g*x)^n/x^n), Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[e*((g*x)^n/x^n), Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !I
ntegerQ[p] && !IntegersQ[n, 2*p]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
```

```
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \frac{2^m(1-x^2)\left(\frac{x}{1+x^2}\right)^{1+m}}{x(a+2bx-ax^2)}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

$$= \frac{2^{1+m}\text{Subst}\left(\int \frac{(1-x^2)\left(\frac{x}{1+x^2}\right)^{1+m}}{x(a+2bx-ax^2)}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

$$= \frac{\left(2^{1+m}\tan^{-m}\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^m\left(1+\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^m\right)\text{Subst}\left(\int \frac{x^m(1-x^2)(1+x^2)^{-1-m}}{a+2bx-ax^2}\right.}{d}$$

$$= \frac{\left(2^{1+m}\tan^{-m}\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^m\left(1+\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^m\right)\text{Subst}\left(\int\left(\frac{x^m(1+x^2)^{-1-m}}{a}-2\right.\right.}{d}$$

$$= \frac{\left(2^{1+m}\tan^{-m}\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^m\left(1+\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^m\right)\text{Subst}\left(\int x^m(1+x^2)^{-1-m}\,d\right.}{ad}$$
$$- \frac{\left(2^{2+m}b\tan^{-m}\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^m\left(1+\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^m\right)\text{Subst}\left(\int \frac{x^{1+m}(1+x^2)^{-1-m}}{a+2bx-ax^2}\right.}{ad}$$

$$= \frac{2^{1+m}\text{Hypergeometric2F1}\left(\frac{1+m}{2}, 1+m, \frac{3+m}{2}, -\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\tan\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^m}{ad(1+m)}$$
$$- \frac{\left(2^{2+m}b\tan^{-m}\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^m\left(1+\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^m\right)\text{Subst}\left(\int\left(-\frac{ax^{1+m}(1-}{\sqrt{a^2+b^2}\left(2b-2\right.}\right.\right.}{ad}$$

$$= \frac{2^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1+m}{2}, 1+m, \frac{3+m}{2}, -\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\tan\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right.}{ad(1+m)}$$
$$+ \frac{\left(2^{2+m}b\tan^{-m}\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^m\left(1+\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^m\right)\text{Subst}\left(\int \frac{x^{1+m}(1+x^2)^{-1-m}}{2b-2\sqrt{a^2+b^2}-2a}\right.}{\sqrt{a^2+b^2}d}$$
$$- \frac{\left(2^{2+m}b\tan^{-m}\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^m\left(1+\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^m\right)\text{Subst}\left(\int \frac{x^{1+m}(1+x^2)^{-1-m}}{2b+2\sqrt{a^2+b^2}-2a}\right.}{\sqrt{a^2+b^2}d}$$

$$= \frac{2^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1+m}{2}, 1+m, \frac{3+m}{2}, -\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\tan\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right.}{ad(1+m)}$$
$$+ \frac{\left(2^{3+m}ab\tan^{-m}\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^m\left(1+\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^m\right)\text{Subst}\left(\int \frac{x^{2+m}(1+x^2)}{\left(2b-2\sqrt{a^2+b^2}\right)}\right.}{\sqrt{a^2+b^2}d}$$
$$- \frac{\left(2^{3+m}ab\tan^{-m}\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^m\left(1+\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^m\right)\text{Subst}\left(\int \frac{x^{2+m}(1+x^2)}{\left(2b+2\sqrt{a^2+b^2}\right)}\right.}{\sqrt{a^2+b^2}d}$$
$$+ \frac{\left(2^{3+m}b\left(b-\sqrt{a^2+b^2}\right)\tan^{-m}\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^m\left(1+\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^m\right)\text{Subst}\left(\right.}{\sqrt{a^2+b^2}d}$$
$$- \frac{\left(2^{3+m}b\left(b+\sqrt{a^2+b^2}\right)\tan^{-m}\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^m\left(1+\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^m\right)\text{Subst}\left(\right.}{\sqrt{a^2+b^2}d}$$

$$= \frac{2^{1+m}\text{Hypergeometric2F1}\left(\frac{1+m}{2},1+m,\frac{3+m}{2},-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\tan\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^{m}}{ad(1+m)}$$

$$+ \frac{2^{1+m}b\,\text{AppellF1}\left(\frac{2+m}{2},1+m,1,\frac{4+m}{2},-\tan^2\left(\frac{1}{2}(c+dx)\right),\frac{a^2\tan^2\left(\frac{1}{2}(c+dx)\right)}{\left(b-\sqrt{a^2+b^2}\right)^2}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^{m}}{\sqrt{a^2+b^2}\left(b-\sqrt{a^2+b^2}\right)d(2+m)}$$

$$- \frac{2^{1+m}b\,\text{AppellF1}\left(\frac{2+m}{2},1+m,1,\frac{4+m}{2},-\tan^2\left(\frac{1}{2}(c+dx)\right),\frac{a^2\tan^2\left(\frac{1}{2}(c+dx)\right)}{\left(b+\sqrt{a^2+b^2}\right)^2}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^{m}}{\sqrt{a^2+b^2}\left(b+\sqrt{a^2+b^2}\right)d(2+m)}$$

$$+ \frac{2^{1+m}ab\,\text{AppellF1}\left(\frac{3+m}{2},1+m,1,\frac{5+m}{2},-\tan^2\left(\frac{1}{2}(c+dx)\right),\frac{a^2\tan^2\left(\frac{1}{2}(c+dx)\right)}{\left(b-\sqrt{a^2+b^2}\right)^2}\right)\tan^3\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^{m}}{\sqrt{a^2+b^2}\left(b-\sqrt{a^2+b^2}\right)^2 d(3+m)}$$

$$- \frac{2^{1+m}ab\,\text{AppellF1}\left(\frac{3+m}{2},1+m,1,\frac{5+m}{2},-\tan^2\left(\frac{1}{2}(c+dx)\right),\frac{a^2\tan^2\left(\frac{1}{2}(c+dx)\right)}{\left(b+\sqrt{a^2+b^2}\right)^2}\right)\tan^3\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^{m}}{\sqrt{a^2+b^2}\left(b+\sqrt{a^2+b^2}\right)^2 d(3+m)}$$

Mathematica [F]

$$\int \frac{\sin^m(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\sin^m(c+dx)}{a+b\tan(c+dx)}\,dx$$

```
[In] Integrate[Sin[c + d*x]^m/(a + b*Tan[c + d*x]),x]
[Out] Integrate[Sin[c + d*x]^m/(a + b*Tan[c + d*x]), x]
```

Maple [F]

$$\int \frac{\sin^m(dx+c)}{a+b\tan(dx+c)}dx$$

```
[In] int(sin(d*x+c)^m/(a+b*tan(d*x+c)),x)
[Out] int(sin(d*x+c)^m/(a+b*tan(d*x+c)),x)
```

Fricas [F]

$$\int \frac{\sin^m(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\sin\left(dx+c\right)^m}{b\tan\left(dx+c\right)+a}\,dx$$

```
[In] integrate(sin(d*x+c)^m/(a+b*tan(d*x+c)),x, algorithm="fricas")
[Out] integral(sin(d*x + c)^m/(b*tan(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{\sin^m(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\sin^m\left(c+dx\right)}{a+b\tan\left(c+dx\right)}\,dx$$

```
[In] integrate(sin(d*x+c)**m/(a+b*tan(d*x+c)),x)
[Out] Integral(sin(c + d*x)**m/(a + b*tan(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sin^m(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\sin\left(dx+c\right)^m}{b\tan\left(dx+c\right)+a}\,dx$$

```
[In] integrate(sin(d*x+c)^m/(a+b*tan(d*x+c)),x, algorithm="maxima")
[Out] integrate(sin(d*x + c)^m/(b*tan(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\sin^m(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\sin\left(dx+c\right)^m}{b\tan\left(dx+c\right)+a}\,dx$$

```
[In] integrate(sin(d*x+c)^m/(a+b*tan(d*x+c)),x, algorithm="giac")
[Out] integrate(sin(d*x + c)^m/(b*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^m(c+dx)}{a+b\tan(c+dx)}\,dx=\int \frac{\sin\left(c+d\,x\right)^m}{a+b\tan\left(c+d\,x\right)}\,dx$$

```
[In] int(sin(c + d*x)^m/(a + b*tan(c + d*x)),x)
[Out] int(sin(c + d*x)^m/(a + b*tan(c + d*x)), x)
```

3.83 $\int \sin^m(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^n\,dx = \text{Int}(\sin^m(c+dx)(a+b\tan(c+dx))^n, x)$$

```
[Out] CannotIntegrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^n,x)
```

Rubi [N/A]

Not integrable

Time = 3.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^n\,dx = \int \sin^m(c+dx)(a+b\tan(c+dx))^n\,dx$$

```
[In] Int[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^n,x]
[Out] Defer[Int][Sin[c + d*x]^m*(a + b*Tan[c + d*x])^n, x]
```

Rubi steps

$$\text{integral} = \int \sin^m(c+dx)(a+b\tan(c+dx))^n\,dx$$

Mathematica [N/A]

Not integrable

Time = 6.55 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^n\,dx=\int \sin^m(c+dx)(a+b\tan(c+dx))^n\,dx$$

```
[In] Integrate[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^n,x]
[Out] Integrate[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^n, x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \left(\sin^m\left(dx+c\right)\right)\left(a+b\tan\left(dx+c\right)\right)^n dx$$

```
[In] int(sin(d*x+c)^m*(a+b*tan(d*x+c))^n,x)
[Out] int(sin(d*x+c)^m*(a+b*tan(d*x+c))^n,x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^n\,dx=\int \left(b\tan\left(dx+c\right)+a\right)^n\sin\left(dx+c\right)^m\,dx$$

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*tan(d*x + c) + a)^n*sin(d*x + c)^m, x)
```

Sympy [N/A]

Not integrable

Time = 54.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^n\,dx=\int \left(a+b\tan\left(c+dx\right)\right)^n\sin^m\left(c+dx\right)dx$$

```
[In] integrate(sin(d*x+c)**m*(a+b*tan(d*x+c))**n,x)
[Out] Integral((a + b*tan(c + d*x))**n*sin(c + d*x)**m, x)
```

Maxima [N/A]

Not integrable

Time = 2.51 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n\sin(dx+c)^m\,dx$$

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^m, x)
```

Giac [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n\sin(dx+c)^m\,dx$$

```
[In] integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^m, x)
```

Mupad [N/A]

Not integrable

Time = 8.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^n\,dx = \int \sin(c+d\,x)^m\,(a+b\tan(c+d\,x))^n\,dx$$

```
[In] int(sin(c + d*x)^m*(a + b*tan(c + d*x))^n,x)
[Out] int(sin(c + d*x)^m*(a + b*tan(c + d*x))^n, x)
```

3.84 $\int \sin^4(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 435

$$\int \sin^4(c+dx)(a+b\tan(c+dx))^n\,dx = -\frac{\left(ab^2n(5a^2+b^2(3+2n))+\sqrt{-b^2}(3a^4+a^2b^2(6+6n-n^2)+b^4(3+4n+n^2))\right)\text{Hypergeometric2F1}\left(1,\right.}{16b\left(a^2+b^2\right)^2\left(a-\sqrt{-b^2}\right)d(1+n)} - \frac{\left(ab^2n(5a^2+b^2(3+2n))-\sqrt{-b^2}(3a^4+a^2b^2(6+6n-n^2)+b^4(3+4n+n^2))\right)\text{Hypergeometric2F1}\left(1,\right.}{16b\left(a^2+b^2\right)^2\left(a+\sqrt{-b^2}\right)d(1+n)} + \frac{\cos^4(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{4\left(a^2+b^2\right)d} - \frac{\cos^2(c+dx)(a+b\tan(c+dx))^{1+n}\left(b(a^2(7-n)+b^2(5+n))+a(5a^2+b^2(3+2n))\tan(c+dx)\right)}{8\left(a^2+b^2\right)^2d}$$

```
[Out] -1/16*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a+(-b^2)^(1/2)))*(a*b^2*n*
(5*a^2+b^2*(3+2*n))-(3*a^4+a^2*b^2*(-n^2+6*n+6)+b^4*(n^2+4*n+3))*(-b^2)^(1/
2))*(a+b*tan(d*x+c))^(1+n)/b/(a^2+b^2)^2/d/(1+n)/(a+(-b^2)^(1/2))-1/16*hype
rgeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a-(-b^2)^(1/2)))*(a*b^2*n*(5*a^2+b^2
*(3+2*n))+(3*a^4+a^2*b^2*(-n^2+6*n+6)+b^4*(n^2+4*n+3))*(-b^2)^(1/2))*(a+b*t
an(d*x+c))^(1+n)/b/(a^2+b^2)^2/d/(1+n)/(a-(-b^2)^(1/2))+1/4*cos(d*x+c)^4*(b
+a*tan(d*x+c))*(a+b*tan(d*x+c))^(1+n)/(a^2+b^2)/d-1/8*cos(d*x+c)^2*(a+b*tan
(d*x+c))^(1+n)*(b*(a^2*(7-n)+b^2*(5+n))+a*(5*a^2+b^2*(3+2*n))*tan(d*x+c))/(
a^2+b^2)^2/d
```

Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3597, 1663, 845, 70}

$$\int \sin^4(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{\cos^4(c+dx)(a\tan(c+dx)+b)(a+b\tan(c+dx))^{n+1}}{4d\,(a^2+b^2)} - \frac{\cos^2(c+dx)\left(a(5a^2+b^2(2n+3))\tan(c+dx)+b(a^2(7-n)+b^2(n+5))\right)(a+b\tan(c+dx))^{n+1}}{8d\,(a^2+b^2)^2} - \frac{\left(ab^2n(5a^2+b^2(2n+3))+\sqrt{-b^2}(3a^4+a^2b^2(-n^2+6n+6)+b^4(n^2+4n+3))\right)(a+b\tan(c+dx))^{n+}}{16bd(n+1)\,(a^2+b^2)^2\left(a-\sqrt{-b^2}\right)} - \frac{\left(ab^2n(5a^2+b^2(2n+3))-\sqrt{-b^2}(3a^4+a^2b^2(-n^2+6n+6)+b^4(n^2+4n+3))\right)(a+b\tan(c+dx))^{n+}}{16bd(n+1)\,(a^2+b^2)^2\left(a+\sqrt{-b^2}\right)}$$

```
[In] Int[Sin[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] -1/16*((a*b^2*n*(5*a^2 + b^2*(3 + 2*n)) + Sqrt[-b^2]*(3*a^4 + a^2*b^2*(6 +
6*n - n^2) + b^4*(3 + 4*n + n^2)))*Hypergeometric2F1[1, 1 + n, 2 + n, (a +
b*Tan[c + d*x])/(a - Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/(b*(a^2 + b
^2)^2*(a - Sqrt[-b^2])*d*(1 + n)) - ((a*b^2*n*(5*a^2 + b^2*(3 + 2*n)) - Sqr
t[-b^2]*(3*a^4 + a^2*b^2*(6 + 6*n - n^2) + b^4*(3 + 4*n + n^2)))*Hypergeome
tric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*(a + b*Tan[
c + d*x])^(1 + n))/(16*b*(a^2 + b^2)^2*(a + Sqrt[-b^2])*d*(1 + n)) + (Cos[c
 + d*x]^4*(b + a*Tan[c + d*x])*(a + b*Tan[c + d*x])^(1 + n))/(4*(a^2 + b^2)
*d) - (Cos[c + d*x]^2*(a + b*Tan[c + d*x])^(1 + n)*(b*(a^2*(7 - n) + b^2*(5
 + n)) + a*(5*a^2 + b^2*(3 + 2*n))*Tan[c + d*x]))/(8*(a^2 + b^2)^2*d)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 845

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 1663

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(-(d + e*x)^(m + 1))*(a + c*x^2)^(p + 1)*((a*(e*f - d*g)
+ (c*d*f + a*e*g)*x)/(2*a*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[1/(2*a*(p +
1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(c*d^2 + a*e^2)*Q + c*d^2*f*(2*p + 3) - a*e*(d*g*m - e*f*(m + 2*p + 3
)) + e*(c*d*f + a*e*g)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, c, d, e, m
}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[m,
0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{b\text{Subst}\left(\int \frac{x^4(a+x)^n}{(b^2+x^2)^3}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{4(a^2+b^2)d} - \frac{\text{Subst}\left(\int \frac{(a+x)^n(b^4(a^2+b^2(1+n))-ab^4(2-n)x-4b^2(a^2+b^2)x^2)}{(b^2+x^2)^2}\,dx, x, b\tan(c+dx)\right)}{4b(a^2+b^2)d}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{4(a^2+b^2)d} - \frac{\cos^2(c+dx)(a+b\tan(c+dx))^{1+n}(b(a^2(7-n)+b^2(5+n))+a(5a^2+b^2(3+2n))\tan(c+dx))}{8(a^2+b^2)^2d} + \frac{\text{Subst}\left(\int \frac{(a+x)^n(b^4(3a^4+a^2b^2(6+6n-n^2)+b^4(3+4n+n^2))+ab^4n(5a^2+b^2(3+2n))x)}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{8b^3(a^2+b^2)^2d}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{4(a^2+b^2)d} - \frac{\cos^2(c+dx)(a+b\tan(c+dx))^{1+n}(b(a^2(7-n)+b^2(5+n))+a(5a^2+b^2(3+2n))\tan(c+dx))}{8(a^2+b^2)^2d} + \frac{\text{Subst}\left(\int \left(\frac{\left(-ab^6n(5a^2+b^2(3+2n))+b^4\sqrt{-b^2}(3a^4+a^2b^2(6+6n-n^2)+b^4(3+4n+n^2))\right)(a+x)^n}{2b^2\left(\sqrt{-b^2}-x\right)} + \frac{\left(ab^6n(5a^2+b^2(3+2n))+b^4\right.}{}\right.\right.}{8b^3(a^2+b^2)^2d}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{4(a^2+b^2)d}$$
$$- \frac{\cos^2(c+dx)(a+b\tan(c+dx))^{1+n}(b(a^2(7-n)+b^2(5+n))+a(5a^2+b^2(3+2n))\tan(c+dx}{8(a^2+b^2)^2 d}$$
$$- \frac{\left(ab^2n(5a^2+b^2(3+2n))-\sqrt{-b^2}(3a^4+a^2b^2(6+6n-n^2)+b^4(3+4n+n^2))\right)\text{Subst}\left(\int \frac{(a+x)^n}{\sqrt{-b^2}-}\right.}{16b(a^2+b^2)^2 d}$$
$$+ \frac{\left(ab^2n(5a^2+b^2(3+2n))+\sqrt{-b^2}(3a^4+a^2b^2(6+6n-n^2)+b^4(3+4n+n^2))\right)\text{Subst}\left(\int \frac{(a+x)^n}{\sqrt{-b^2}+}\right.}{16b(a^2+b^2)^2 d}$$
$$=$$
$$- \frac{\left(ab^2n(5a^2+b^2(3+2n))+\sqrt{-b^2}(3a^4+a^2b^2(6+6n-n^2)+b^4(3+4n+n^2))\right)\text{Hypergeometri}}{16b(a^2+b^2)^2\left(a-\sqrt{-b^2}\right)d(1+n)}$$
$$- \frac{\left(ab^2n(5a^2+b^2(3+2n))-\sqrt{-b^2}(3a^4+a^2b^2(6+6n-n^2)+b^4(3+4n+n^2))\right)\text{Hypergeometri}}{16b(a^2+b^2)^2\left(a+\sqrt{-b^2}\right)d(1+n)}$$
$$+ \frac{\cos^4(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{4(a^2+b^2)d}$$
$$- \frac{\cos^2(c+dx)(a+b\tan(c+dx))^{1+n}(b(a^2(7-n)+b^2(5+n))+a(5a^2+b^2(3+2n))\tan(c+dx}{8(a^2+b^2)^2 d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 910 vs. 2(435) = 870.

Time = 6.81 (sec) , antiderivative size = 910, normalized size of antiderivative = 2.09

$$\int \sin^4(c+dx)(a+b\tan(c+dx))^n\,dx$$

$$= \frac{b\left(\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)(a+b\tan(c+dx))^{1+n}}{2\sqrt{-b^2}\left(a-\sqrt{-b^2}\right)(1+n)} - \frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)(a+b\tan(c+dx)}{2\sqrt{-b^2}\left(a+\sqrt{-b^2}\right)(1+n)}\right.}{}$$

```
[In] Integrate[Sin[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (b*((Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2
])]*(a + b*Tan[c + d*x])^(1 + n))/(2*Sqrt[-b^2]*(a - Sqrt[-b^2])*(1 + n)) -
 (Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + Sqrt[-b^2])]
*(a + b*Tan[c + d*x])^(1 + n))/(2*Sqrt[-b^2]*(a + Sqrt[-b^2])*(1 + n)) - (C
```

```
os[c + d*x]^2*(a + b*Tan[c + d*x])^(1 + n)*(b^2 + a*b*Tan[c + d*x]))/(b^2*(
a^2 + b^2)) + (Cos[c + d*x]^4*(a + b*Tan[c + d*x])^(1 + n)*(b^2 + a*b*Tan[c
 + d*x]))/(4*b^2*(a^2 + b^2)) + (((Sqrt[-b^2]*(a^2 + b^2*(1 - n)) - a*b^2*n
)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2])]
*(a + b*Tan[c + d*x])^(1 + n))/(b^2*(a - Sqrt[-b^2])*(1 + n)) - ((a^2*Sqrt[
-b^2] - (-b^2)^(3/2)*(1 - n) + a*b^2*n)*Hypergeometric2F1[1, 1 + n, 2 + n,
(a + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/(b^2*(
a + Sqrt[-b^2])*(1 + n)))/(2*(a^2 + b^2)) - (b^2*((Cos[c + d*x]^2*(a + b*Ta
n[c + d*x])^(1 + n)*(b^2*(-3*a^2 - b^2*(3 - n)) + a^2*b^2*(2 - n) + b*(a*(-
3*a^2 - b^2*(3 - n)) - a*b^2*(2 - n))*Tan[c + d*x]))/(2*b^4*(a^2 + b^2)) -
(((a*b^2*(3*a^2 + b^2*(5 - 2*n))*n - Sqrt[-b^2]*(3*a^4 + a^2*b^2*(6 - 2*n -
 n^2) + b^4*(3 - 4*n + n^2)))*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan
[c + d*x])/(a - Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/(2*b^2*(a - Sqrt
[-b^2])*(1 + n)) + ((a*b^2*(3*a^2 + b^2*(5 - 2*n))*n + Sqrt[-b^2]*(3*a^4 +
a^2*b^2*(6 - 2*n - n^2) + b^4*(3 - 4*n + n^2)))*Hypergeometric2F1[1, 1 + n,
 2 + n, (a + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n)
)/(2*b^2*(a + Sqrt[-b^2])*(1 + n)))/(2*b^2*(a^2 + b^2))))/(4*(a^2 + b^2))))
/d
```

Maple [F]

$$\int \left(\sin^{4}\left(dx+c\right)\right)\left(a+b\tan\left(dx+c\right)\right)^{n}dx$$

```
[In] int(sin(d*x+c)^4*(a+b*tan(d*x+c))^n,x)
[Out] int(sin(d*x+c)^4*(a+b*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int \sin^4(c+dx)(a+b\tan(c+dx))^n\,dx=\int \left(b\tan\left(dx+c\right)+a\right)^{n}\sin\left(dx+c\right)^{4}\,dx$$

```
[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
[Out] integral((cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*(b*tan(d*x + c) + a)^n, x)
```

Sympy [F]

$$\int \sin^4(c+dx)(a+b\tan(c+dx))^n\,dx = \int (a+b\tan{(c+dx)})^n\sin^4{(c+dx)}\,dx$$

```
[In] integrate(sin(d*x+c)**4*(a+b*tan(d*x+c))**n,x)
[Out] Integral((a + b*tan(c + d*x))**n*sin(c + d*x)**4, x)
```

Maxima [F]

$$\int \sin^4(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan{(dx+c)}+a)^n\sin{(dx+c)}^4\,dx$$

```
[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^4, x)
```

Giac [F]

$$\int \sin^4(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan{(dx+c)}+a)^n\sin{(dx+c)}^4\,dx$$

```
[In] integrate(sin(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^4(c+dx)(a+b\tan(c+dx))^n\,dx = \int {\sin{(c+d\,x)}}^4\,{(a+b\tan(c+d\,x))}^n\,dx$$

```
[In] int(sin(c + d*x)^4*(a + b*tan(c + d*x))^n,x)
[Out] int(sin(c + d*x)^4*(a + b*tan(c + d*x))^n, x)
```

3.85 $\int \sin^2(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 276

$$\begin{aligned}\int \sin^2(c+dx)(a+b\tan(c+dx))^n\,dx = \\ &-\frac{\left(ab^2n+\sqrt{-b^2}(a^2+b^2(1+n))\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)(a+b\tan(c+dx))^{1+}}{4b\,(a^2+b^2)\left(a-\sqrt{-b^2}\right)d(1+n)} \\ &-\frac{\left(ab^2n-\sqrt{-b^2}(a^2+b^2(1+n))\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)(a+b\tan(c+dx))^{1+}}{4b\,(a^2+b^2)\left(a+\sqrt{-b^2}\right)d(1+n)} \\ &-\frac{\cos^2(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{2\,(a^2+b^2)\,d}\end{aligned}$$

```
[Out] -1/4*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a+(-b^2)^(1/2)))*(a*b^2*n-(
a^2+b^2*(1+n))*(-b^2)^(1/2))*(a+b*tan(d*x+c))^(1+n)/b/(a^2+b^2)/d/(1+n)/(a+
(-b^2)^(1/2))-1/4*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a-(-b^2)^(1/2)
))*(a*b^2*n+(a^2+b^2*(1+n))*(-b^2)^(1/2))*(a+b*tan(d*x+c))^(1+n)/b/(a^2+b^2
)/d/(1+n)/(a-(-b^2)^(1/2))-1/2*cos(d*x+c)^2*(b+a*tan(d*x+c))*(a+b*tan(d*x+c
))^(1+n)/(a^2+b^2)/d
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used

$= \{3597, 1663, 845, 70\}$

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^n\,dx =$$
$$-\frac{\left(\sqrt{-b^2}(a^2+b^2(n+1))+ab^2n\right)(a+b\tan(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right.}{4bd(n+1)\left(a^2+b^2\right)\left(a-\sqrt{-b^2}\right)}$$
$$-\frac{\left(ab^2n-\sqrt{-b^2}(a^2+b^2(n+1))\right)(a+b\tan(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right.}{4bd(n+1)\left(a^2+b^2\right)\left(a+\sqrt{-b^2}\right)}$$
$$-\frac{\cos^2(c+dx)(a\tan(c+dx)+b)(a+b\tan(c+dx))^{n+1}}{2d\left(a^2+b^2\right)}$$

```
[In] Int[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] -1/4*((a*b^2*n + Sqrt[-b^2]*(a^2 + b^2*(1 + n)))*Hypergeometric2F1[1, 1 + n
, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n
))/(b*(a^2 + b^2)*(a - Sqrt[-b^2])*d*(1 + n)) - ((a*b^2*n - Sqrt[-b^2]*(a^2
 + b^2*(1 + n)))*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a
 + Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/(4*b*(a^2 + b^2)*(a + Sqrt[-b
^2])*d*(1 + n)) - (Cos[c + d*x]^2*(b + a*Tan[c + d*x])*(a + b*Tan[c + d*x])
^(1 + n))/(2*(a^2 + b^2)*d)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 845

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 1663

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(-(d + e*x)^(m + 1))*(a + c*x^2)^(p + 1)*((a*(e*f - d*g)
+ (c*d*f + a*e*g)*x)/(2*a*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[1/(2*a*(p +
1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(c*d^2 + a*e^2)*Q + c*d^2*f*(2*p + 3) - a*e*(d*g*m - e*f*(m + 2*p + 3
)) + e*(c*d*f + a*e*g)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, c, d, e, m
```

```
}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[m,
0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b\text{Subst}\left(\int \frac{x^2(a+x)^n}{(b^2+x^2)^2}\,dx, x, b\tan(c+dx)\right)}{d} \\
&= -\frac{\cos^2(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{2\left(a^2+b^2\right)d} \\
&\quad -\frac{\text{Subst}\left(\int \frac{(a+x)^n\left(-b^2\left(a^2+b^2(1+n)\right)-ab^2nx\right)}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{2b\left(a^2+b^2\right)d} \\
&= -\frac{\cos^2(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{2\left(a^2+b^2\right)d} \\
&\quad -\frac{\text{Subst}\left(\int \left(\frac{\left(ab^4n-b^2\sqrt{-b^2}\left(a^2+b^2(1+n)\right)\right)(a+x)^n}{2b^2\left(\sqrt{-b^2}-x\right)}+\frac{\left(-ab^4n-b^2\sqrt{-b^2}\left(a^2+b^2(1+n)\right)\right)(a+x)^n}{2b^2\left(\sqrt{-b^2}+x\right)}\right)dx, x, b\tan(c+dx)\right)}{2b\left(a^2+b^2\right)d} \\
&= -\frac{\cos^2(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{2\left(a^2+b^2\right)d} \\
&\quad -\frac{\left(ab^2n-\sqrt{-b^2}\left(a^2+b^2(1+n)\right)\right)\text{Subst}\left(\int \frac{(a+x)^n}{\sqrt{-b^2}-x}\,dx, x, b\tan(c+dx)\right)}{4b\left(a^2+b^2\right)d} \\
&\quad +\frac{\left(ab^2n+\sqrt{-b^2}\left(a^2+b^2(1+n)\right)\right)\text{Subst}\left(\int \frac{(a+x)^n}{\sqrt{-b^2}+x}\,dx, x, b\tan(c+dx)\right)}{4b\left(a^2+b^2\right)d} \\
&= \\
&\quad -\frac{\left(ab^2n+\sqrt{-b^2}\left(a^2+b^2(1+n)\right)\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)(a+b\tan(c+\ldots}{4b\left(a^2+b^2\right)\left(a-\sqrt{-b^2}\right)d(1+n)} \\
&\quad -\frac{\left(ab^2n-\sqrt{-b^2}\left(a^2+b^2(1+n)\right)\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)(a+b\tan(c+\ldots}{4b\left(a^2+b^2\right)\left(a+\sqrt{-b^2}\right)d(1+n)} \\
&\quad -\frac{\cos^2(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{2\left(a^2+b^2\right)d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.98

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{\left(\left(a^3\sqrt{-b^2}+a^2b^2(-1+n)-b^4(1+n)-a(-b^2)^{3/2}(1+2n)\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan}{a-}\right.}{}$$

```
[In] Integrate[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (((a^3*Sqrt[-b^2] + a^2*b^2*(-1 + n) - b^4*(1 + n) - a*(-b^2)^(3/2)*(1 + 2*
n))*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2]
)] - (a^3*Sqrt[-b^2] - a^2*b^2*(-1 + n) + b^4*(1 + n) - a*(-b^2)^(3/2)*(1 +
 2*n))*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + Sqrt[-b
^2])] + 2*b*(a^2 + b^2)*(1 + n)*Cos[c + d*x]*(b*Cos[c + d*x] + a*Sin[c + d*
x]))*(a + b*Tan[c + d*x])^(1 + n))/(4*b*(a^2 + b^2)*(-a + Sqrt[-b^2])*(a +
Sqrt[-b^2])*d*(1 + n))
```

Maple [F]

$$\int \left(\sin^2(dx+c)\right)(a+b\tan(dx+c))^n\,dx$$

```
[In] int(sin(d*x+c)^2*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(sin(d*x+c)^2*(a+b*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n\sin(dx+c)^2\,dx$$

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)*(b*tan(d*x + c) + a)^n, x)
```

Sympy [F]

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^n\,dx = \int (a+b\tan{(c+dx)})^n \sin^2{(c+dx)}\,dx$$

```
[In] integrate(sin(d*x+c)**2*(a+b*tan(d*x+c))**n,x)
[Out] Integral((a + b*tan(c + d*x))**n*sin(c + d*x)**2, x)
```

Maxima [F]

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan{(dx+c)}+a)^n \sin{(dx+c)}^2\,dx$$

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^2, x)
```

Giac [F]

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan{(dx+c)}+a)^n \sin{(dx+c)}^2\,dx$$

```
[In] integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^n\,dx = \int \sin{(c+d\,x)}^2\,(a+b\tan(c+d\,x))^n\,dx$$

```
[In] int(sin(c + d*x)^2*(a + b*tan(c + d*x))^n,x)
[Out] int(sin(c + d*x)^2*(a + b*tan(c + d*x))^n, x)
```

3.86 $\int \csc^2(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 48

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{b\,\text{Hypergeometric2F1}\left(2,1+n,2+n,1+\frac{b\tan(c+dx)}{a}\right)(a+b\tan(c+dx))^{1+n}}{a^2d(1+n)}$$

```
[Out] b*hypergeom([2, 1+n],[2+n],1+b*tan(d*x+c)/a)*(a+b*tan(d*x+c))^(1+n)/a^2/d/(
1+n)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3597, 67}

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{b(a+b\tan(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(2,n+1,n+2,\frac{b\tan(c+dx)}{a}+1\right)}{a^2d(n+1)}$$

```
[In] Int[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (b*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a]*(a + b*Tan[c
+ d*x])^(1 + n))/(a^2*d*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
```

```
d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{b\text{Subst}\left(\int \frac{(a+x)^n}{x^2}\,dx, x, b\tan(c+dx)\right)}{d}$$
$$= \frac{b\,\text{Hypergeometric2F1}\left(2, 1+n, 2+n, 1+\frac{b\tan(c+dx)}{a}\right)(a+b\tan(c+dx))^{1+n}}{a^2d(1+n)}$$

Mathematica [A] (verified)

Time = 6.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^n\,dx$$
$$= \frac{b\,\text{Hypergeometric2F1}\left(2, 1+n, 2+n, 1+\frac{b\tan(c+dx)}{a}\right)(a+b\tan(c+dx))^{1+n}}{a^2d(1+n)}$$

```
[In] Integrate[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
[Out] (b*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a]*(a + b*Tan[c
+ d*x])^(1 + n))/(a^2*d*(1 + n))
```

Maple [F]

$$\int \left(\csc^2(dx+c)\right)(a+b\tan(dx+c))^n\,dx$$

```
[In] int(csc(d*x+c)^2*(a+b*tan(d*x+c))^n,x)
[Out] int(csc(d*x+c)^2*(a+b*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n \csc(dx+c)^2\,dx$$

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*tan(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

Sympy [F]

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^n\,dx = \int (a+b\tan(c+dx))^n \csc^2(c+dx)\,dx$$

```
[In] integrate(csc(d*x+c)**2*(a+b*tan(d*x+c))**n,x)
[Out] Integral((a + b*tan(c + d*x))**n*csc(c + d*x)**2, x)
```

Maxima [F]

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n \csc(dx+c)^2\,dx$$

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

Giac [F]

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n \csc(dx+c)^2\,dx$$

```
[In] integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^n\,dx=\int \frac{(a+b\tan(c+d\,x))^n}{\sin(c+d\,x)^2}\,dx$$

```
[In] int((a + b*tan(c + d*x))^n/sin(c + d*x)^2,x)
[Out] int((a + b*tan(c + d*x))^n/sin(c + d*x)^2, x)
```

3.87 $\int \csc^4(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 140

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{b(2-n)\cot^2(c+dx)(a+b\tan(c+dx))^{1+n}}{6a^2d} - \frac{\cot^3(c+dx)(a+b\tan(c+dx))^{1+n}}{3ad} + \frac{b(6a^2+b^2(2-3n+n^2))\,\text{Hypergeometric2F1}\left(2,1+n,2+n,1+\frac{b\tan(c+dx)}{a}\right)(a+b\tan(c+dx))^{1+n}}{6a^4d(1+n)}$$

```
[Out] 1/6*b*(2-n)*cot(d*x+c)^2*(a+b*tan(d*x+c))^(1+n)/a^2/d-1/3*cot(d*x+c)^3*(a+b
*tan(d*x+c))^(1+n)/a/d+1/6*b*(6*a^2+b^2*(n^2-3*n+2))*hypergeom([2, 1+n],[2+
n],1+b*tan(d*x+c)/a)*(a+b*tan(d*x+c))^(1+n)/a^4/d/(1+n)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3597, 964, 79, 67}

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{b(2-n)\cot^2(c+dx)(a+b\tan(c+dx))^{n+1}}{6a^2d} + \frac{b(6a^2+b^2(n^2-3n+2))\,(a+b\tan(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(2,n+1,n+2,\frac{b\tan(c+dx)}{a}+1\right)}{6a^4d(n+1)} - \frac{\cot^3(c+dx)(a+b\tan(c+dx))^{n+1}}{3ad}$$

```
[In] Int[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (b*(2 - n)*Cot[c + d*x]^2*(a + b*Tan[c + d*x])^(1 + n))/(6*a^2*d) - (Cot[c
+ d*x]^3*(a + b*Tan[c + d*x])^(1 + n))/(3*a*d) + (b*(6*a^2 + b^2*(2 - 3*n +
 n^2))*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a]*(a + b*Ta
n[c + d*x])^(1 + n))/(6*a^4*d*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 964

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + c*x^2)^p, d + e*x
, x], R = PolynomialRemainder[(a + c*x^2)^p, d + e*x, x]}, Simp[R*(d + e*x)
^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))), x] + Dist[1/((m + 1)*(e
*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*ExpandToSum[(m + 1)*(e*f - d*
g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{b\text{Subst}\left(\int \frac{(a+x)^n(b^2+x^2)}{x^4}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{\cot^3(c+dx)(a+b\tan(c+dx))^{1+n}}{3ad} - \frac{b\text{Subst}\left(\int \frac{(a+x)^n(b^2(2-n)-3ax)}{x^3}\,dx, x, b\tan(c+dx)\right)}{3ad}$$

$$= \frac{b(2-n)\cot^2(c+dx)(a+b\tan(c+dx))^{1+n}}{6a^2d} - \frac{\cot^3(c+dx)(a+b\tan(c+dx))^{1+n}}{3ad} - \frac{(b(-6a^2+b^2(2-n)(-1+n)))\,\text{Subst}\left(\int \frac{(a+x)^n}{x^2}\,dx, x, b\tan(c+dx)\right)}{6a^2d}$$

$$= \frac{b(2-n)\cot^2(c+dx)(a+b\tan(c+dx))^{1+n}}{6a^2d} - \frac{\cot^3(c+dx)(a+b\tan(c+dx))^{1+n}}{3ad} + \frac{b(6a^2+b^2(1-n)(2-n))\,\text{Hypergeometric2F1}\left(2,1+n,2+n,1+\frac{b\tan(c+dx)}{a}\right)(a+b\tan(c+d}{6a^4d(1+n)}$$

Mathematica [A] (verified)

Time = 7.97 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.56

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{b\Big(a^2\,\text{Hypergeometric2F1}\left(2,1+n,2+n,1+\frac{b\tan(c+dx)}{a}\right)+b^2\,\text{Hypergeometric2F1}\left(4,1+n,2+n,1+\frac{b}{}\right.}{a^4d(1+n)}$$

```
[In] Integrate[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (b*(a^2*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a] + b^2*Hy
pergeometric2F1[4, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a])*(a + b*Tan[c + d*
x])^(1 + n))/(a^4*d*(1 + n))
```

Maple [F]

$$\int \left(\csc^4\left(dx+c\right)\right)\left(a+b\tan\left(dx+c\right)\right)^n\,dx$$

```
[In] int(csc(d*x+c)^4*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(csc(d*x+c)^4*(a+b*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^n\,dx = \int \left(b\tan\left(dx+c\right)+a\right)^n\csc\left(dx+c\right)^4\,dx$$

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

Sympy [F]

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^n\,dx = \int (a+b\tan(c+dx))^n \csc^4(c+dx)\,dx$$

```
[In] integrate(csc(d*x+c)**4*(a+b*tan(d*x+c))**n,x)
[Out] Integral((a + b*tan(c + d*x))**n*csc(c + d*x)**4, x)
```

Maxima [F]

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n \csc(dx+c)^4\,dx$$

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

Giac [F]

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n \csc(dx+c)^4\,dx$$

```
[In] integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^n\,dx = \int \frac{(a+b\tan(c+d\,x))^n}{\sin(c+d\,x)^4}\,dx$$

```
[In] int((a + b*tan(c + d*x))^n/sin(c + d*x)^4,x)
[Out] int((a + b*tan(c + d*x))^n/sin(c + d*x)^4, x)
```

3.88 $\int \sin^3(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^n\,dx = \text{Int}\left(\sin^3(c+dx)(a+b\tan(c+dx))^n,x\right)$$

```
[Out] CannotIntegrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

Rubi [N/A]

Not integrable

Time = 2.32 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^n\,dx = \int \sin^3(c+dx)(a+b\tan(c+dx))^n\,dx$$

```
[In] Int[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] Defer[Int][Sin[c + d*x]^3*(a + b*Tan[c + d*x])^n, x]
```

Rubi steps

$$\text{integral} = \int \sin^3(c+dx)(a+b\tan(c+dx))^n\,dx$$

Mathematica [N/A]

Not integrable

Time = 4.96 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^n\,dx = \int \sin^3(c+dx)(a+b\tan(c+dx))^n\,dx$$

```
[In] Integrate[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]
[Out] Integrate[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^n, x]
```

Maple [N/A] (verified)

Not integrable

Time = 3.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \left(\sin^3\left(dx+c\right)\right)\left(a+b\tan\left(dx+c\right)\right)^n dx$$

```
[In] int(sin(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
[Out] int(sin(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.52

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^n\,dx = \int \left(b\tan\left(dx+c\right)+a\right)^n \sin\left(dx+c\right)^3\,dx$$

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
[Out] integral(-(cos(d*x + c)^2 - 1)*(b*tan(d*x + c) + a)^n*sin(d*x + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^n\,dx = \text{Timed out}$$

```
[In] integrate(sin(d*x+c)**3*(a+b*tan(d*x+c))**n,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 4.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^n\,dx=\int (b\tan(dx+c)+a)^n\sin(dx+c)^3\,dx$$

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^3, x)
```

Giac [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^n\,dx=\int (b\tan(dx+c)+a)^n\sin(dx+c)^3\,dx$$

```
[In] integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^3, x)
```

Mupad [N/A]

Not integrable

Time = 7.38 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^n\,dx=\int \sin(c+d\,x)^3\,(a+b\tan(c+d\,x))^n\,dx$$

```
[In] int(sin(c + d*x)^3*(a + b*tan(c + d*x))^n,x)
[Out] int(sin(c + d*x)^3*(a + b*tan(c + d*x))^n, x)
```

3.89 $\int \sin(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \sin(c+dx)(a+b\tan(c+dx))^n\,dx = \text{Int}(\sin(c+dx)(a+b\tan(c+dx))^n,x)$$

```
[Out] CannotIntegrate(sin(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

Rubi [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sin(c+dx)(a+b\tan(c+dx))^n\,dx = \int \sin(c+dx)(a+b\tan(c+dx))^n\,dx$$

```
[In] Int[Sin[c + d*x]*(a + b*Tan[c + d*x])^n,x]
[Out] Defer[Int][Sin[c + d*x]*(a + b*Tan[c + d*x])^n, x]
```

Rubi steps

$$\text{integral} = \int \sin(c+dx)(a+b\tan(c+dx))^n\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.77 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \sin(c+dx)(a+b\tan(c+dx))^n\,dx = \int \sin(c+dx)(a+b\tan(c+dx))^n\,dx$$

```
[In] Integrate[Sin[c + d*x]*(a + b*Tan[c + d*x])^n,x]
[Out] Integrate[Sin[c + d*x]*(a + b*Tan[c + d*x])^n, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.57 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \sin(dx+c)(a+b\tan(dx+c))^n\,dx$$

```
[In] int(sin(d*x+c)*(a+b*tan(d*x+c))^n,x)
[Out] int(sin(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \sin(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n\sin(dx+c)\,dx$$

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*tan(d*x + c) + a)^n*sin(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 4.72 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \sin(c+dx)(a+b\tan(c+dx))^n\,dx = \int (a+b\tan(c+dx))^n\sin(c+dx)\,dx$$

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))**n,x)
[Out] Integral((a + b*tan(c + d*x))**n*sin(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 1.58 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \sin(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n \sin(dx+c)\,dx$$

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c), x)
```

Giac [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \sin(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n \sin(dx+c)\,dx$$

```
[In] integrate(sin(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^n*sin(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 5.48 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \sin(c+dx)(a+b\tan(c+dx))^n\,dx = \int \sin(c+d\,x)\,\left(a+b\tan(c+d\,x)\right)^n\,dx$$

```
[In] int(sin(c + d*x)*(a + b*tan(c + d*x))^n,x)
[Out] int(sin(c + d*x)*(a + b*tan(c + d*x))^n, x)
```

3.90 $\int \csc(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \csc(c+dx)(a+b\tan(c+dx))^n\,dx = \text{Int}(\csc(c+dx)(a+b\tan(c+dx))^n,x)$$

```
[Out] CannotIntegrate(csc(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

Rubi [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \csc(c+dx)(a+b\tan(c+dx))^n\,dx = \int \csc(c+dx)(a+b\tan(c+dx))^n\,dx$$

```
[In] Int[Csc[c + d*x]*(a + b*Tan[c + d*x])^n,x]
[Out] Defer[Int][Csc[c + d*x]*(a + b*Tan[c + d*x])^n, x]
```

Rubi steps

$$\text{integral} = \int \csc(c+dx)(a+b\tan(c+dx))^n\,dx$$

Mathematica [N/A]

Not integrable

Time = 4.42 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \csc(c+dx)(a+b\tan(c+dx))^n\,dx = \int \csc(c+dx)(a+b\tan(c+dx))^n\,dx$$

```
[In] Integrate[Csc[c + d*x]*(a + b*Tan[c + d*x])^n,x]
[Out] Integrate[Csc[c + d*x]*(a + b*Tan[c + d*x])^n, x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.53 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \csc\left(dx+c\right)\left(a+b\tan\left(dx+c\right)\right)^{n}dx$$

```
[In] int(csc(d*x+c)*(a+b*tan(d*x+c))^n,x)
[Out] int(csc(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \csc(c+dx)(a+b\tan(c+dx))^n\,dx = \int \left(b\tan\left(dx+c\right)+a\right)^{n}\csc\left(dx+c\right)\,dx$$

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*tan(d*x + c) + a)^n*csc(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 3.70 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \csc(c+dx)(a+b\tan(c+dx))^n\,dx = \int \left(a+b\tan\left(c+dx\right)\right)^{n}\csc\left(c+dx\right)dx$$

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))**n,x)
[Out] Integral((a + b*tan(c + d*x))**n*csc(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \csc(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n\csc(dx+c)\,dx$$

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^n*csc(d*x + c), x)
```

Giac [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \csc(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n\csc(dx+c)\,dx$$

```
[In] integrate(csc(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^n*csc(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 4.56 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \csc(c+dx)(a+b\tan(c+dx))^n\,dx = \int \frac{(a+b\tan(c+d\,x))^n}{\sin(c+d\,x)}\,dx$$

```
[In] int((a + b*tan(c + d*x))^n/sin(c + d*x),x)
[Out] int((a + b*tan(c + d*x))^n/sin(c + d*x), x)
```

3.91 $\int \csc^3(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^n\,dx = \mathrm{Int}\left(\csc^3(c+dx)(a+b\tan(c+dx))^n,x\right)$$

```
[Out] CannotIntegrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

Rubi [N/A]

Not integrable

Time = 1.98 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^n\,dx = \int \csc^3(c+dx)(a+b\tan(c+dx))^n\,dx$$

```
[In] Int[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] Defer[Int][Csc[c + d*x]^3*(a + b*Tan[c + d*x])^n, x]
```

Rubi steps

$$\text{integral} = \int \csc^3(c+dx)(a+b\tan(c+dx))^n\,dx$$

Mathematica [N/A]

Not integrable

Time = 23.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^n\,dx = \int \csc^3(c+dx)(a+b\tan(c+dx))^n\,dx$$

```
[In] Integrate[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]
[Out] Integrate[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^n, x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.57 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \left(\csc^3\left(dx+c\right)\right)\left(a+b\tan\left(dx+c\right)\right)^n dx$$

```
[In] int(csc(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
[Out] int(csc(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan\left(dx+c\right)+a)^n \csc\left(dx+c\right)^3\,dx$$

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*tan(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

Sympy [N/A]

Not integrable

Time = 34.95 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^n\,dx = \int \left(a+b\tan\left(c+dx\right)\right)^n \csc^3\left(c+dx\right)dx$$

```
[In] integrate(csc(d*x+c)**3*(a+b*tan(d*x+c))**n,x)
[Out] Integral((a + b*tan(c + d*x))**n*csc(c + d*x)**3, x)
```

Maxima [N/A]

Not integrable

Time = 2.96 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n\csc(dx+c)^3\,dx$$

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

Giac [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n\csc(dx+c)^3\,dx$$

```
[In] integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

Mupad [N/A]

Not integrable

Time = 6.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^n\,dx = \int \frac{(a+b\tan(c+d\,x))^n}{\sin(c+d\,x)^3}\,dx$$

```
[In] int((a + b*tan(c + d*x))^n/sin(c + d*x)^3,x)
[Out] int((a + b*tan(c + d*x))^n/sin(c + d*x)^3, x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                  end if
              else #result contains complex but optimal is not
                  if debug then
                          print("result contains complex but optimal is not");
                  fi;
                  return "C","Result contains complex when optimal does not.";
              fi;
          else # result do not contain complex
               # this assumes optimal do not as well. No check is needed here.
              if debug then
                     print("result do not contain complex, this assumes optimal do not as well")
              fi;
              if leaf_count_result<=2*leaf_count_optimal then
                  if debug then
                      print("leaf_count_result<=2*leaf_count_optimal");
                  fi;
                  return "A"," ";
              else
                  if debug then
                      print("leaf_count_result>2*leaf_count_optimal");
                  fi;
                  return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                    convert(leaf_count_result,string),"$ vs. $2(",
                                    convert(leaf_count_optimal,string),")=",convert(2*leaf_count
              fi;
          fi;
      else #ExpnType(result) > ExpnType(optimal)
          if debug then
              print("ExpnType(result) > ExpnType(optimal)");
          fi;
          return "C",cat("Result contains higher order function than in optimal. Order ",
                        convert(ExpnType_result,string)," vs. order ",
                        convert(ExpnType_optimal,string),".");
      fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.a
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
          #is checked before calling the grading function that is passed.
          #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):



    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)



    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```